Quantum feedback for preparation and protection of quantum states of light

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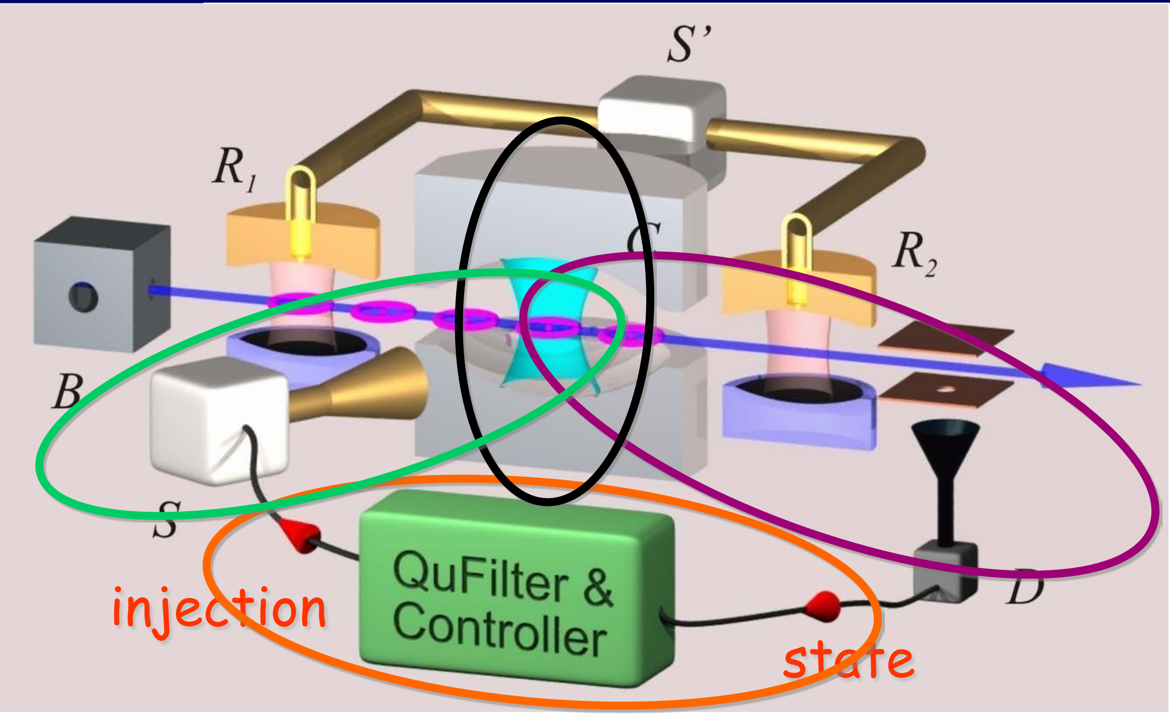
Cavity QED quantum feedback scheme



Goal:

- **Steering the trapped microwave field (harmonic oscillator) to a desired quantum state**
- **Preserving this state from decoherence**

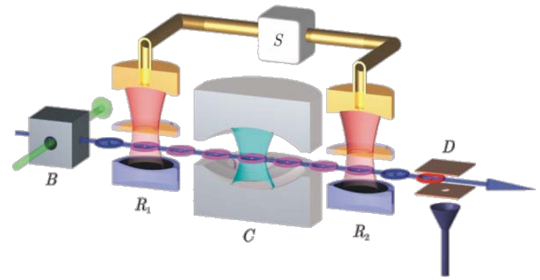
Elements of feedback loop

- **Quantum measurement:** performed with (spin ½) atoms followed by cavity state estimation
- **Quantum filter:** estimation of what is best to do for becoming closer to the target
- **Actuator – Classical:** microwave injection with a classical source

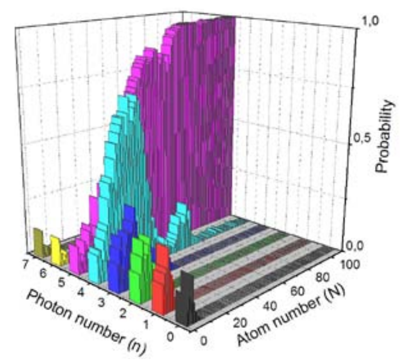
  Quantum: resonant interaction with a single two-level atom

Outline

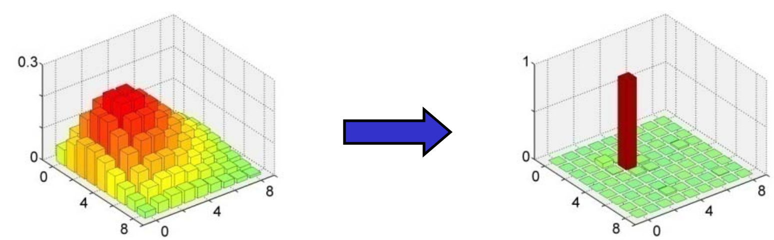
- Cavity QED setup



- Quantum non-demolition measurement



- Quantum feedback proposal:



