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Convergence of monotone finite volume schemes for hyperbolic scalar conservation laws with a multiplicative stochastic force

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SUMMARY

We are interested in the Cauchy problem for a nonlinear hyperbolic scalar conservation law in d space dimensions with a multiplicative stochastic perturbation of type:

$$\begin{cases} du + \operatorname{div}_x \left[\vec{v}(x,t) f(u)\right] dt &= g(u) dW \quad \text{in } \Omega \times \mathbb{R}^d \times (0,T), \\ u(\omega, x, 0) &= u_0(x), \qquad \omega \in \Omega,\ x \in \mathbb{R}^d, \end{cases} \tag{1}$$

where div_x is the divergence operator with respect to the space variable, d is a positive integer, $T > 0$ and $W = \{W_t, \mathcal{F}_t; 0 \leqslant t \leqslant T\}$ is a standard adapted one-dimensional continuous Brownian motion defined on a probability space $(\Omega, \mathcal{F}, P)$. Note that this problem, (written in a probabilistic's way) has to be understood in the following manner: for almost all ω in Ω and for all φ in $\mathcal{C}_c^\infty\left(\mathbb{R}^d \times [0,T)\right)$

$$\begin{aligned} &\int_{\mathbb{R}^d} u_0(x)\varphi(x,0)dx + \int_0^T \int_{\mathbb{R}^d} u(\omega,x,t)\partial_t\varphi(x,t) + \vec{v}(x,t) f(u(\omega,x,t)).\nabla_x\varphi(x,t) dxdt \\ = \quad &\int_0^T \int_{\mathbb{R}^d} \int_0^t g(u(\omega,x,s)) dW(s) \partial_t\varphi(x,t) dxdt. \end{aligned}$$

In this talk , I will present the discretization of Problem (**??**) by monotone finite volume schemes under the following hypotheses:

- H_1: $u_0 \in L^2(\mathbb{R}^d)$.
- H_2: $f : \mathbb{R} \to \mathbb{R}$ is a Lipschitz-continuous function with $f(0) = 0$.
- H_3: $g : \mathbb{R} \to \mathbb{R}$ is a Lipschitz-continuous function with $g(0) = 0$.
- H_4: $\vec{v} \in \mathcal{C}^1(\mathbb{R}^d \times [0,T], \mathbb{R}^d)$ and $\operatorname{div}_x[\vec{v}(x,t)] = 0$, $\forall (x,t) \in \mathbb{R}^d \times [0,T]$.
- H_5: There exists $V < \infty$ such that $|\vec{v}(x,t)| \leqslant V$, $\forall (x,t) \in \mathbb{R}^d \times [0,T]$.
- H_6: g is a bounded function.

Firstly, I will introduce the well-posedness theory for solutions of such kind of stochastic problems. Then, the main part of the talk will be devoted to the study of the monotone numerical scheme used to approximate the solution of (**??**). I will show that under a stability condition on the time step, one is able to show the convergence of the finite volume approximation towards the unique stochastic entropy solution of the problem.

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