SISM2 - MAP 561
Answers for the 2013 MAP pale

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Exercise 1

A. xxx

B. xxx

C. xxx

D. xxx

Exercise 2

A. xxx

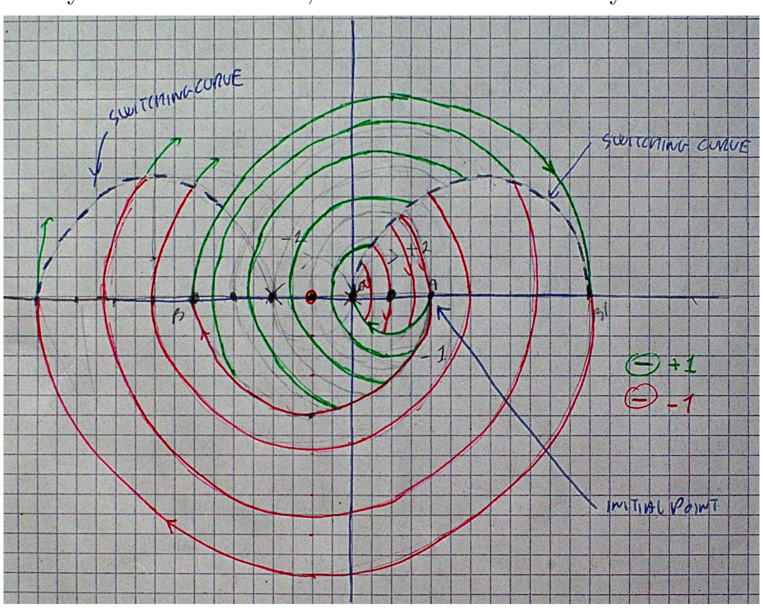
B. xxx

C. xxx

Exercise 3

A. The system is controllable, but not small time locally controllable.

B.



Exercise 4

A. If we start from $y = 0$ we cannot leave the axes. On the upper semi-plane it is LBG. The same on the lower semi-plane.

B. On the axis $y = 0$ there are abnormal extremals.

C. The abnormal extremal found at the previous point is also normal.

Exercise 5

The properties of controllability are the same of those of the system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix} + u \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u \in \mathbf{R}$$

This is due to the fact that the function $f(x)$ does not change the integral curve of the drift, but only its parameterisation. Hence the system is controllable and small time locally controllable (from Kalman condition).

Exercise 6

A. The controlled part is LBG and the controls are not a priori bounded. Hence it is controllable and also small time locally controllable

B. No, the proof is similar to the sub-Riemannian case and uses the fact that the controlled part is LBG.

C. Direct computation with the PMP.