generation of photon-number states

Microwave superconducting cavity: Storage box for photons

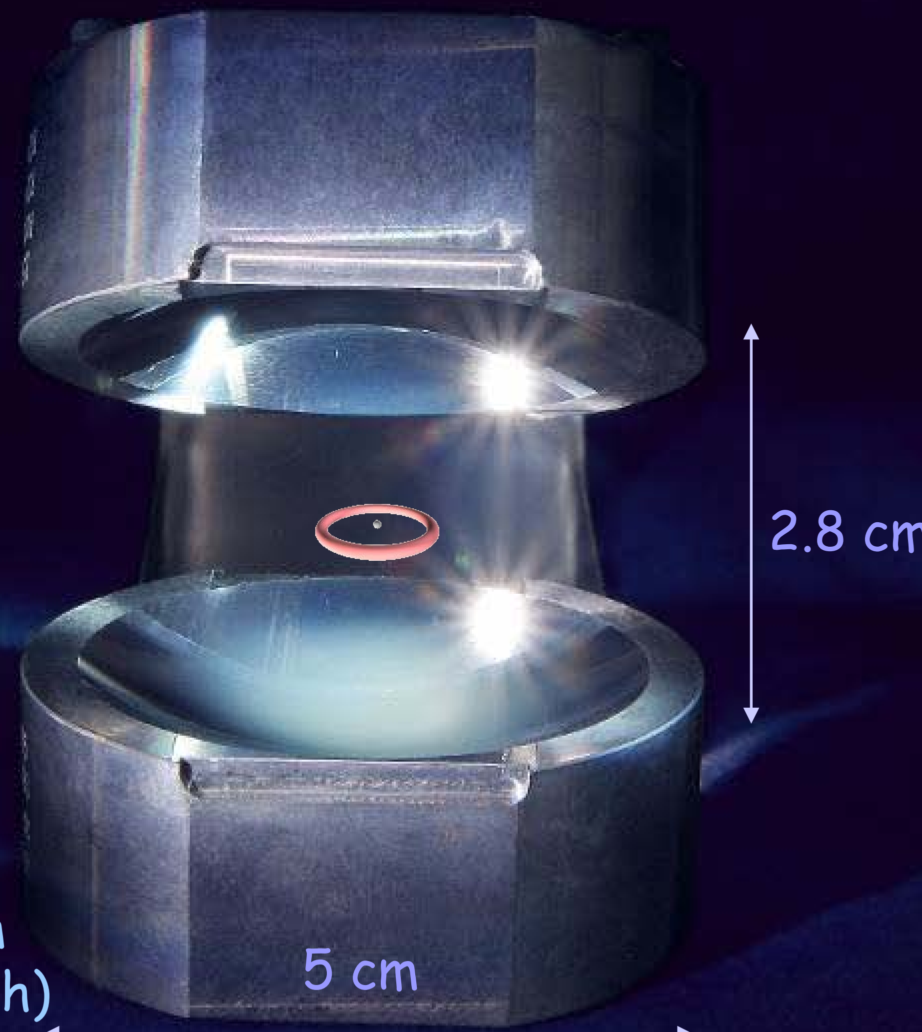
- Resona
- Lifeti ns
- Q fac

$$T_{\mathrm{cav}} = 130\,\mathrm{ms}$$

- best Fabry-Pérot resonator so far

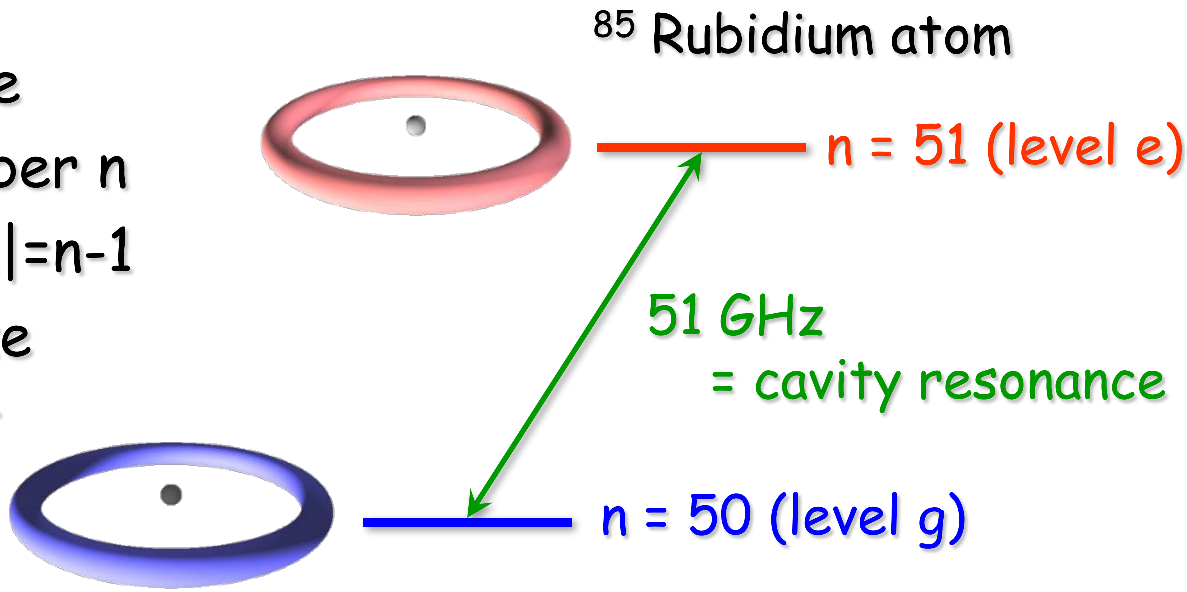
- 1.4 billion bounces on the mirrors

- a light travel distance of 39 000 km
  (one full turn around the Earth)



