Ray-tracing for room acoutics

Master 2 Acoustical engineering - Session 7

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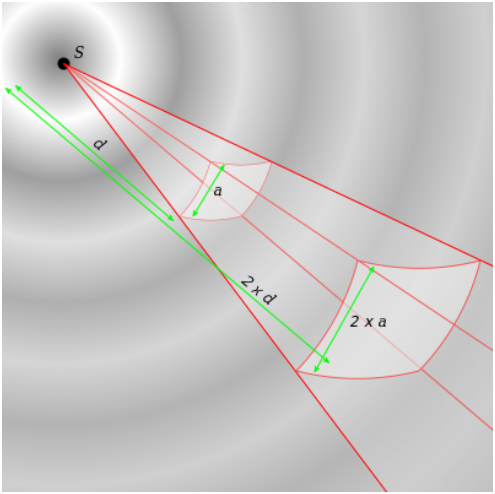
Punctual sound source propagation



Acoustic modelization

Following the conservation law, the energy of a punctual sound source centered at origin is given by :

$$E = \int_{S_r} \vec{\mathbf{I}} \cdot \vec{\mathbf{ds}} \quad (= \delta_0),$$

where S_r is a r-radius sphere and $\vec{\mathbf{I}}$ is the acoustic intensity (flow) accros the sphere, such as :

$$\vec{\mathbf{I}} = \frac{\vec{\mathbf{r}}}{4\pi|\vec{\mathbf{r}}|^3}.$$

Useful property : Energy is constant along solid angles :

$$E_\sigma = \int_\sigma \frac{\vec{\mathbf{r}}}{4\pi|\vec{\mathbf{r}}|^3} \cdot \vec{\mathbf{ds}} = \frac{\Omega_\sigma}{4\pi}.$$

Solid angles basis

For a spherical partition, such as entire surface is subdivided into N elementary parts :

$$S = \sum_{n=1}^{N} \sigma_n,$$

energy can be rewriten into :

$$E = \sum_{i=1}^{N} E_i = \frac{1}{4\pi} \sum_{i=1}^{N} \Omega_i.$$

Useful property : Solid angles form a basis for propagation directions.

Discretization

Solid angles are geometric objects that can be represented by N rays, defined by :

- Starting point : $\mathbf{x}_i$,
- Direction vector (unitary) : $\vec{\mathbf{u}}_i$,
- Energy : Ω_i,

in order to form **beams**.

For example, with a 3-D omnidirectional source $\sigma(\mathbf{x}_0)$ centered at the point $\mathbf{x}_0 = (x_0, y_0, z_0)$:

- $\mathbf{x}_i = (x_0, y_0, z_0)$,
- $\vec{\mathbf{u}}_i = (u_i^x, u_i^y, u_i^z)$ uniformly spread over S^2,
- $\Omega_i = \frac{4\pi}{N}$.

3D energy measurement

Coupled to spherical source $\sigma(\mathbf{x}_0)$, we define a spherical measurement $\nu(\mathbf{x_m}, a)$, centered at a listening point $\mathbf{x}_m$ of radius a. Energy of the measure is given **statistically** by a beam, formed by the n rays crossing $\nu(\mathbf{x}_m, a)$:

$$E_\nu \approx \frac{n}{N} \left(= \frac{1}{4\pi} \sum_{i=1}^{n} \Omega_i \right) .$$

We can show in 3D that :

$$\frac{n}{N} \approx \frac{\pi a^2}{4\pi |\mathbf{x}_s - \mathbf{x}_m|^2},$$

wich leads to the distance evaluation :

$$|\mathbf{x}_s - \mathbf{x}_m| \approx \frac{a}{2} \sqrt{\frac{N}{n}}.$$

Geometrical reminders

Using parametric representation, each ray i is described by :

$$\mathbf{x}_i + \delta \vec{\mathbf{u}}_i \quad \text{with} \quad \delta \in \mathbb{R}.$$

Considering the carthesian representation for the measurement sphere $\nu(\mathbf{x_m}, a)$:

$$(\mathbf{x} - \mathbf{x}_m)^2 - a^2 = 0 \quad \forall \mathbf{x} \in \nu,$$

quadratic equations in δ has to be solved to define the beam :

$$|\vec{\mathbf{u}}|^2 \delta^2 + 2(\mathbf{x}_i - \mathbf{x}_m) \cdot \vec{\mathbf{u}} \delta + |\mathbf{x}_i - \mathbf{x}_m|^2 - a^2 = 0.$$

Measured rays are given by positive discriminant and positive solutions δ^+ and δ^-.

2D case

- Punctual source energy :

$$E = \int_{C(t)} \frac{\vec{\mathbf{r}}}{2\pi|\vec{\mathbf{r}}(t)|^2} \cdot \vec{\mathbf{ds}}.$$

- Energy is constant along angle α :

$$E_\sigma = \int_\sigma \frac{\vec{\mathbf{r}}}{2\pi|\vec{\mathbf{r}}(t)|^2} \cdot \vec{\mathbf{ds}} = \frac{1}{2\pi}\alpha.$$

