Geometry of Quantum Transport

Yosi Avron, Martin Fraas, Gian Michele Graf, Oded Kenneth

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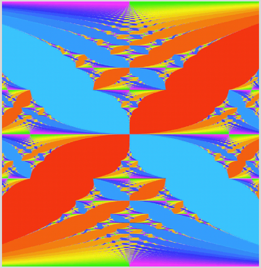
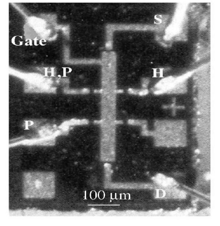
1. Outline

- Motivation: QHE, adiabatic transport in open q-system
- Control and Response
- Geometry: ω Symplectic structure, g metric
- Main result: $f^{-1} = \gamma g + \omega, \ \gamma = \textit{dephasing}$
- Lindbladians and dephasing
- Adiabatic evolutions
- Kähler structure
- Examples

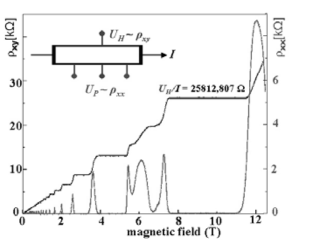


2. Motivation: Quantum Hall effect

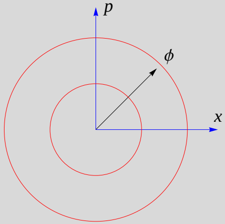
- Ill characterized microscopically
- Quantized Hall resistivity $\frac{h}{e^2 n}, \quad n \in \mathbb{Z}$
- Accurate to 12 significant digits



- Resolution: Integer is a Chern number of g.s bundle $P(\phi)$ in Hilbert space
- Assumption: $\epsilon\dot{\rho} = -i[H(\phi), \rho]$ Unitary evolution
- Adiabatic theory: $\rho \approx P$
- What about open q-system?



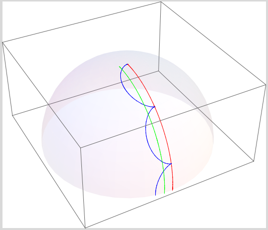
3. Response and control



- Controls: $\phi = (\phi_p, \phi_x)$;
- Driving= control rates $\dot{\phi}$
- Response: $\nabla_\phi H = (\partial_{\phi_p} H, \partial_{\phi_x} H)$
- Example 1: Harmonic oscillator
  $H(\phi) = \frac{1}{2}(p - \phi_p)^2 + \frac{1}{2}(x - \phi_x)^2$

  controls=(momentum, position), response=(velocity,force)
- Example 2: Spin in magnetic field $\phi = \hat{B}, \quad H(B) = \hat{B} \cdot \sigma$
- Control=Orientation of $\hat{B}$,

  Response= Magnetic moment



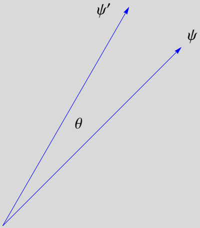
4. Geometry: Metric and symplectic structure

- Suppose $P(\phi)$ is the ground state bundle of $H(\phi)$
- Example spin 1/2: $P(\phi) = \frac{1-\hat{B}\cdot\sigma}{2}, \quad \phi = \hat{B}$
- Fubini-Study metric on control space

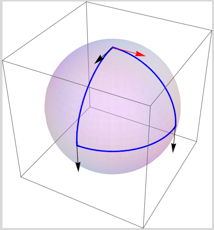
$$g_{\mu\nu}(\phi) = \mathrm{Tr}\; P_\perp \{\partial_\nu P, \partial_\mu P\}$$

- Symplectic structure on control space

$$\omega_{\mu\nu}(\phi) = i\,\mathrm{Tr}\; P_\perp [\partial_\nu P, \partial_\mu P]$$



- Endows control space with geometry
- Geometry of q-origin



5. Adiabatic transport coefficients

- Adiabatic evolutions $\epsilon\dot{\rho} = \mathcal{L}(\rho), \quad \epsilon \to 0$
- Transport coefficients $f_{\mu\nu}$
- Response & driving: $\mathrm{Tr}(\rho\partial_\mu H) = \cdots + f_{\mu\nu}\dot{\phi}_\nu + \dots$

- Geometry of q-origin