Circular Rydberg atoms: Field microprobes

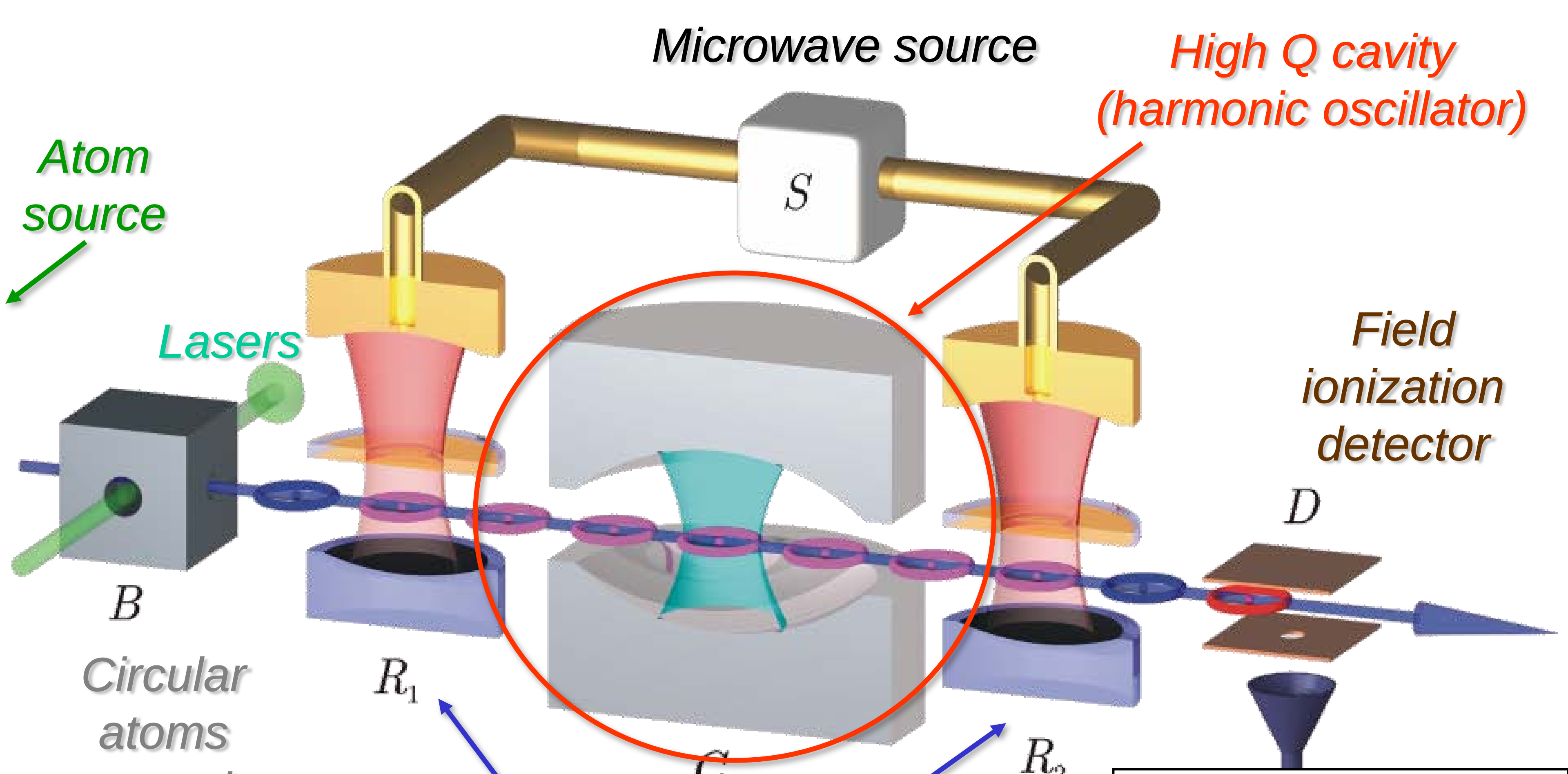
- Rydberg atoms: large principle quantum number n
- Circular states: l=|m|=n-1
- Mesoscopic orbit size
- Large dipole moment



Advantages:

- Almost ideal two-level system
- Long lifetime (30 ms)
- Tunable via the Stark effect
- Large coupling to radiation (orbit diameter of 0.25 μm)
- Efficient state sensitive detection by ionization

Meeting atoms and photons

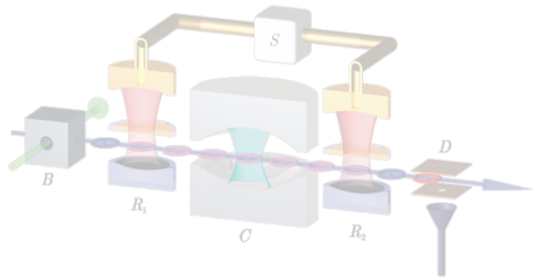


Low Q cavities: classical field pulses
(Ramsey interferometer to manipulate spin ½ ator

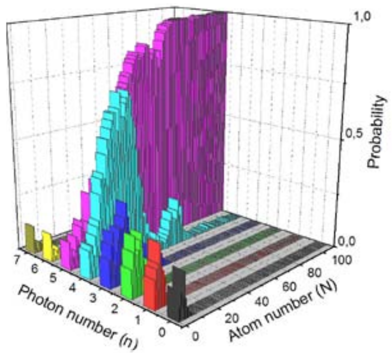
cooled to 0.8 K; thermal, acoustic, magnetic & electric isolation