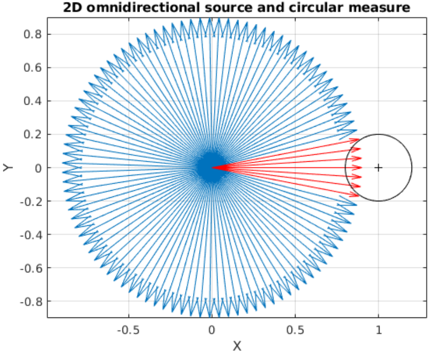
- An omnidirectional source has an energy repartition given by :

$$\alpha_i = \frac{2\pi}{N}.$$

- Energy is measured by circle $\nu(\mathbf{x}_m, a)$ following :

$$E_\nu \approx \frac{n}{N} \approx \frac{2a}{2\pi|\mathbf{x}_s - \mathbf{x}_m|} \quad \Rightarrow \quad |\mathbf{x}_s - \mathbf{x}_m| \approx \frac{a}{\pi}\frac{N}{n}.$$

Numerical application

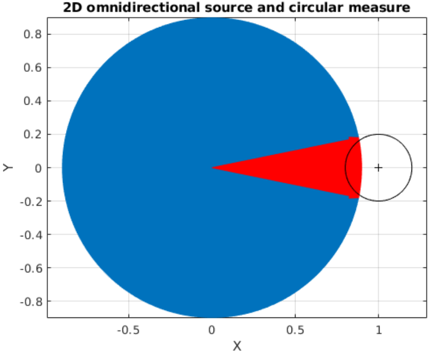


n	dist (m)	time (ms)
1000	0.0064	< 1
100	0.064	< 1
10	0.64	2
1	6.4	20

$N = 100$ rays, measurement radius
$a = 0.2m$ and sound speed $c = 340m/s$

Note : Maximum reverberation time computable is 20ms.

Numerical application

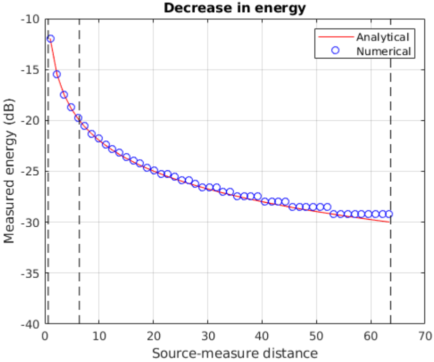


n	dist (m)	time (ms)
1000	0.64	1.8
100	6.4	18
10	64	180
1	640	1800

$N = 10\ 000$ rays, measurement radius $a = 0.2m$ and sound speed $c = 340m/s$

Note : Maximum reverberation time computable is now 2s.

Numerical application



$N = 10\ 000$ rays, measurement radius $a = 0.2m$ and sound speed $c = 340m/s$

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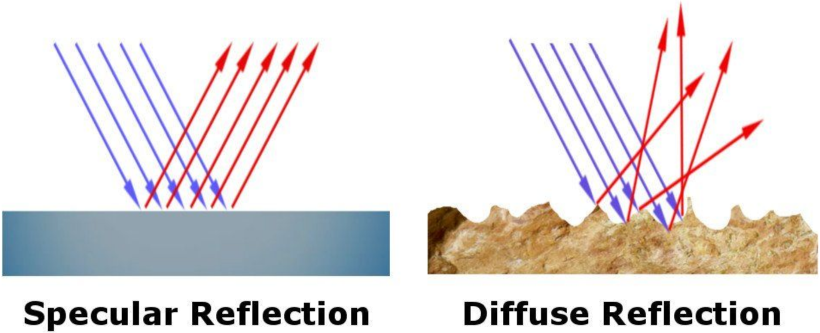
Specular reflections



Following high frequency approximation, we consider only **specular reflexions** with **material absorption** when a ray encounter an obstacle :

$$E_{ref}(f) = E_{inc}(f)(1 - \beta(f)).$$

Wall absorption coefficients

Absorption of Reflected Sound at Various Frequencies						
Material	**125 Hz**	**250 Hz**	**500 Hz**	**1 kHz**	**2 kHz**	**4 kHz**
Brick	3%	3%	3%	4%	5%	7%
Carpet *(on concrete w/foam rubber pad)*	8%	24%	57%	69%	71%	73%
Drapes *(heavy velour)*	14%	35%	55%	72%	70%	65%
Drywall *(1/2" on 2x4s)*	29%	10%	5%	4%	7%	9%
Linoleum *(on concrete)*	2%	3%	3%	3%	3%	2%
Paneling *(3/8" on 2x4s)*	28%	22%	17%	9%	10%	11%
Plaster *(rough finish, over lath)*	14%	10%	6%	5%	4%	9%
Window Glass	35%	25%	18%	12%	7%	4%
Wood	15%	11%	10%	7%	7%	4%

Examples of absorption coefficients $\beta(f)$ in function of frequency.

Geometrical reminders (2D)

Discretizing the room with a lineic mesh of edges, each ray i and edge e can respectively be represented by right (A_i) and (B_e) :

$$\begin{aligned} (A_i) &\quad : \quad \mathbf{A}_i + \delta \vec{\mathbf{u}}_i \quad \text{with} \quad \delta \in \mathbb{R}, \\ (B_e) &\quad : \quad \mathbf{B}_e + \lambda \vec{\mathbf{v}}_e \quad \text{with} \quad \lambda \in \mathbb{R}. \end{aligned}$$

Intersections between N rays and M edges conduct to solve MxN equations :

$$\vec{\mathbf{OA}} + \delta \vec{\mathbf{u}} = \vec{\mathbf{OB}} + \lambda \vec{\mathbf{v}},$$

equivalent to linear systems :

$$\begin{pmatrix} u_x & -v_x \\ u_y & -v_y \end{pmatrix} \begin{pmatrix} \delta \\ \lambda \end{pmatrix} = \begin{pmatrix} B_x - A_x \\ B_y - A_y \end{pmatrix}.$$

Solutions are unique $\iff$ $0 < \lambda \leq 1$, $\delta > 0$ and minimal.

