# Natural Evolution Strategies for Direct Search

PGMO-COPI 2014
Recent Advances on Continuous Randomized black-box optimization

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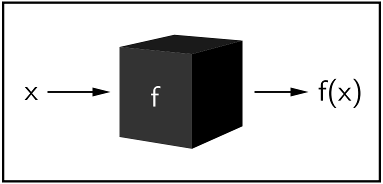

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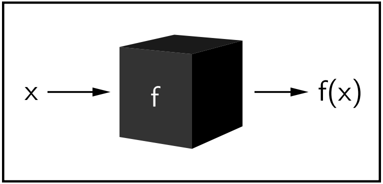


4. Don't ever rely on function values, only comparisons $f(x) < f(x')$, e.g., for ranking.

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- observations of objective values perturbed by noise
- high dimensionality, e.g., $d \gg 1000$
- black-box constraints (possibly non-smooth, ...)

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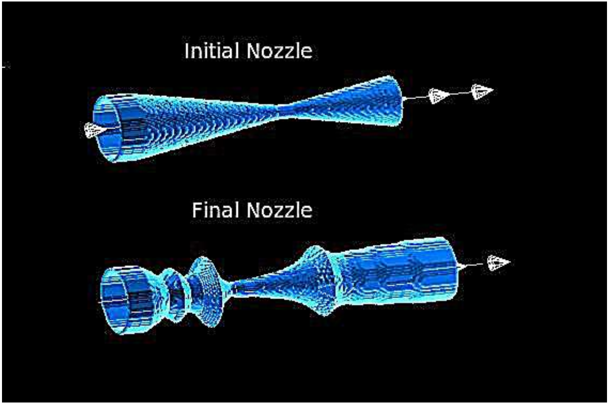

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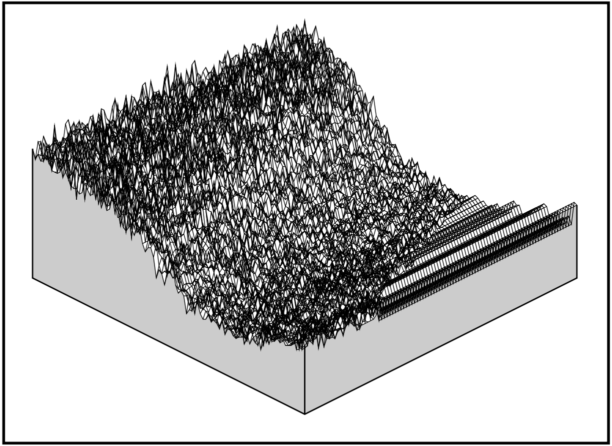

# Evolution Strategies

**input** $m \in \mathbb{R}^d$, $\sigma > 0$, $C (= \mathrm{I}) \in \mathbb{R}^{d \times d}$
**loop**
  sample "offspring" $x_1, \dots, x_\lambda \sim \mathcal{N}(m, \sigma^2 C)$
  evaluate $f(x_1), \dots, f(x_\lambda)$
  select new "population" of size $\mu$ (e.g., best offspring)
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- randomized
- population-based
- rank-based (function-value free)
- step size control
- metric learning (covariance matrix adaptation, CMA)

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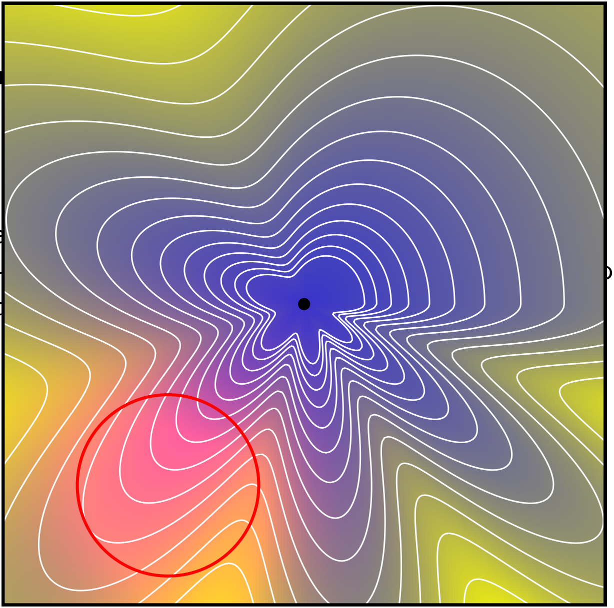