Outline

- Cavity QED setup

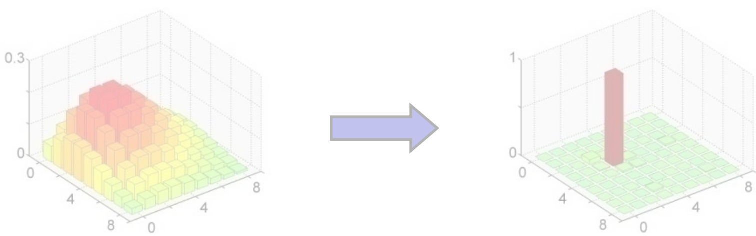


- Quantum non-demolition measurement

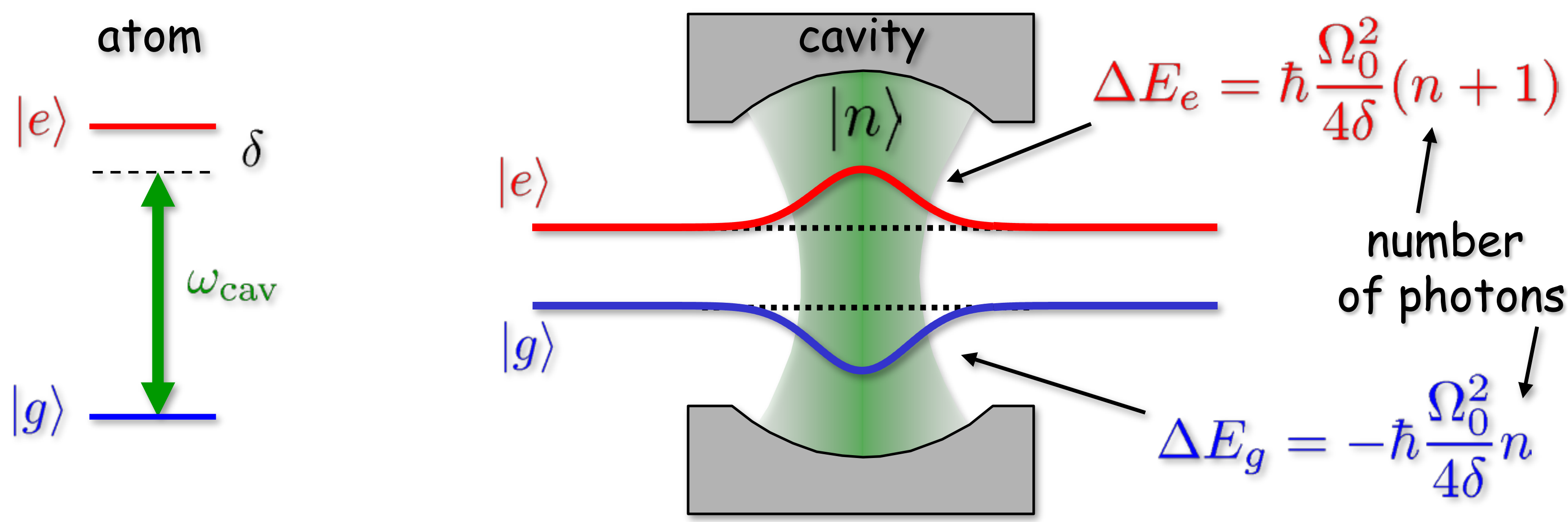


- Quantum feedback proposal:

generation of photon-number states



Dispersive interaction



Phase shift of atomic coherence (light shift)

$$\varphi(n) = (n + 1/2)\varphi_0$$

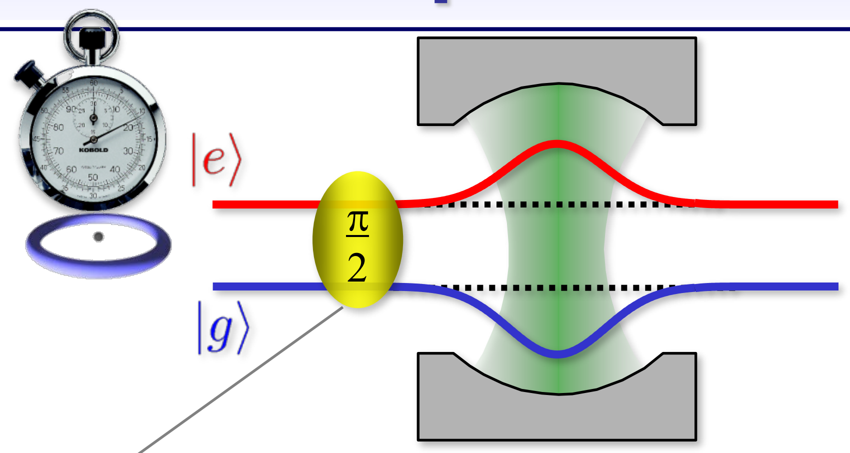
$$\varphi_0 = \frac{\Omega_0^2}{2\delta} t_{\text{int}}$$ phase shift per photon

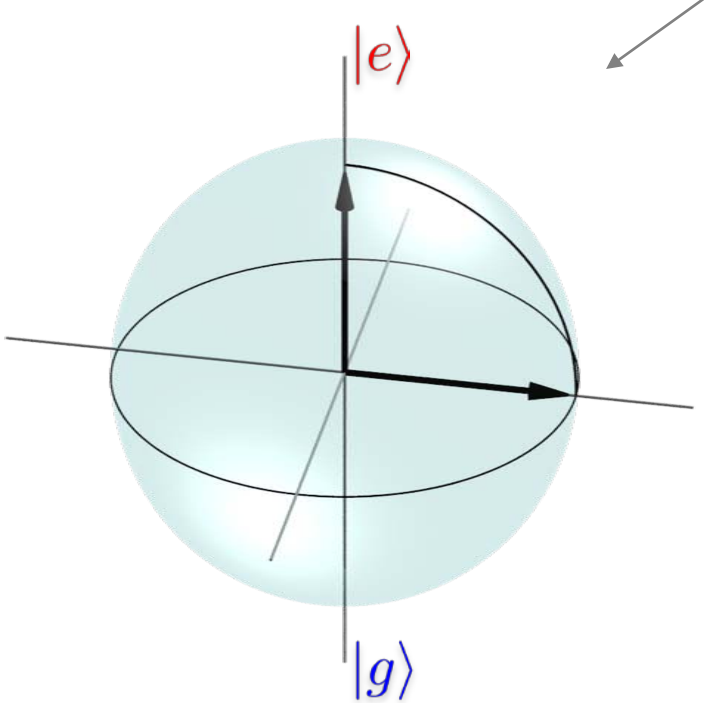
Energy conservation + adiabatic coupling

$\Rightarrow$ the field (i.e. photon number) is preserved

QND measurement of photon number

1. Trigger of the atom clock:
resonant $\pi/2$ pulse



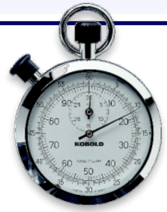


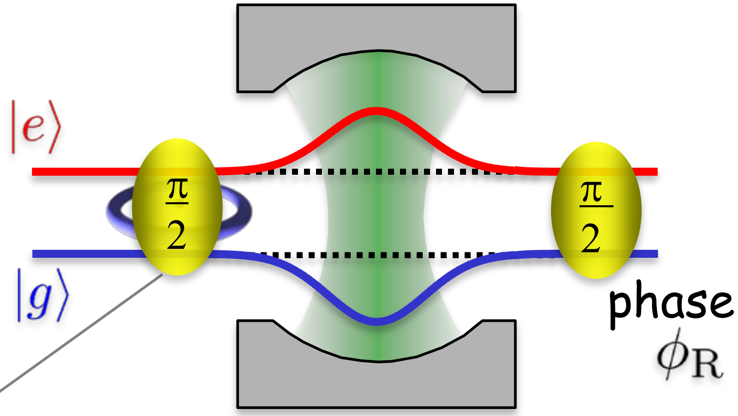
Bloch vector representation
for spin $\frac{1}{2}$ particle