Geometrical reminders (3D)

Discretizing the room with a surfacic mesh of triangles, triangle e can be represented by plane (C_e), such as :

$$(C_e) : \mathbf{C}_e + \lambda \vec{\mathbf{v}}_e + \mu \vec{\mathbf{w}}_e, \quad \text{with} \quad \lambda, \mu \in \mathbb{R}.$$

Intersections between N rays and M triangles conduct to solve MxN linear equations :

$$\vec{\mathbf{OA}} + \delta \vec{\mathbf{u}} = \vec{\mathbf{OC}} + \lambda \vec{\mathbf{v}} + \mu \vec{\mathbf{w}},$$

equivalent to linear systems :

$$\begin{pmatrix} u_x & -v_x & -w_x \\ u_y & -v_y & -w_y \\ u_z & -v_z & -w_z \end{pmatrix} \begin{pmatrix} \delta \\ \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} C_x - A_x \\ C_y - A_y \\ C_z - A_z \end{pmatrix}.$$

Solutions are unique $\iff 0 < \lambda, \mu \leq 1$ and $\delta > 0$ and minimal.

Geometrical reminders (nD)

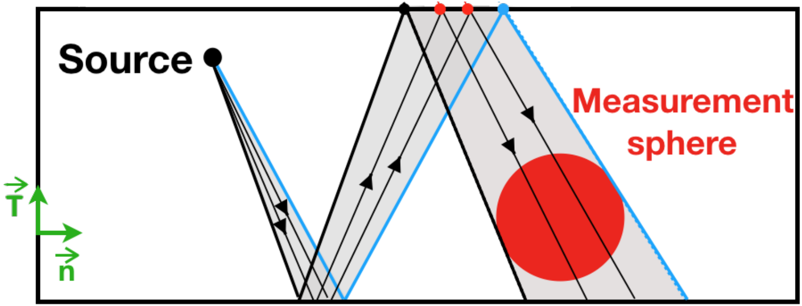
Once each ray i is paired with an unique element e of the mesh, all coefficient δ_i are determined. Specular reflections lead to update rays parameters, such as :

- Starting point : $\mathbf{x}_i = \mathbf{x}_i + \delta_i \vec{\mathbf{u}}_i$
- Direction vector : $\vec{\mathbf{u}}_i = (\vec{\mathbf{u}}_i \cdot \vec{\mathbf{T}}_e)\vec{\mathbf{T}}_e - (\vec{\mathbf{u}}_i \cdot \vec{\mathbf{n}}_e)\vec{\mathbf{n}}_e$
- Energy : $\Omega_i(f) = \Omega_i(f)(1 - \beta_e(f))$.

where $\vec{\mathbf{T}}_e$ stand for the tangantial basis of each element and $\vec{\mathbf{n}}_e$ the associated normal.

Note : Using this geometric approach, mesh no needs to be compliant and oriented...

Multiple reflections



In case of m reflections from source to measure, energy measured is given by the law :

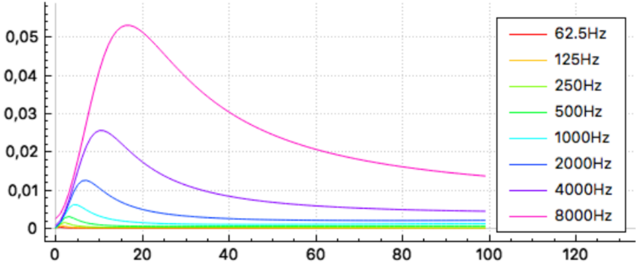
$$E_\mu(f) = \frac{n}{N} \prod_{e}^{m} (1 - \beta_e(f)).$$

Air absorption coefficients

For long distance (time) propagation, air absorption has to be considered. For example, considering a ray starting from a source point $\mathbf{x}_s$ to a measurement point $\mathbf{x}_m$:

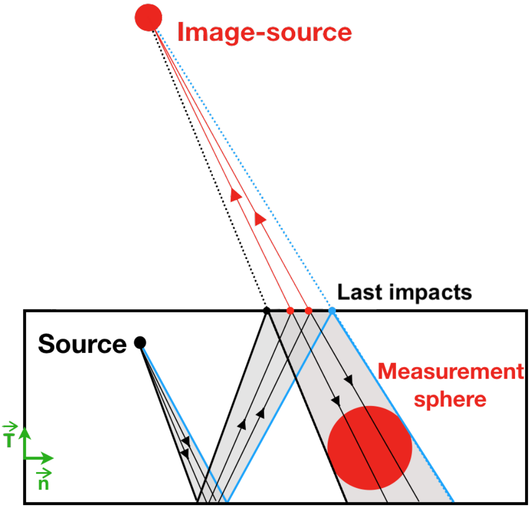
$$E_\mu(f) = \frac{n}{N} e^{-m(f)|\mathbf{x_s} - \mathbf{x_m}|} \prod_e^m (1 - \beta_e(f)).$$

where absorption coeffiscient $m(f)$ is measured or computed by physical laws :



Air absorption depending on relative humidity (%), octave bands.

