Final Exam - Derivative Free Optimization (Part I)
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The documents are not allowed.
Length of the exam: 1h30

Exercice 1

1. We have seen during the class different methods to adapt the step-size in Evolution Strategy (ES) algorithms. Explain what means step-size adaptation and why it is important to adapt the step-size.
2. Give the pseudo-code of the $(\mu/\mu, \lambda)$-ES with step-size adapted with the Path Length Control or Cumulative step-size adaptation (CSA). We assume that the covariance matrix is fixed to the identity.
3. Which type of convergence will you observe if you use this algorithm to optimize the sphere function $x \mapsto \sum_{i=1}^{n} x_i^2$.
4. Same question if you optimize the function $x \mapsto \sqrt{\sum_{i=1}^{n} x_i^2}$.

Exercice 2

Three algorithms have been tested within the Comparing Continuous Optimizers (COCO) platform:

- the (1+1)-ES with step-size adapted with one-fifth success rule
- the CMA-ES algorithm seen during the class (Covariance Matrix Adaptation Evolution Strategy)
- the diag-CMA-ES algorithm, a specific version of CMA-ES where the covariance matrix is diagonal.

We investigate the performance of the three algorithms using Empirical Cumulative Distribution Functions (ECDF) plots displayed in Figure 1. The ECDF plots display the performance on the sphere function $f(x) = \sum_{i=1}^{n} x_i^2$ (left), on the ellipsoid function $f_{\text{elli}}(x) = \sum_{i=1}^{n} (10^6)^{\frac{i-1}{n-1}} x_i^2$ (middle) and the rotated ellipsoid $f_{\text{elli}}(Rx)$ (right) where R is a rotation matrix different from the identity in dimension $n = 10$.

1. Give the geometric shape of the iso-density lines of the Gaussian vector used to sampled candidate solutions in the diag-CMA-ES algorithm.

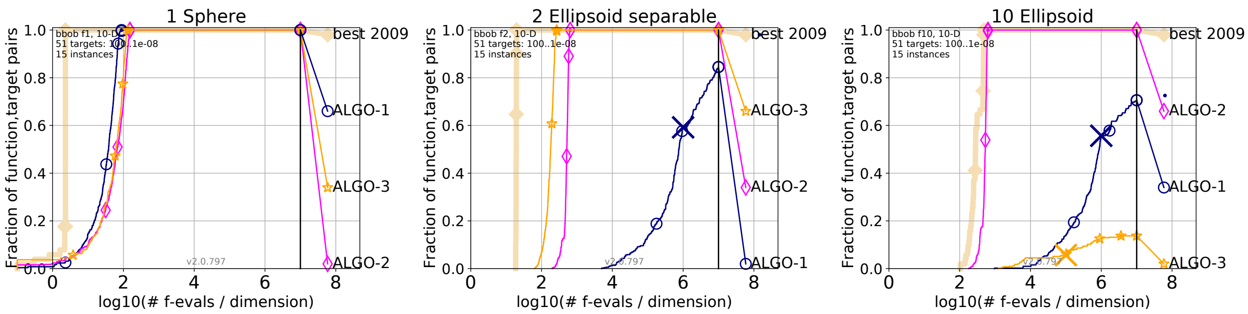

Figure 1: ECDF plot displaying the performance of three algorithms to be identified on the sphere, ellipsoid and rotated ellipsoid in dimension 10.

2. Explain the motivation and advantages of displaying performance measure of algorithms with ECDF plots. Explain how to read a ECDF plot.
3. Identify looking at the ECDFs in Figure 1 which algorithm is ALGO-1 ; which algorithm is ALGO-2 and which algorithm is ALGO-3. Explain carefully your reasoning.

Exercice 3

We consider the following five test functions

$$f_1(x) = \frac{1}{2}\left(10^{-4}x_1^2 + 10^4 x_2^2 + \sum_{i=3}^n x_i^2\right) \quad f_3(x) = \sqrt{f_2(x)} \quad f_5(x) = f_1(Rx)$$

$$f_2(x) = \frac{1}{2}\sum_{i=1}^n (10^6)^{\frac{i-1}{n-1}} x_i^2 \qquad f_4(x) = f_2(Rx)$$

where for the f_4 and f_5 functions, the matrix R is a rotation matrix different from the identity.