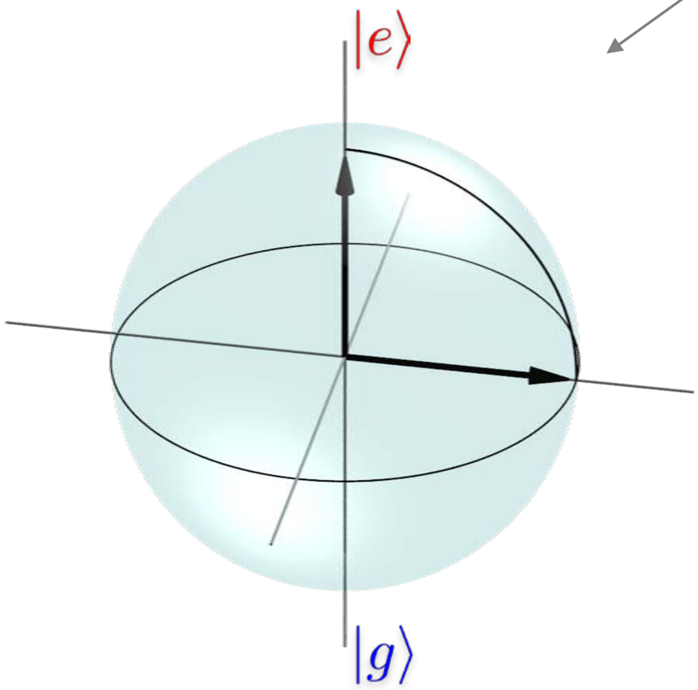
QND measurement of photon number

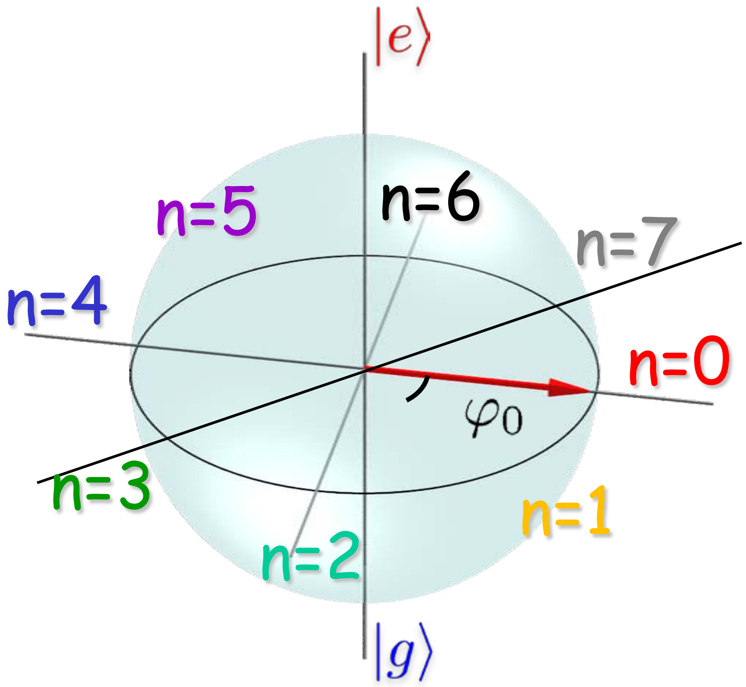
1. Trigger of the atom clock: resonant $\pi/2$ pulse