Source-image, representation and energy



$$E_{\mu}^{is}(f) = \left(\frac{n}{N}\right)\left(e^{-m(f)|\mathbf{x_s}-\mathbf{x_m}|}\right)\left(\prod_{e}^{m}(1-\beta_e(f))\right)$$

Room Impulse Response

Source-image can be computed recursively, until beams become statistically unmeasurables by a ray basis :

$$(2D) \quad : \quad |\mathbf{x}_s - \mathbf{x}_m| \leq \frac{a}{\pi}\frac{N}{n_0},$$

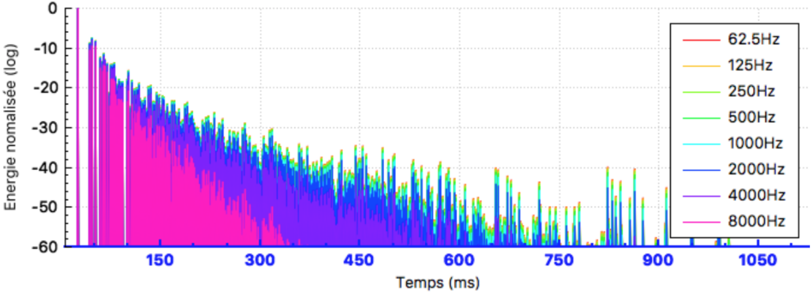
$$(3D) \quad : \quad |\mathbf{x}_s - \mathbf{x}_m| \approx \frac{a}{2}\sqrt{\frac{N}{n_0}},$$

with n_0 fixed by users. Finally, it gives P couples of :

- Spatial poistion : $\mathbf{x_i}$,
- Energy in band-frequency : $E_i(f)$.

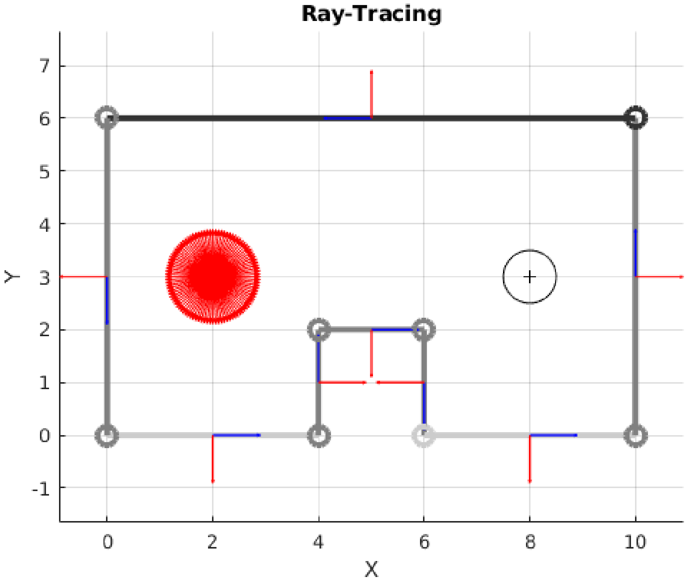
Converting spatial position to arrival time ($t_i = c|\mathbf{x_i}|$), a finite impulse response of the room is computed.

Room Impulse Response



Moreover, converting energy to signal pressure ($p_i(f) = \sqrt{E_i}$), convolution with audio signal gives the room acoustics rendering.

Rir computation with openRir2D (100 rays)



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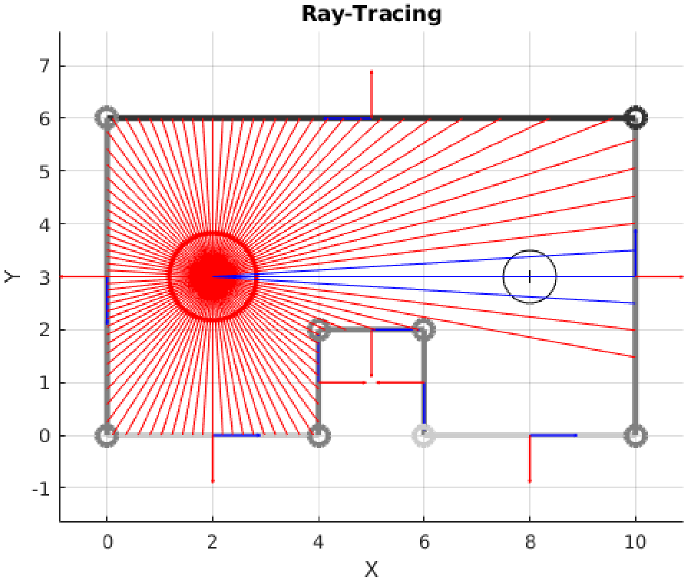

