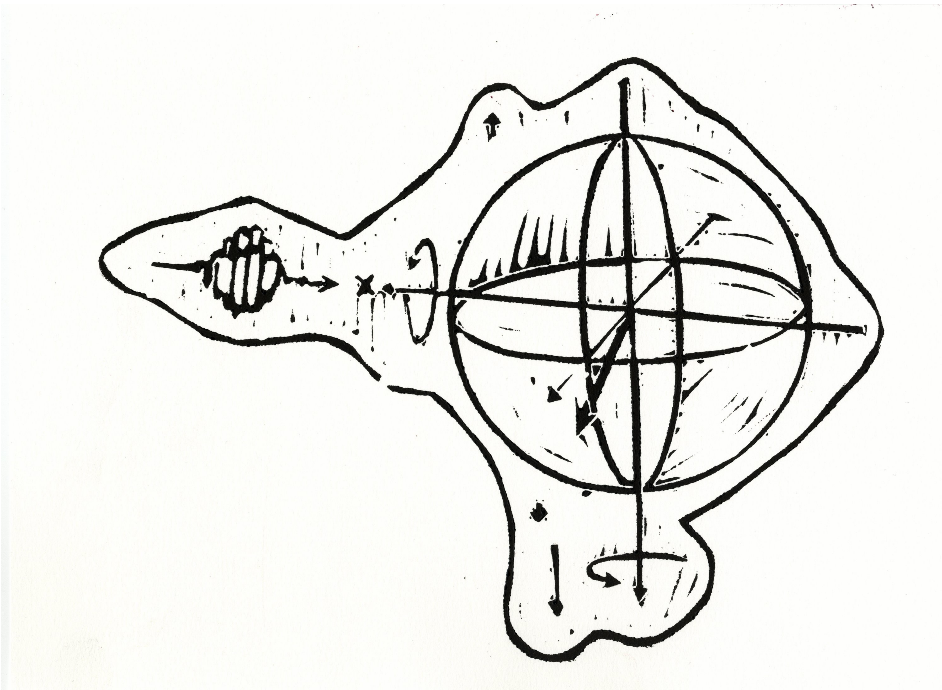
Workshop on Quantum Control

IHP, Paris Dec. 8-11, 2010



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BOOK OF ABSTRACTS

Part of the

Control of Partial and Differential Equations and Applications Trimester

Aims of the Workshop

One of the most challenging and modern applications of the control of PDEs is the control of Quantum Mechanical Systems. Indeed the problem of inducing transitions between molecular levels or spin states is relevant to many applications of chemical physics: from Nuclear Magnetic Resonance, to spectroscopy, to the realization of Quantum gates in quantum information science. These experiments are often very delicate and expensive. With no doubt, control theory can bring new perspectives.

The purpose of this workshop is to create a link among the mathematical community of control of PDEs, the community of finite dimensional quantum control, the community of quantum mechanics and the community of physicists and engineers carrying out real experiments.

The aim is twofold. First, this will be an opportunity for mathematicians to know what "real problems" are. Then, engineers and physicists can learn new mathematical techniques and optimization methods.

The conference will cover the following topics:

- Controllability of the Schrodinger equation
- Optimal control for spin systems
- Controllability of Lindblad type equations
- Adiabatic Theory
- Quantum Feedback Control and trajectories
- Numerical methods in quantum control

List of Speakers

Roger Brockett (Harvard)
Yosi Avron (Technion, Haifa)
Hans-Rudolf Jauslin (Dijon)
Stefan Teufel (Tübingen)
Steffen Glaser (München)
Bernard Bonnard (Dijon)
Alfio Borzì (Benevento)
Claudio Altafini (SISSA, Trieste)
Mazyar Mirrahimi (INRIA Rocquencourt)
Vahagn Nersesyan (Versailles Saint-Quentin)
Karine Beauchard (ENS Cachan)
Gianluca Panati (La Sapienza, Rome)
Holger Teismann (Acadia University, Canada)
Mario Sigalotti (INRIA, Nancy)
Thomas Chambrion (Nancy)
Gian Michele Graf (Zürich)
Chitra Rangan (Windsor, Canada)
Martin Fraas (Technion, Haifa)
Dominique Sugny (Dijon)
Gabriel Turinici (Dauphine, Paris)
Gunther Dirr (Würzburg)
Andrey Sarychev (Florence)
Igor Dotskenko (ENS, Paris)
Sylvain Ervedoza (CNRS, Toulouse)
Francesca Chittaro (L2S, Paris)
Camille Laurent (CMAP, Ecole Polytechnique, Paris)