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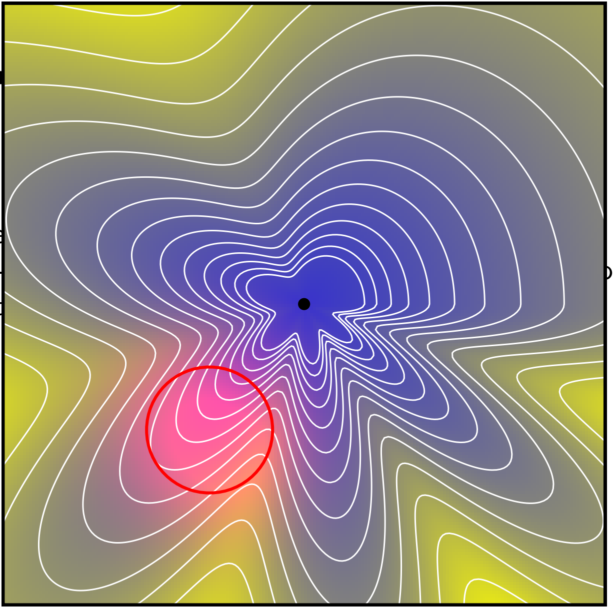

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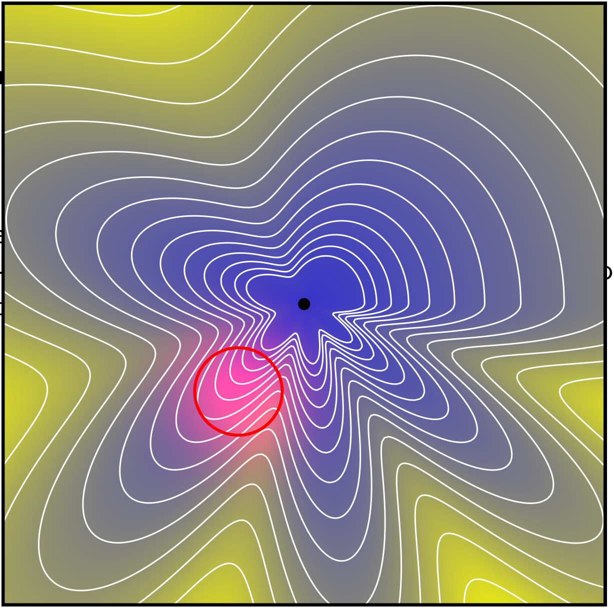

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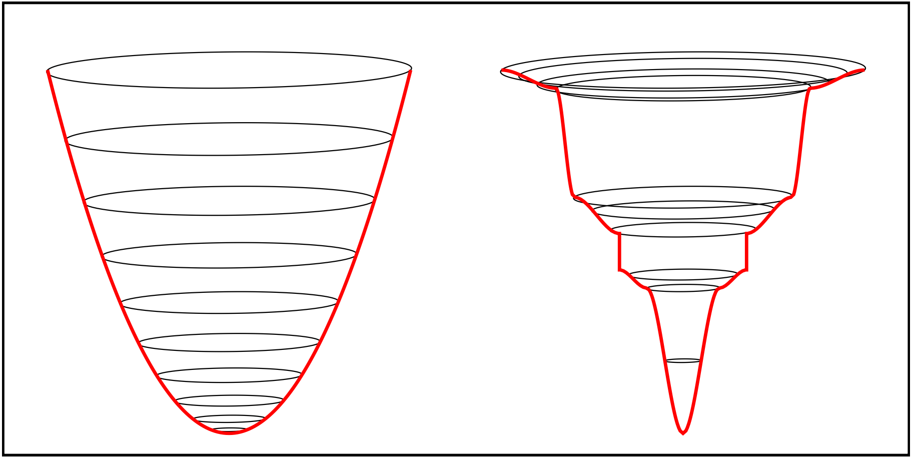