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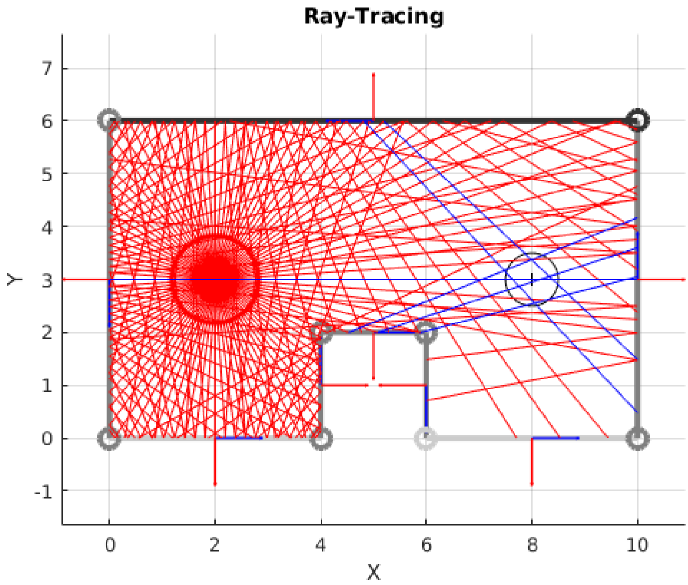

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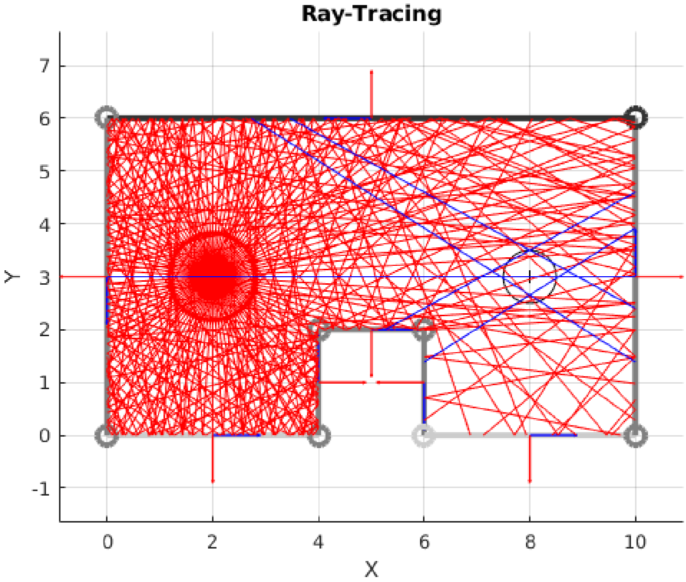

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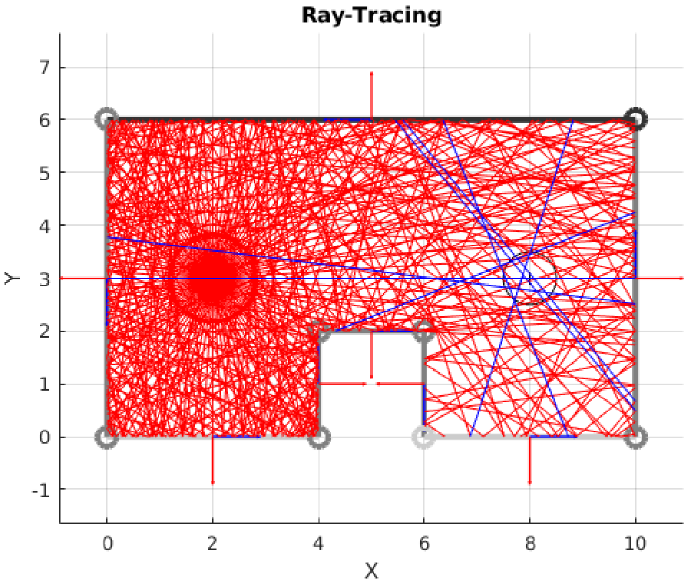

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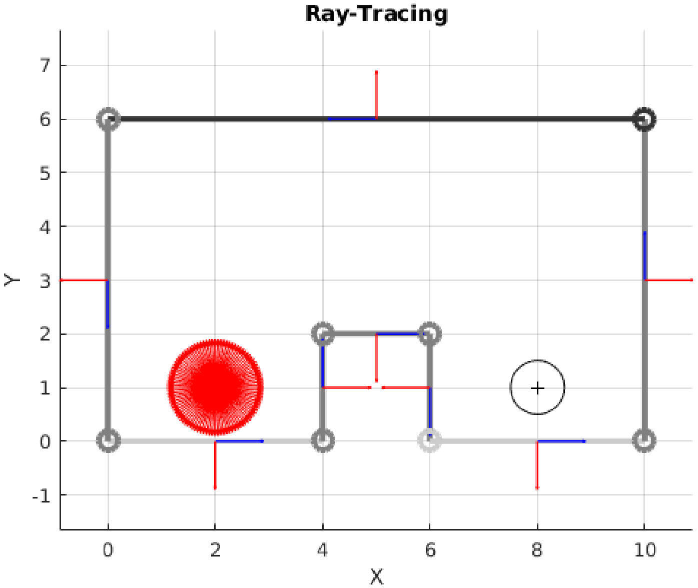