2. Dephasing of the clock: interaction with the cavity field

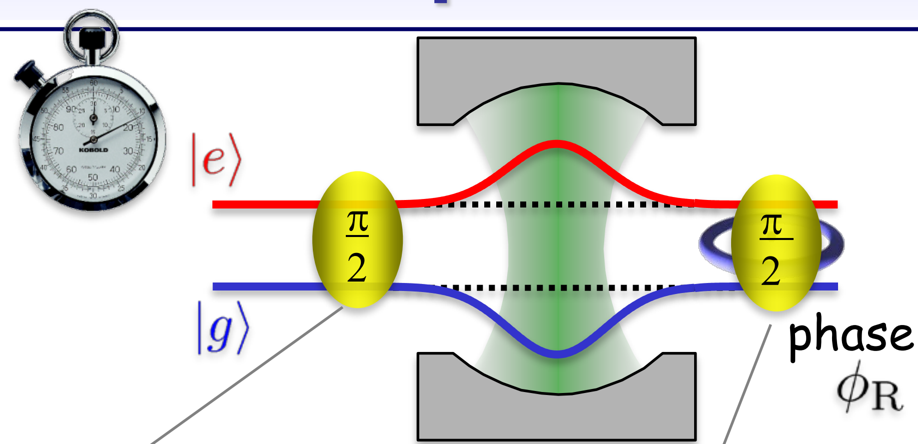








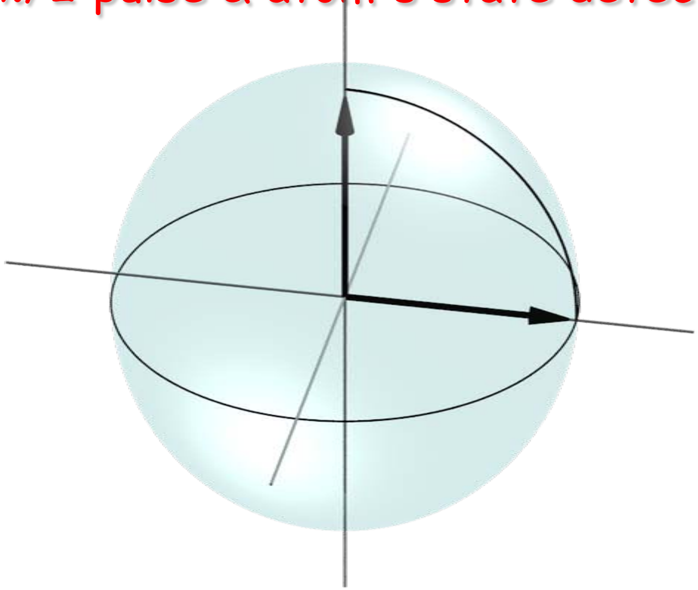
QND measurement of photon number

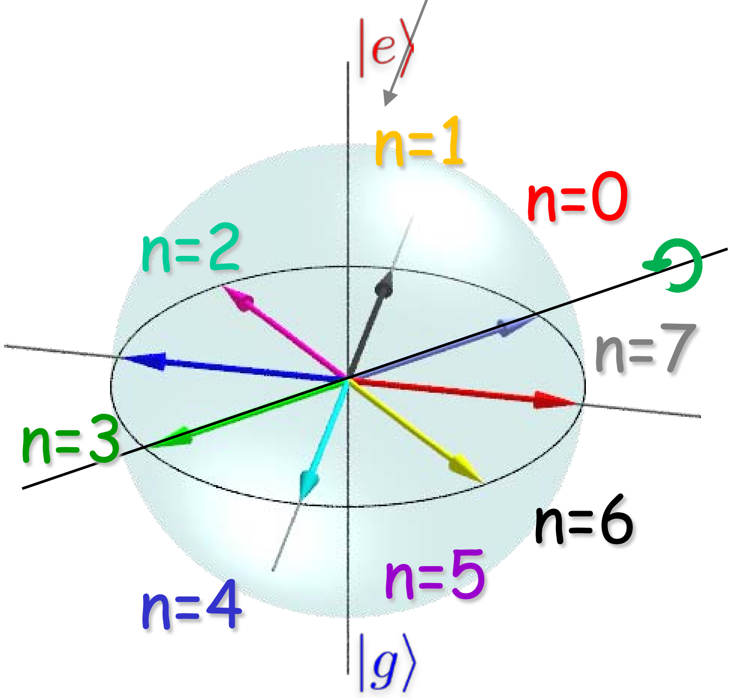


1. Trigger of the atom clock: resonant $\pi/2$ pulse

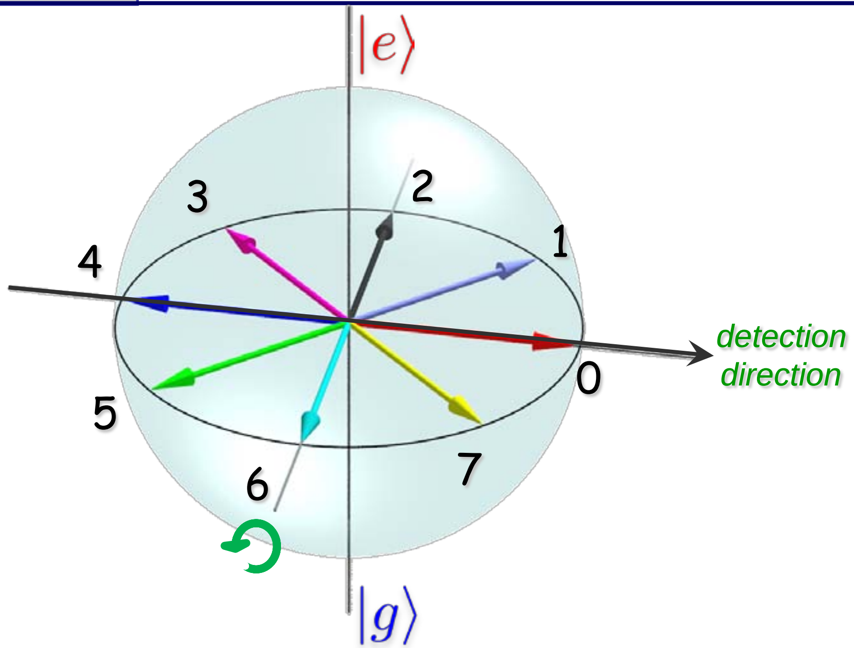
2. Dephasing of the clock: interaction with the cavity field