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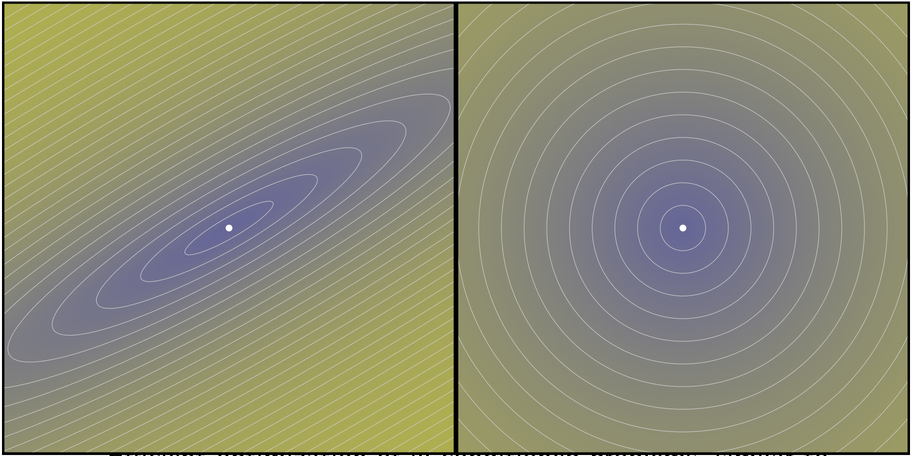

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# This Talk

**Given samples and their ranks,**

**how to update the distribution parameters?**

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$$\Theta = \mathbb{R}^d \times \mathcal{P}_d$$

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- Equipped with intrinsic (Riemannian) geometry; metric tensor given by Fisher information matrix $\mathcal{I}(\theta)$.

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- More flexible choice:

$$W_f(\theta) = \mathbb{E}_{x \sim P_\theta}\Big[w\big(f(x)\big)\Big]$$

  with monotonic weight function $w : \mathbb{R} \to \mathbb{R}$.

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- "log-likelohood trick":

$$\begin{aligned} \nabla_\theta W_f(\theta) &= \nabla_\theta \mathbb{E}_{x \sim P_\theta}\Big[w\big(f(x)\big)\Big] \\ &= \mathbb{E}_{x \sim P_\theta}\Big[\nabla_\theta \log\big(p_\theta(x)\big) \cdot w\big(f(x)\big)\Big] \end{aligned}$$

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- We have to replace the "plain" gradient on (Euclidean) parameter space $\Theta$ with the **natural gradient** respecting the intrinsic geometry of the statistical manifold:

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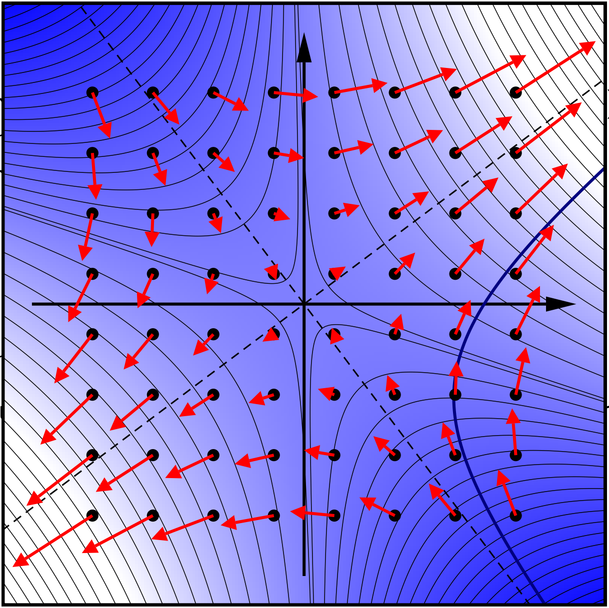

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- Optimization: follow inverse flow curves $t \mapsto \phi^{-t}(\theta)$ from $\theta$ into (local) minimum of $W_f$.