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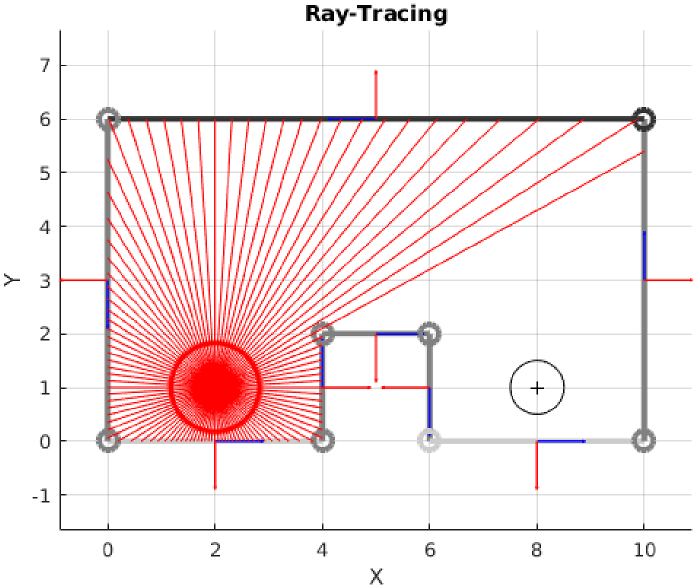

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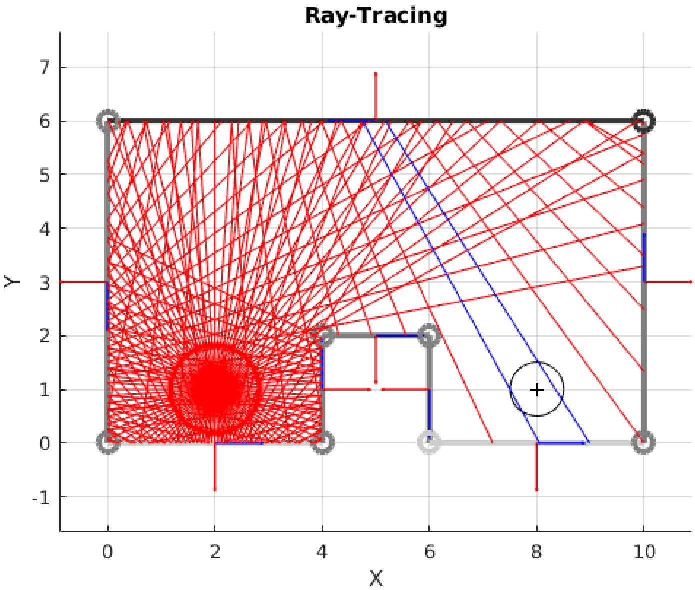

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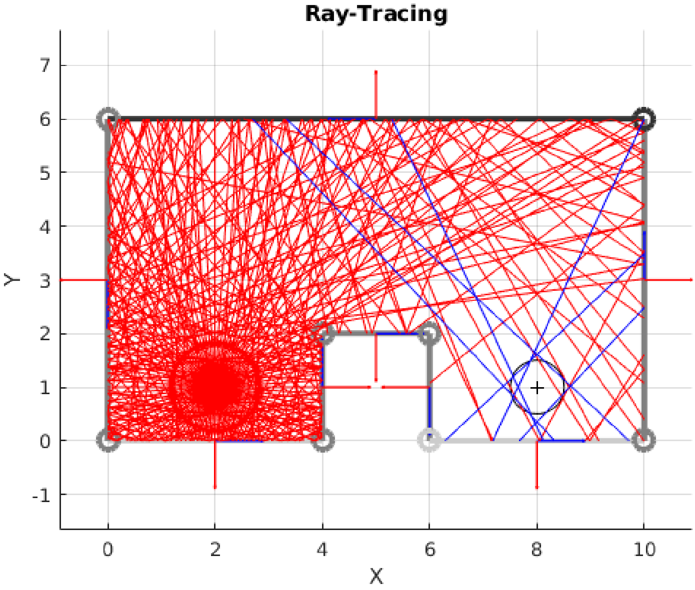