1. Compute the Hessian matrices and condition numbers for the functions f_1, f_2, f_4 and f_5.
2. The CMA-ES algorithm has been used to optimize the five functions. Four of the graphical outputs of CMA-ES optimizing the functions are displayed in Figure 2. They are identified as A, B, C, D below the plots. Identify which plot correspond to which function. Justify carefully your reasoning to arrive to this conclusion.

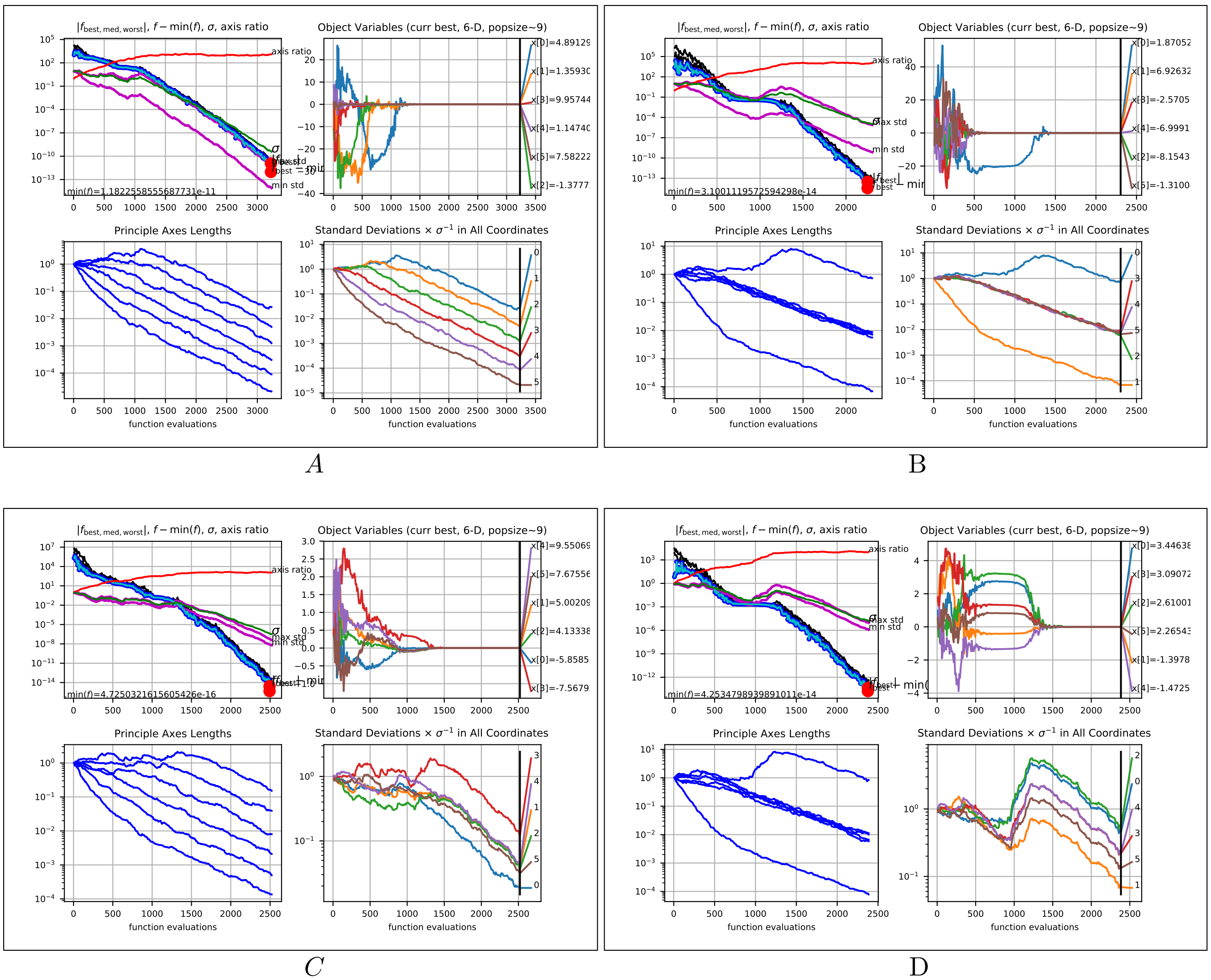

Figure 2: Graphical output of CMA-ES optimizing four of the five functions from Exercice 3.