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- Black-box setting: expectation

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- **Monte Carlo (MC)** approximation

$$\nabla_\theta W_f(\theta) \approx G(\theta) = \frac{1}{\lambda} \sum_{i=1}^{\lambda} \Big[ \nabla_\theta \log \big( p_\theta(x_i) \big) \cdot w\big( f(x_i) \big) \Big]$$

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- Yields natural gradient MC approximation

$$\tilde{G}(\theta) = \big( \mathcal{I}(\theta) \big)^{-1} \cdot G(\theta) \ .$$

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Ollivier et al. Information-Geometric Optimization Algorithms: A Unifying Picture via Invariance Principles. *arXiv:1106.3708*, 2011.

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- NES approach: optimization of expected objective value with Gaussian distributions.
- Offspring $x_1, \dots, x_\lambda$ act as MC sample for the estimation of $G(\theta)$.
- Closed form Fisher tensor for $\mathcal{N}(m, C)$:

$$\mathcal{I}_{i,j}(\theta) = \frac{\partial m^T}{\partial \theta_i} C^{-1} \frac{\partial m}{\partial \theta_j} + \frac{1}{2} \operatorname{tr}\left( C^{-1} \frac{\partial C}{\partial \theta_i} C^{-1} \frac{\partial C}{\partial \theta_j} \right)$$

## NES algorithm

**input** $\theta \in \Theta$, $\lambda \in \mathbb{N}$, $\eta > 0$
**loop**
  sample $x_1, \dots, x_\lambda \sim P_\theta$
  evaluate $f(x_1), \dots, f(x_\lambda)$
  $G(\theta) \leftarrow \frac{1}{\lambda} \sum_{i=1}^{\lambda} \nabla_\theta \log \big( p_\theta(x_i) \big) \cdot f(x_i)$
  $\tilde{G}(\theta) \leftarrow \big( \mathcal{I}(\theta) \big)^{-1} \cdot G(\theta)$
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Wierstra et al. Natural Evolution Strategies. *CEC*, 2008 and *JMLR*, 2014.

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- This turns NES into a function value free algorithm.
- Benefits:
  - invariance under monotonic transformations of objective values
  - linear convergence on scale invariant problems.

## xNES (exponential NES) algorithm

**input** $(m, A) \in \Theta$, $\lambda \in \mathbb{N}$, $\eta_m, \eta_A > 0$
**loop**
  sample $z_1, \dots, z_\lambda \sim \mathcal{N}(0, \mathrm{I})$
  transform $x_k, \dots, x_\lambda \leftarrow A z_k + m$
  evaluate $f(x_1), \dots, f(x_\lambda)$
  $\tilde{G}_m(\theta) \leftarrow \frac{1}{\lambda} \sum_{i=1}^{\lambda} w(f(x_i)) \cdot z_i$
  $\tilde{G}_C(\theta) \leftarrow \frac{1}{\lambda} \sum_{i=1}^{\lambda} w(f(x_i)) \cdot \frac{1}{2}(z_i z_i^T - \mathrm{I})$
  $m \leftarrow m - \eta_m \cdot A \cdot \tilde{G}_m(\theta)$
  $A \leftarrow A \cdot \exp\left(-\eta_A \cdot \frac{1}{2} \tilde{G}_C(\theta)\right)$
**until** stopping criterion met

Glasmachers et al. Exponential Natural Evolution Strategies. *GECCO*, 2010.

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- SNGD has only a single mechanism.
- Astonishing insight: all (most) parameters can be updated with a single mechanism.

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- This is insightful: it means that CMA-ES is (essentially) a SNGD algorithm.

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- SNGD on a stochastically relaxed problem results in a direct search algorithm: the Natural-gradient Evolution Strategy (NES) algorithm.
- The SNGD parameter update is by no means restricted to Gaussian distributions. It is a general construction template for update equations of continuous distribution parameters.

# Thank you!
# Questions?