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Titles and Abstracts

Plenary Talks

Geometry of quantum transport of open systems

Yosi Avron

Technion, Haifa

I shall outline a geometric theory of transport in adiabatically driven open systems where the coefficients of dissipative transport are determined by the Fubini-Study metric and the coefficients of non-dissipative transport by the adiabatic curvature. When the metric and symplectic form are compatible non-dissipative terms in the inverse matrix of transport coefficients are immune to dephasing. Examples of compatible systems shall be described. Based on joint work with Fraas, Graf and Kenneth.

In what sense are Law-Eberly transfers optimal?

Roger W. Brockett

Harvard

The Law-Eberly scheme for transferring quantum states in an infinite dimensional setting has the unusual property of proceeding through a highly restricted sequence of finite dimensional subspaces rather than making use of the full Hilbert space. In this talk we will explore the optimality of such schemes and also investigate related questions involving transfers restricted to decoherence free subspaces and related matters.

Approximate controllability of the bilinear Schroedinger equation: some estimates

Thomas Chambrion

Nancy

he bilinear Schroedinger equation with discrete spectrum is known to be approximately controllable provided certain coupling conditions and some distinctness assumptions on the gaps between two eigenvalues are satisfied. In this talk, we present some estimates on the norm of the control and the time needed to steer such a control system from a given source to (a neighborhood of) a given goal.

Optimal Control of Spin Dynamics in Magnetic Resonance

Steffen Glaser

München

Based on principles of optimal control theory, the physical limits of quantum control can be explored and robust time-optimal and relaxation-optimized pulse sequences can be designed to control the dynamics of spin systems (1-3). Furthermore, efficient numerical algorithms are available for the development of robust pulse sequences, compensating experimental imperfections and taking into account experimental limitations. Recent advances include: (A) The experimental realization of a general solution for the optimal control of one or two uncoupled spins with limited rf amplitude and in the presence of relaxation. This is illustrated for the saturation problem in the absence of B0 or B1 gradients, where the optimal solution (4,5) is significantly faster than conventional sequences based on inversion recovery. (B) A set of robust 90° and 180° pulses has been numerically optimized, which makes it possible to simply replace rectangular pulses in most multi-dimensional experiments in a straight-forward way, providing increased bandwidth and tolerance with respect to rf inhomogeneity. (C) Numerically optimized robust pulses to compensate for large differences in the scalings of the rf amplitude in toroid probes, enabling 2D NMR spectroscopy for this hardware platform. (D)The application of optimal control methods to the problem of heteronuclear decoupling(6) yields not only significantly improved broadband decoupling performance but also unprecedented flexibility in the design of tailored decoupling sequences.
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Transport in quantum devices and its geometry

Gian Michele Graf

Zürich

This talk is about mesoscopic devices, which are driven slowly and periodically in time, and known as quantum pumps. In a first part I will present a discussion of transport properties, concerning charge, noise, and dissipation. Pump processes, which are "optimal" in this respect, may be characterized geometrically using the Hopf map. These considerations will be based on the scattering approach to pumping due to Buettiker. The system is viewed as consisting of a finite device connected to leads. It allows for scattering states at Fermi energy and is hence gapless. In a second part I will review an alternative approach due to Thouless, in which the system is idealized as being of infinite extent and gapped. The charge transported in a cycle is realized as a Chern number. It will be shown how to relate the seemingly disjoint approaches and in particular how they yield the same transported charge.

Robust control of quantum processes in adiabatic Floquet theory

Hans- Rudolf Jauslin

Dijon

We will present some general concepts of quantum control by adiabatic processes, in the framework of adiabatic Floquet theory. We will discuss the robustness of adiabatic schemes based on geometrical and topological properties of instantaneous eigenenergy surfaces. Furthermore we will discuss the relation between Floquet theory and the formalism of atoms dressed by quantized electromagnetic fields.

