Données haute fréquence
Analyse et modélisation statistique multi-échelle de séries chronologiques financières

Cours de Master - Paris 6
Transparents Partie II
Dynamique des Carnets d’ordre

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An orderbook snapshot

<table>
<thead>
<tr>
<th>Time</th>
<th>Price</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>13:11:56</td>
<td>112.43</td>
<td>697</td>
</tr>
<tr>
<td>-0.32</td>
<td>112.42</td>
<td>2042</td>
</tr>
<tr>
<td>&lt;Default&gt;</td>
<td>112.41</td>
<td>772</td>
</tr>
<tr>
<td>518613</td>
<td>112.40</td>
<td>928</td>
</tr>
<tr>
<td>0</td>
<td>112.39</td>
<td>1718</td>
</tr>
<tr>
<td>0</td>
<td>112.38</td>
<td>1231</td>
</tr>
<tr>
<td>1</td>
<td>112.37</td>
<td>1038</td>
</tr>
<tr>
<td>5</td>
<td>112.36</td>
<td>579</td>
</tr>
<tr>
<td>10</td>
<td>112.36</td>
<td>249</td>
</tr>
<tr>
<td>20</td>
<td>112.36</td>
<td>743</td>
</tr>
<tr>
<td>50</td>
<td>112.36</td>
<td>830</td>
</tr>
<tr>
<td>100</td>
<td>112.36</td>
<td>805</td>
</tr>
<tr>
<td>CLR</td>
<td>112.35</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>112.35</td>
<td></td>
</tr>
<tr>
<td>SL</td>
<td>112.33</td>
<td></td>
</tr>
<tr>
<td>SM</td>
<td>112.33</td>
<td></td>
</tr>
<tr>
<td>Limit</td>
<td>112.31</td>
<td></td>
</tr>
<tr>
<td>Del All</td>
<td>112.30</td>
<td></td>
</tr>
<tr>
<td>Delete 3</td>
<td>112.29</td>
<td></td>
</tr>
<tr>
<td>Delete 3</td>
<td>112.28</td>
<td></td>
</tr>
<tr>
<td>Delete 3</td>
<td>112.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>112.26</td>
<td></td>
</tr>
</tbody>
</table>
Basic notions

- Bid/ask spread
- Different types of orders
  - Limit orders
  - Cancel orders
  - Marker orders (generally better to send a limit order!)
- Different rules for execution matching
  - FIFO
  - Proportional
  - ...
  - Mixed rules

⇒ Replayer: hard task
Movie showing two order books (both “large” ticks)

- Bund
- Future on EuroStoxx
"Kerviel" day

- open/close = −6.2%
- open/high = 0.3%
- open/low = −7.6%

and the day after ...

- close/open = −2%
- open/close = 3.4%
- open/high = 4.5%
- open/low = −3.9%
### Table 5.4. Some data for the two liquid French stocks in February 2001. The transaction volume is in number of shares.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>France-Telecom</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial/final price (Euros)</td>
<td>90-65</td>
<td>157-157</td>
</tr>
<tr>
<td>Tick size (Euros)</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>Total # orders</td>
<td>270,000</td>
<td>94,350</td>
</tr>
<tr>
<td># trades</td>
<td>176,000</td>
<td>60,000</td>
</tr>
<tr>
<td>Transaction volume</td>
<td>$75,6 \times 10^6$</td>
<td>$23,4 \times 10^6$</td>
</tr>
<tr>
<td>Average bid-ask (ticks)</td>
<td>2.0</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Bouchaud J.-Ph. and Potters M.
For the BUND (1 month):

- Average Spread: $\approx 1.02$
- Numbers of transactions: $\approx 350000$
- Numbers of limit orders (2 $\times$ 4 levels): $\approx 1.800.000$
- Numbers of cancel orders (2 $\times$ 4 levels): $\approx 1.000.000$
Some Statistical elements: Order book profile

Fig. 5.3. Average size of the queue in the order book as a function of the distance from the best price, in a log-linear plot for three liquid French stocks. The ‘buy’ and ‘sell’ sides of the distribution are found to be identical (up to statistical fluctuations). Both axis have been rescaled such as to superimpose the data. Interestingly, the shape is found to be very similar for all three stocks studied. Inset: same in log-log coordinates, showing a power-law behaviour.

Bouchaud J.-Ph. and Potters M.
Some Statistical elements : Order book profile

Bouchaud J.-Ph. and Potters M.

E. Bacry, CMAP Ecole Polytechnique, 2015
Some Statistical elements: Heavy intraday seasonality

BUND 2007: ♯ Market orders ($\Delta t = 5$mn)
Data from BNP-Paribas FIRST-ETG, London.
Some Statistical elements: Heavy intraday seasonality

BUND 2007: \# orders ($\Delta t = 5\text{mn}$)
- **Blue**: limit orders,
- **Red**: cancel orders,
- **Black**: market orders.

Data from BNP-Paribas FIRST-ETG, London.
Some Statistical elements: A ”better” proxy of Liquidity

BUND 2007 “Liquidity”
Tick size is 0.01
Worst price for buying/selling 600 shares within $\Delta t = 5$mn
Data from BNP-Paribas FIRST-ETG, London.
- broker’s fee (depends on "your size")
  - individuals \(\approx 1\%\) on stocks!
  - e.g., (small hedge fund): 1/4 tick for futures (SXE and FX)

- bid/ask spread: cost of a buy/sell strategy

- Market impact
At high frequency: "large tick" ➔ very strong mean reversion! ➔ build an arbitrage strategy?