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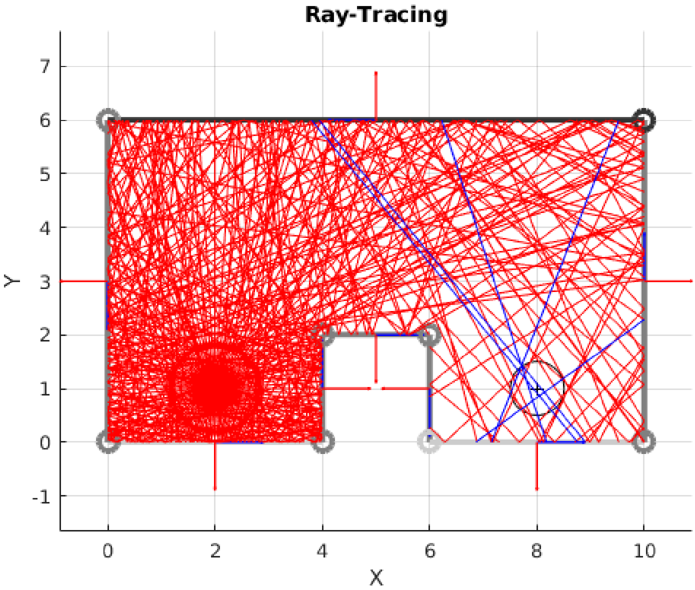
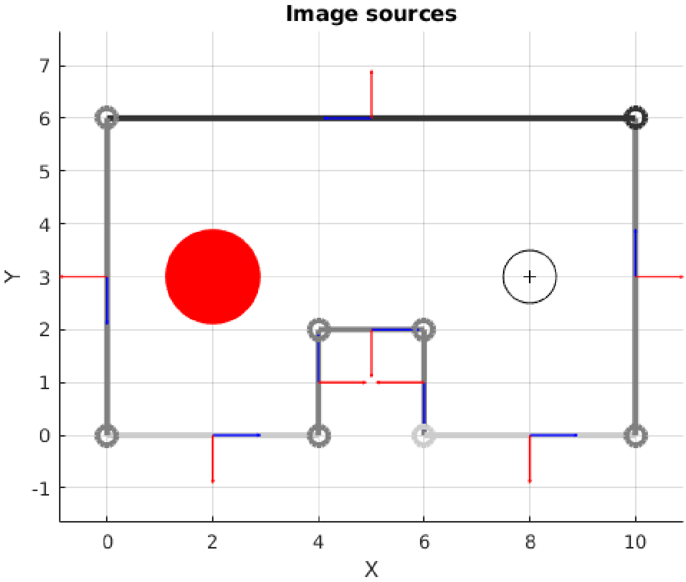
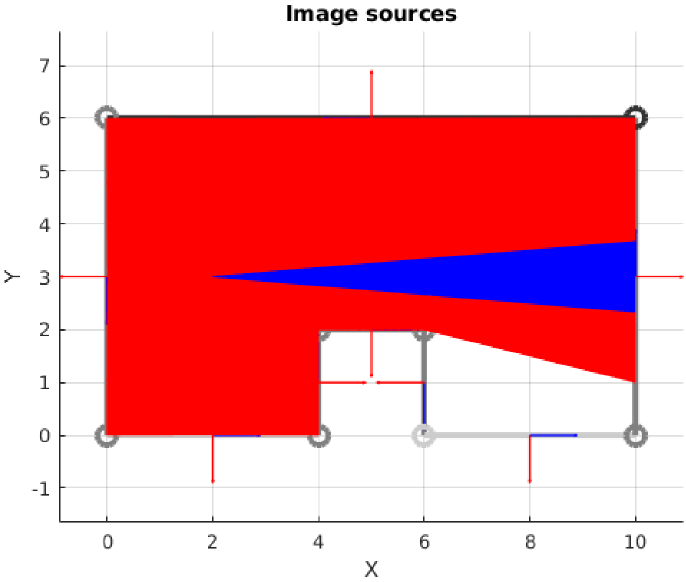


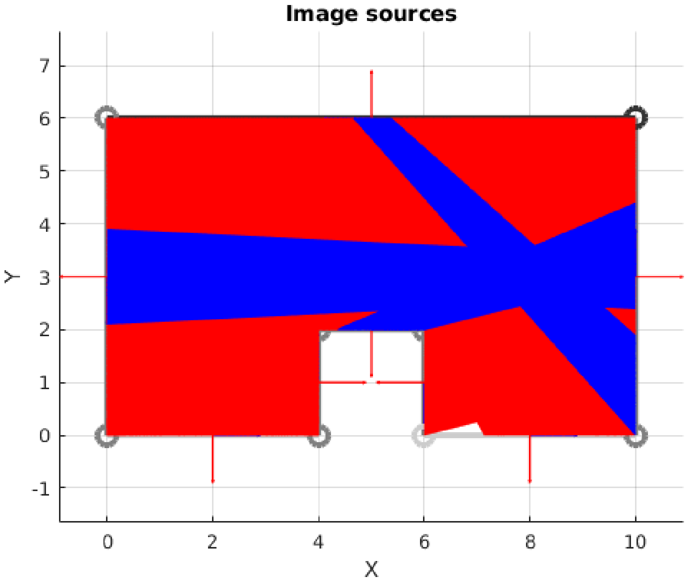
Rir computation with openRir2D (10 000 rays)



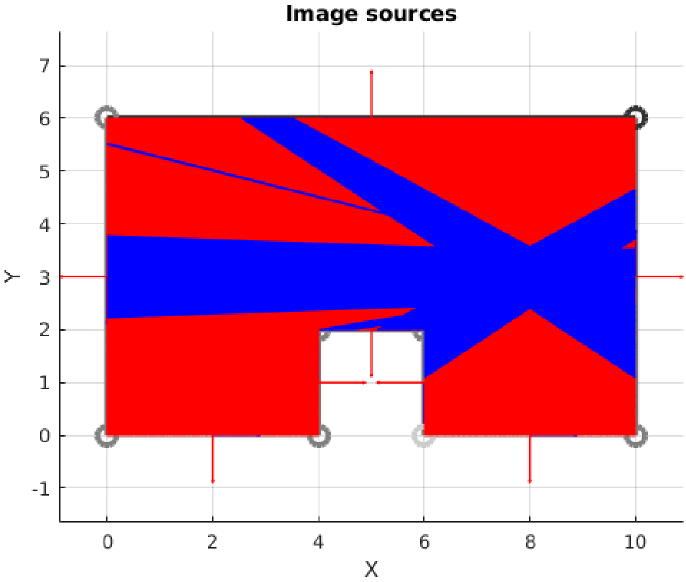
Rir computation with openRir2D (10 000 rays)



Rir computation with openRir2D (10 000 rays)



Rir computation with openRir2D (10 000 rays)