Approximate controllability of the bilinear Schroedinger equation: sufficient conditions

Mario Sigalotti

INRIA, Nancy

We present an approximate controllability result for the bilinear Schroedinger equation. In a general framework (incuding unbounded control operators) we give sufficient conditions on the spectrum of the uncontrolled Schroedinger operator and on the coupling of the controlled term that ensure approximate controllability. The non-resonance assumptions on the uncontrolled term are expressed as distinctness conditions between some eigenvalue gaps. They do not apriori require the spectrum of the uncontrolled Schroedinger operator to be simple.

Partial semiclassical limits in quantum dynamics

Stefan Teufel

Tübingen

We consider quantum systems in which only some degrees of freedom behave semiclassically. After explaining the general structure, we present an Egorov-type Theorem for the partial semiclassical limit of such systems. One interesting aspect is, how the quantum degrees of freedom modify the symplectic form for the semiclassical dynamics. As an application we show how to derive the semiclassical model in solids starting from the Bloch-Landau Hamiltonian for a particle in a periodic potential and a strong magnetic field.

Semiplenary Talks

Environment-assisted stabilization of conditional quantum dynamics

Claudio Altafini

Sissa, Trieste

In quantum-mechanical systems, the use of feedback is complicated by the so-called measurement back-action i.e., a stochastic modification of the dynamics in presence of a measurement. Globally stabilizing the resulting Stochastic Master Equation, conditioned on the outcome of a continuous measurement, is a challenging task. In this work we study how dissipation can be used to simplify this problem, and in particular we obtain a characterization of the semigroups that can be used to stabilize a desired pure state or subspace under open-loop and filtering-based feedback. Joint work with F. Ticozzi (Univ. of Padova) and K. Nishio (Tokyo Inst. of Technology).

Control and stabilization of the Bloch equation.

Karine Beauchard

ENS Cachan

We consider an ensemble of non-interacting half spins, described by the Bloch equation. This system may be seen as a propotype for infinite dimensional bilinear control systems, with continuous spectrum. We will see that this equation is not exactly controllable with a priori bounded controls. However, unbounded controls (such as sums of Dirac masses) allow to recover good controllability properties. For instance,

- the Bloch equation is approximately controllable, in finite time, with unbounded controls,
- one may propose explicit unbounded controls for the exact controllability, in infinite time. Finally, we will present explicit feedback laws for
- the stabilization of the uniform state of spin +1/2,
- or the stabilization of any variable profile, close enough to this uniform state.

These are joint works with Jean-Michel Coron, Pierre Rouchon and Paulo Sergio Pereira da Silva.

Geometric and Numerical Optimal Control with Application to NMR

Bernard Bonnard

Dijon

In this talk we present some techniques from optimal control developed to analyzed dissipative spin 1/2 control systems. The main tool is the Pontryagin's maximum principle and second order optimality conditions which lead to analyse geometrically and numerically Hamiltonian dynamics . The robustness with respect to the parameters of the optimal control laws are discussed using a continuation method. They are currently applied in NMR in collabaoration with S. Glaser.

Advanced numerical schemes for solving infinite-dimensional quantum control problems.

Alfio Borzì

Benevento

The control of quantum electronic states in physical systems has a host of challenging and foreseen applications in nano-sciences that requires the accurate and fast solution of quantum control problems governed by infinite-level Schroedinger-type models. This task involves the development of solution methodologies that accommodate the nonlinear structure of the control mechanisms and the complex functional spaces where the control problems are usually formulated.

Recent advances in computational techniques are discussed that improve accuracy and efficiency through multilevel strategies, suitable discretization schemes, and appropriate choice of the functional spaces where the controls are sought. In this framework, first-order necessary optimality conditions and second-order sufficient optimality conditions are investigated for representative quantum systems arising in quantum optics, dipole transition, and in the transport of Bose–Einstein condensates.

To investigate the influence of uncertainty in the measurements and realization of controls, a framework for solving quantum control problems with random perturbation is discussed.

Quantum control via adiabatic theory and intersection of eigenvalues

Francesca C. Chittaro

L2S, Paris

In this talk, we expose a new method for controlling a quantum dynamical system driven by a Hamiltonian depending on two controls.
If the Hamiltonian has a discrete spectrum that presents conical intersections between the eigenvalues, we can take advantage of the adiabatic theory to induce transfers of population between the energy levels. This strategy permits to approximately control the occupation probability.