3. Measurement of the clock: second $\pi/2$ pulse & atom's state detection

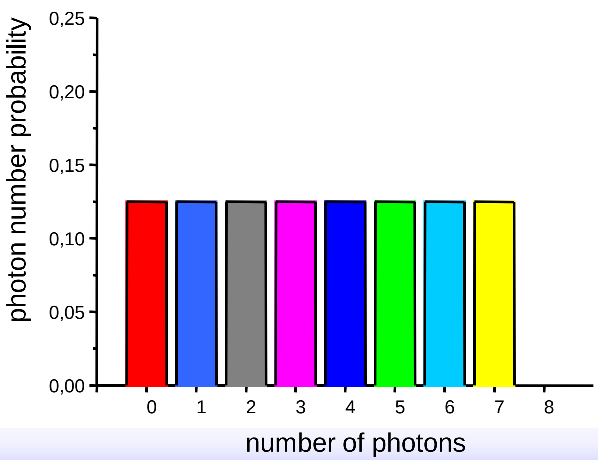




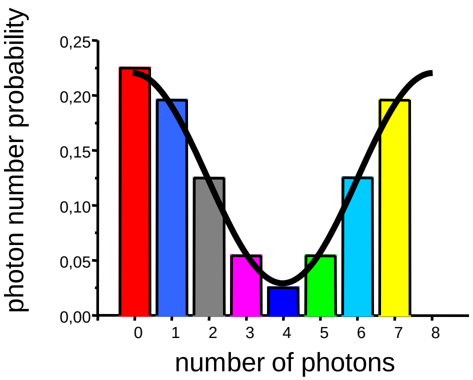
Single atom detection



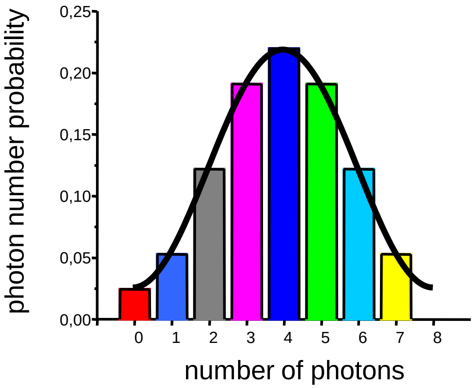
initial knowledge



atom detection changes photon-number distribution

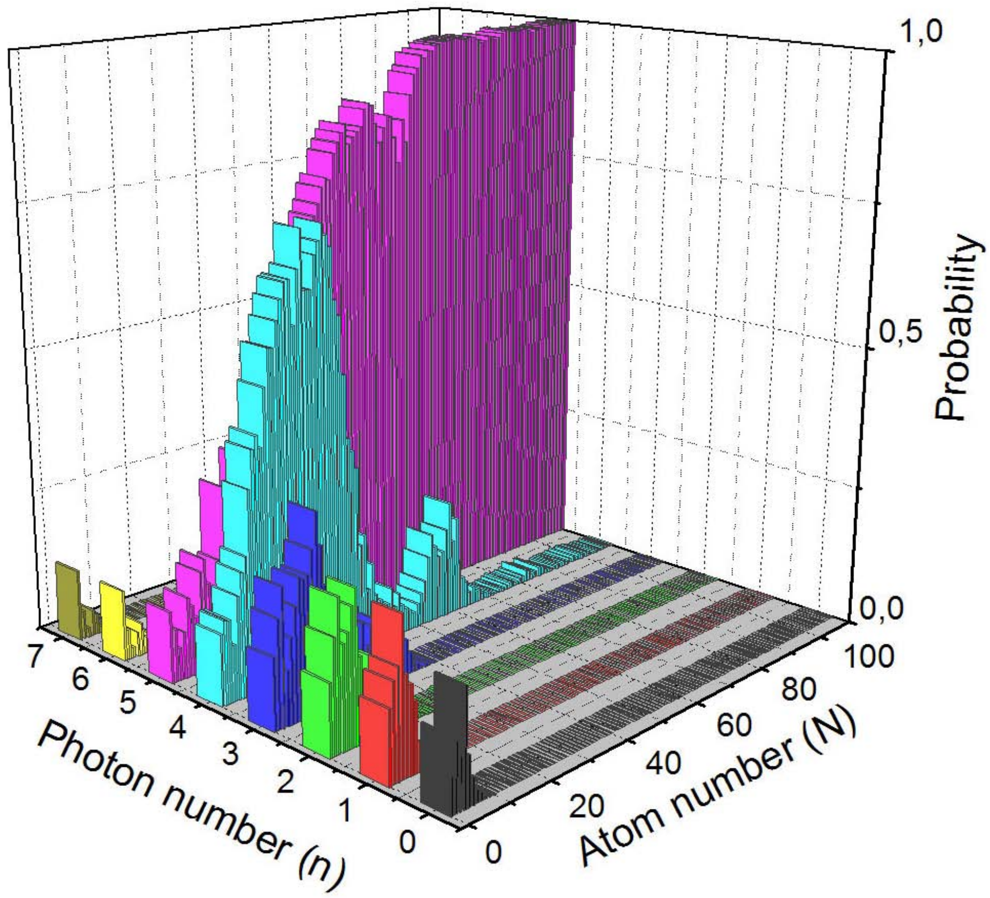


atom in |***e***⟩



atom in |***g***⟩

From weak to projective measurement

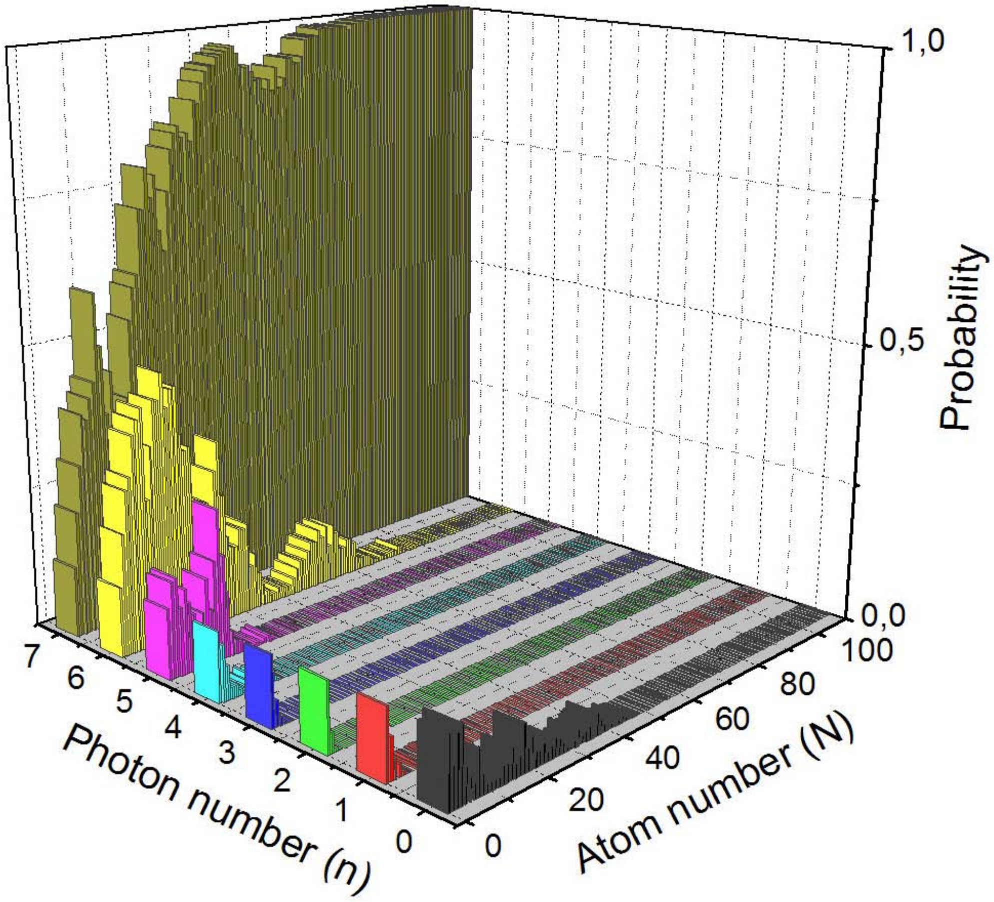


• Initial coherent field with 3.7 photon

• Progressive collapse of the field state vector during information acquisition

Many repeated **weak** measurements result in the ideal **projective** measurement of the photon number