- The model:
  Ask price at time $n$: $a_n$
  Bid price at time $n$: $b_n = a_n - 1$
  The model for the dynamics:

  $$a_n = a_{n-1} + Ne_n$$

  where the $e_n \in \{-1, 1\}$ are known to be strongly anti-correlated.
  At each time $n$, we play the mean-reversion strategy.
A naive arbitrage strategy

<table>
<thead>
<tr>
<th>$N$</th>
<th>$P_-$</th>
<th>#samples</th>
<th>$P^{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80%</td>
<td>17600</td>
<td>100%</td>
</tr>
<tr>
<td>2</td>
<td>60%</td>
<td>1830</td>
<td>75%</td>
</tr>
<tr>
<td>3</td>
<td>55%</td>
<td>680</td>
<td>66.6%</td>
</tr>
<tr>
<td>4</td>
<td>52%</td>
<td>374</td>
<td>62.5%</td>
</tr>
<tr>
<td>5</td>
<td>53%</td>
<td>228</td>
<td>60%</td>
</tr>
<tr>
<td>6</td>
<td>53%</td>
<td>145</td>
<td>58.3%</td>
</tr>
</tbody>
</table>

Bund February 2007, Data BNP-Paris Bas FIRST-ETG, London.
Some order book models

- 0-intelligence (Farmer et al. 2005)
- Cont, de Larrard (2012)
  - "Level 1" model
  - Spread is 1 tick
  - Successive iid processes
- Huang, Lehalle, Rosenbaum (2013)
  - "Level n" generalization of Cont de Larrad

→ Some related Key questions
- Execution of a limit order
- Link macro quantities (e.g., volatility) with micro parameters
- Market impact
- Market making
Key question:

**How does an order impact the price?**

- Capacity of an arbitrage strategy
- Optimal execution
A naive proxy for the impact: the Response function

- $n$ : trading time (i.e., $n^{th}$ transaction) or physical time
- $\epsilon_n$ : Sign of the transaction
  + +1 if buy market order and -1 if sell market order
- $v_n$ : The volume of the transaction

A naive proxy for the impact:
- Response function

$$R_{n,v} = E_k (\epsilon_k (P_{k+n} - P_k) | v_k = v)$$
A naive proxy for the impact: the Response function

Bund Future February 2007: Response function $R_{n,v}$
Data BNP-Paribas FIRST-ETG, London.
Volume buckets are power of 2
$\Rightarrow$ Strong permanent impact??
Auto-correlation function \( Cor(\epsilon_k, \epsilon_{k+n}) \)

Bouchaud J.-Ph. and Potters M.
Some Statistical elements: Correlation of the signs of the trades

Auto-correlation function $Cor(\epsilon_k, \epsilon_{k+n})$ for the Bund

Data from BNP-Paribas FIRST-ETG, London.
Market impact profile of a (meta)order: variation of the price during and after the execution of the (meta)order.

Market impact profile is generally estimated by aggregating all executions of a certain type (unconditionally to the market conditions) after time and price rescaling.
Market impact profile curve

E.B., M.Hoffmann, A.Iuga, M.Lasnier, C.A.Lehalle (working paper : Estimation on a pool of European stocks
Data from CA Chevreux.

See also : Moro et al. (2009)
Concave impact while trading
Top point (temporary market impact) - "Square-root" law (Gatheral, 2008)

\[ I \sim C \sigma \sqrt{\frac{\nu}{\hat{V}}} \]

Relaxation after trading
Is impact permanent?
• **WARNING**: confusion between market impact and response function!
  \[ \Rightarrow \text{a priori one needs labelled data} \]

• **WARNING**: Very hard to estimate due to correlation of orders arrivals and price movements
Market impact estimation: Temporary market impact

E.B., M.Hoffmann, A.Iuga, M.Lasnier, C.A.Lehalle (working paper)

84794 meta-orders on all European stocks from Chevreux VWAP or Percentage of Volume strategies
Market impact estimation: Temporary market impact

E.B., M. Hoffmann, A. Iuga, M. Lasnier, C.A. Lehalle (working paper)

- $v$: Volume of the meta-order
- $T$: Duration of the meta-order
- $V$: Daily volume

$$I \approx \left( \frac{v}{TV} \right)^{0.7}$$
Market impact estimation: Impact profile

E.B., M.Hoffmann, A.Iuga, M.Lasnier, C.A.Lehalle (working paper)

(a) fit exponent 0.96
(b) fit exponent 0.63
(c) fit exponent 0.57
(d) fit exponent 0.58
(e) fit exponent 0.37
(f) fit exponent 0.21
Market impact estimation: Decay profile

E.B., M.Hoffmann, A.Iuga, M.Lasnier, C.A.Lehalle (working paper)

\[ r \in [0.1, 0.4346], \ T \in [4.8, 7.4] \]

\[ r \in [0.4346, 0.8619], \ T \in [3, 7.85] \]

\[ r \in [0.8619, 1.6869], \ T \in [3, 13.6] \]

\[ r \in [1.6869, 15], \ T \in [57, 96] \]

(a) \( \beta_1 - \beta_2 = 0.87 \)

(b) \( \beta_1 - \beta_2 = 0.98 \)

(c) \( \beta_1 - \beta_2 = 0.98 \)

(d) \( \beta_1 - \beta_2 = 1.01 \)
Permanent impact of Trend followers

Figure 3.8: Market impact extraday for trend-follower investors (in spreads).
Market impact estimation: permanent impact

E.B., M.Hoffmann, A.Iuga, M.Lasnier, C.A.Lehalle (working paper)
Permanent impact of Mean reverters

Figure 3.9: Market impact extraday for investors playing mean reversion (in spreads).