New estimates and bounds on the reachable sets of controlled Lindblad Kossakowski equations

Gunther Dirr

Würzburg

In the talk, a sharp estimate of the reachable sets of Lindblad Kossakowski equations (LKE) with Hamiltonian controls will be provided. In particular, depending on the controllability properties of the associated closed quantum system, new bounds for "dissipative" LKE are presented. In doing so, we will also touch the delicate issue of dissipation in open quantum system.

Quantum feedback for preparation and protection of quantum states of light

Igor Dotsenko

ENS, Paris

Aiming on preparation a given physical system in a particular quantum state, we can follow one of several standard approaches. The most straight-forward one is to expose a system to a specific coherent evolution that will deterministically bring it into a desired final state. However, in case of microscopic few-particle systems this approach requires very good control of many physical parameters at the single particle level. Another method of the state production is based on the projective quantum measurement of the system. Quantum mechanics postulates that the system will be then randomly projected onto one of the eigenstates of the measurement operator. Being less demanding this method is however inherently non-deterministic. Our approach relies on repetitive quantum measurement in combination with deterministic coherent and easy-to-realize evolution of the system's state. In this way we plan to realize a quantum feedback strategy. Our system under study is a microwave field confined in a high-quality cavity with a long lifetime of 130ms. In the current work we aim on generating small photon number (Fock) states of the trapped field. The measurement is realized by single atoms flying across the cavity mode one by one. They are prepared in a highly-excited Rydberg level and interact dispersively with the field, thus preventing absorption or emission of the field's photons. The detection of the final atomic state provides us with partial information on the photon number present in the cavity and thus projects the field into a new definite state. Next, in order to guide the field towards the desired Fock state, we calculate and inject into the cavity mode a small coherent field that increases the population in the desired photon number. The described steps constitute a feedback loop. Being repeated many times, they steer the field to the desired quantum state. In order to investigate the proposed feedback strategy, we have performed detailed theoretical study of the feedback convergence as well as extensive Monte-Carlo simulations taking into account all known and relevant parameters and imperfections of our experimental system. We have found that after 10ms of the feedback operation the fidelity e.g. of the target 3-photon state overcomes 50% and during first 100ms more that 95% of trajectories successfully converge. Moreover, our method shows the stability with respect to sudden quantum jumps of light due to its unavoided coupling to the environment (decoherence). As soon as a change in the photon number is noticed, the feedback corrects for it within about 20ms. Our next goal is the experimental realization and optimization of the proposed quantum feedback scheme. Recently we have launched a series of the corresponding experiment in our laboratory.

Approximate controllability for models of coupled Schrödinger equations modeling a single trapped ion and generalizations.

Sylvain Ervedoza

CNRS, Toulouse

In this talk, we analyze the approximate controllability properties for a system of Schrödinger equations modeling a single trapped ion. The control we use has a special form, which takes its origin from practical limitations. Our approach is based on the controllability of an approximate finite dimensional system proposed by Law-Eberly for which one can design explicitly exact controls. We then justify the approximations which link up the complete and approximate systems. This yields approximate controllability results in the natural space $L^2(\mathbb{R})^2$ and also in stronger spaces corresponding to the domains of powers of the harmonic operator. We also comment on possible extensions on the simplest physical spin-spring model for two trapped ions and some open problems concerning this latest system.

Optimization of quantum adiabatic algorithms

Martin Fraas

Technion, Haifa

In the theory of quantum adiabatic computation, one is interested in reaching a target state from a initial state with high fidelity, as quickly as possible, subject to given cap on the available energy. An algorithm is given by a Hamiltonian path. I will show a local optimization procedure for finding the optimal speed how to traverse the path. The procedure is based on the joint work with Y.Avron, G.M.Graf and P.Grech.

Local controllability of 1D linear and nonlinear Schrödinger equation

Camille Laurent

CMAP, Ecole Polytechnique, Paris

In this talk, we will discuss about the exact local controllability of the Schrödinger equation on an interval. Thanks to a "regularizing effect", we are able to prove the bilinear controllability with a proof much simpler than the previous one of K. Beauchard. This proof is enough robust to be extended to the nonlinear Schrödinger and wave equation. This is a joint work with Karine Beauchard.