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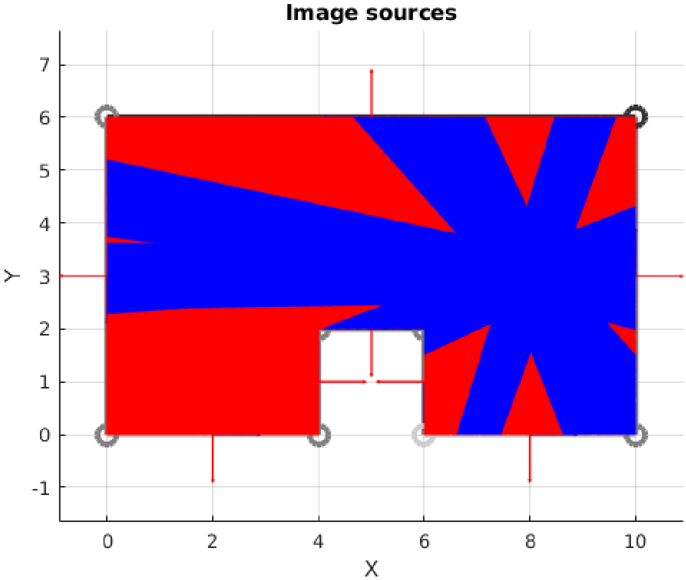
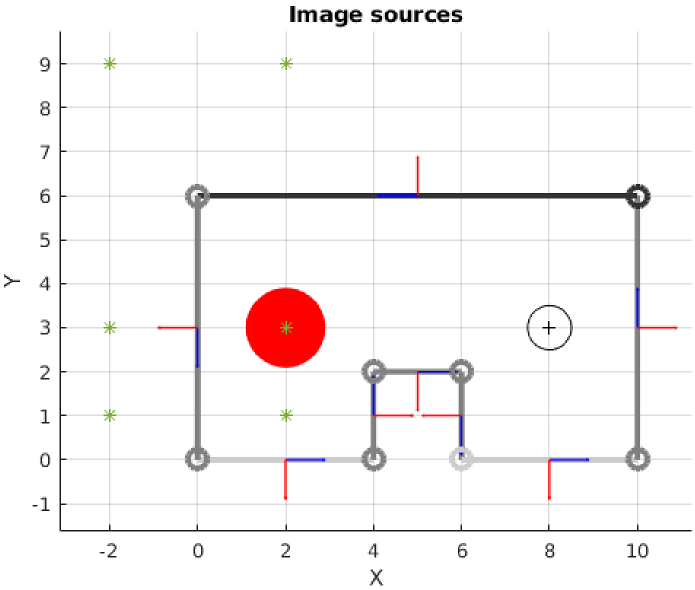
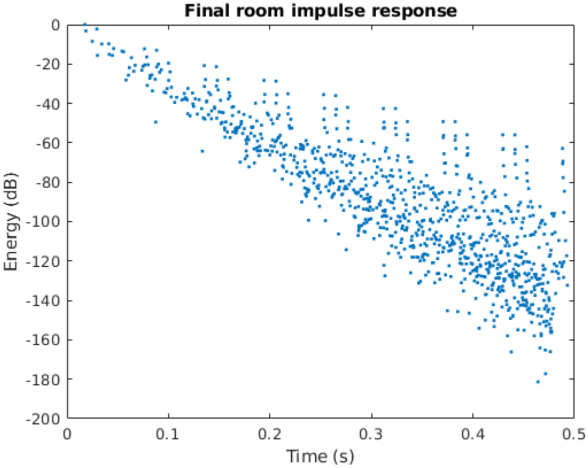


Rir computation with openRir2D



Rir computation with openRir2D

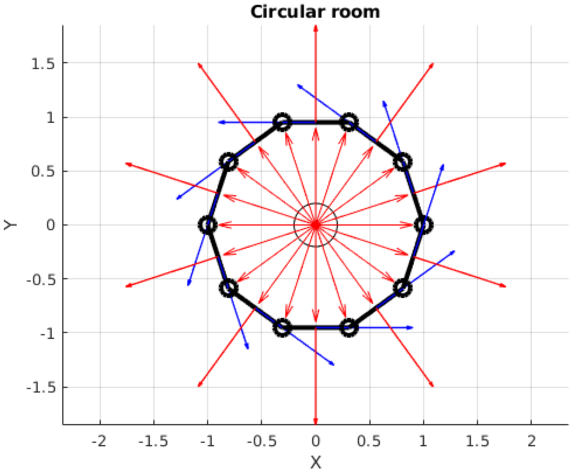


2Droom.wav big2Droom.wav

Execute script *roomImpulseResponse.m* associated to openRir2D.

Exercise

Using openRir2D, what is happening for an unitary circular room, perfecly reflecting, when measure and source are centered ?



Solution

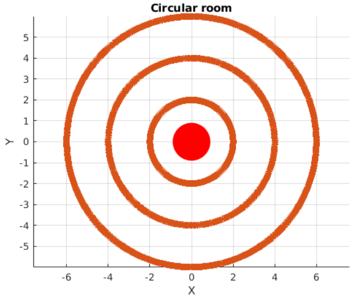
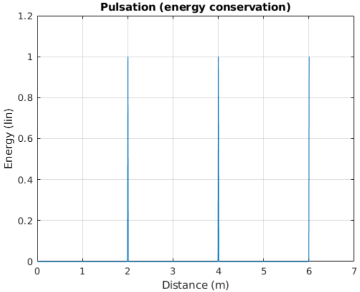


Image sources for a perfectly reflecting circular room.



Focalisation gives a comb of dirac, traducing the energy conservation.

This solution were computed with $N = 10^4$ rays, $M = 10^3$ edges, microphone radius $a = 0.2$m and statistics fixed at $n_0 = 100$.