Feedback preparation of quantum entangled states

Mazyar Mirrahimi

INRIA Rocquencourt

Two particles are said to be in an entangled state if the result of some measurement on one determines the result of some measurement on the other. These states, which play a central role in the quantum information theory, are, in general, very hard to obtain in experiments. In this talk, I will consider the preparation of these states for a system consisting of two qubits. I will show that a collective measurement of the qubits together with individual controls on each of them must allow us to prepare, in a deterministic way, any arbitrary entangled state.

Global exact controllability in infinite time for Schrödinger equation

Vahagn Nersesyan

Versailles Saint-Quentin

This talk is devoted to the study of the controllability of Schrödinger equation. We prove that the system is exactly controllable in infinite time to any position. The proof is based on an inverse mapping theorem for multivalued functions. We show also that the system is not exactly controllable in finite time in lower Sobolev spaces.

Adiabatic methods in Quantum Control theory

Gianluca Panati

La Sapienza, Roma

The separation between slow and fast degrees of freedom has always played an important role in the understanding of complex dynamics. At the level of Quantum Mechanics, the goal is to approximate the solution to the original Schrödinger equation with the solution to a simpler equation, involving the slow degrees of freedom alone (adiabatic decoupling, or adiabatic reduction).
After reviewing the classical adiabatic theorems, I will illustrate an algorithmic procedure to obtain the effective Hamiltonian to higher degrees of accuracy.
Finally, I will discuss how the adiabatic methods, combined with a good control on the structure of eigenvalue crossings, might be useful to Quantum Control Theory.

The impulsive control of Rydberg electron wavepackets

Chitra Rangan

Windsor

Multi-level quantum systems such as alkali atoms have long been as a testbed for the study of classical-to-quantum transitions. I will describe the theory of the control of electron dynamics in highly-excited (Rydberg) atoms on picosecond (10^{-12}s) time scales. Using a single teraherz Half-Cycle Pulse (HCP) as an impulsive (delta function) control, it is possible to perform quantum algorithms such as searching. By using two HCPs, we devise a robust information hiding and retrieval scheme with the relative phases between states in a Rydberg wave packet acting as the bits of a data register. Finally I will discuss a recent study where two time-delayed HCPs can be used to assist ionization and recombination of Rydberg electrons in hydrogen. The latter two studies demonstrate the classical and quantum behaviours of the same control problem.

Cubic Schroedinger equation: controllability in projections by small-dimensional source term

Andrey Sarychev

Florence

In this talk we present results of recent work on controllability in projections of cubic Schroedinger equation on 2D torus with control entering as a source term, which affects few modes. We use method of iterated Lie extensions for establishing criteria of controllability in finite-dimensional projections.

Geometric optimal control of spin systems with applications in Nuclear Magnetic Resonance

Dominique Sugny

Dijon

We apply recent developments in geometric optimal control to analyze the optimal control of spin systems in Nuclear Magnetic Resonance. We consider the case where the spins are in interaction with their environment, their dynamics being governed by the Bloch equation. Geometric optimal control is a vast domain based on the application of the Pontryagin Maximum Principle (PMP) where the idea is to use the methods of differential geometry and Hamiltonian dynamics to solve the optimal control prob- lems. This geometric framework leads to a global analysis of the control problem which completes and guides the numerical computations. We will consider different basic problems of NMR as the saturation of a spin 1/2 particle in minimum time [1, 2, 3, 4], the energy minimization problem of a spin [6] or the simultaneous control of the inversion of two spins with different detunings [5]. The robustness of the optimal control laws with respect to initial conditions and dissipative parameters will be analyzed. The geometric optimal solutions will be also compared to purely numerical optimal control laws computed with the GRAPE algorithm [7]. Finally, experimental results using techniques of Nuclear Magnetic Resonance will illustrate this theoretical work.

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Local controllability of a BEC in a time-varying box.

Holger Teismann

Acadia University

In this talk we consider the "condensate-in-time-varying-box" problem in one space dimension, Taking the length of the box to be the control, we show that the nonlinear Schroedinger equ. is controllable in the vicinity of the nonlinear ground state. This is joint work with Karine Beauchard and Horst Lange.

Multi-polarization quantum control of rotational motion

Gabriel Turinici

Dauphine, Paris

In this talk we present recent results concerning the control of quantum rotational motion under the influence of an external laser field coupled through a permanent dipole moment. The analysis takes into consideration up to three polarization fields, but we also discuss the consequences for working with fewer polarized fields.