Another sequence



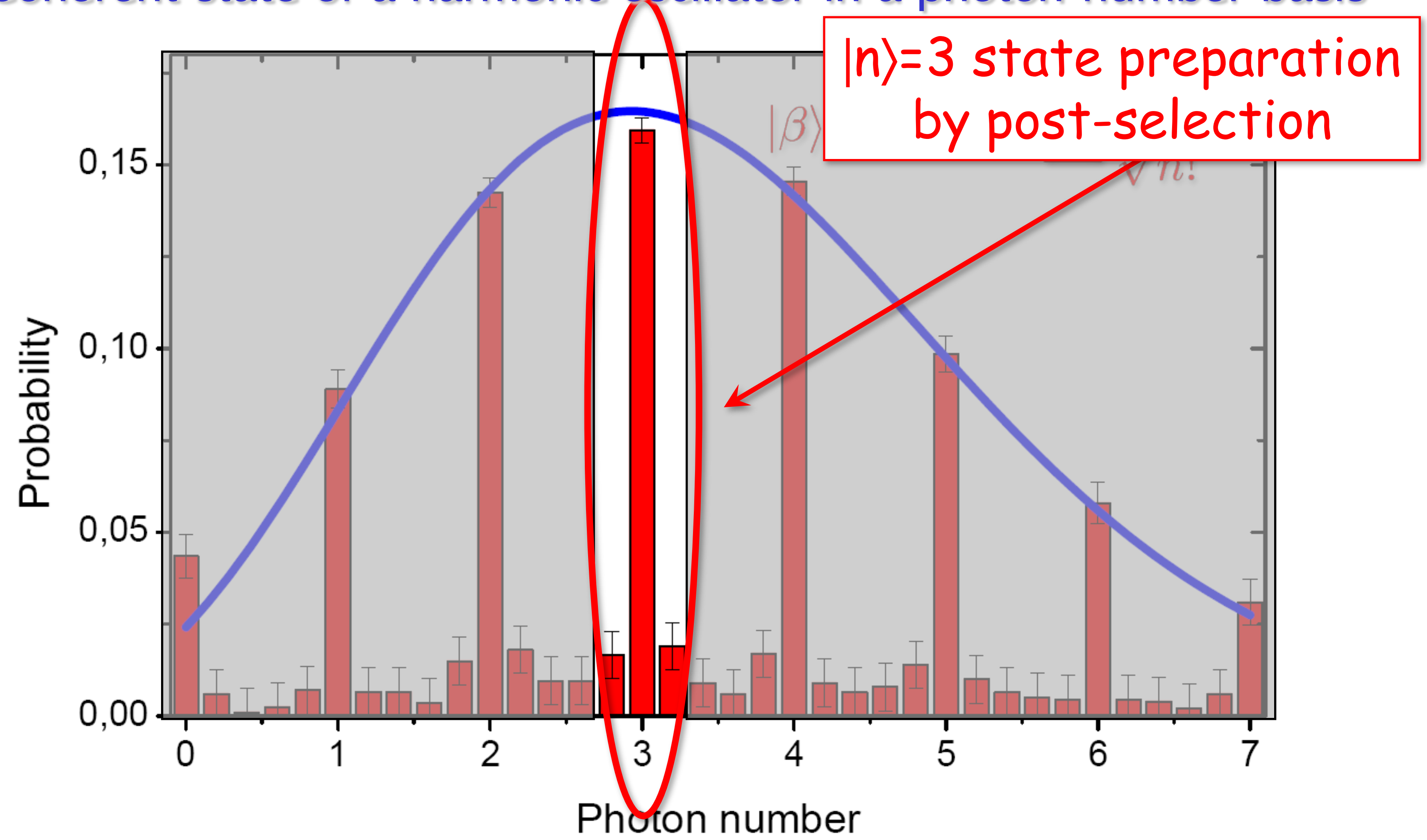
- Final photon number fluctuates randomly from sequence to sequence

- Statistics of final photon number should reveal the statistics of the initial quantum field

Many repeated **weak** measurements result in
the ideal **projective** measurement of the photon number

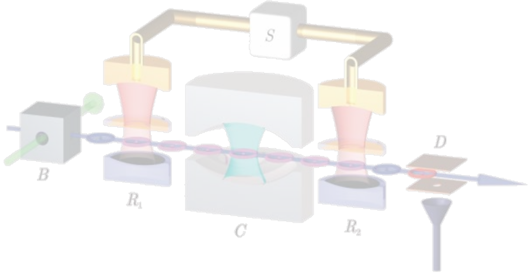
Photon number statistics

Coherent state of a harmonic oscillator in a photon-number basis

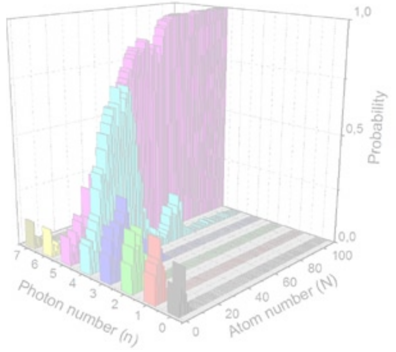


Outline

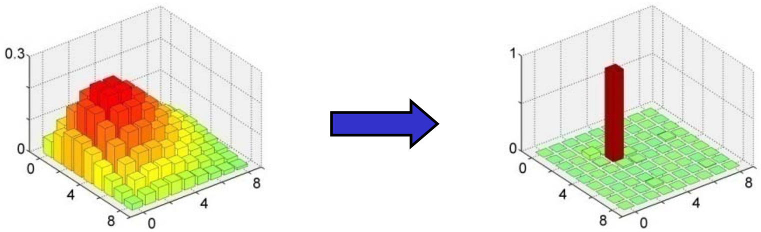
- Cavity QED setup



- Quantum non-demolition measurement

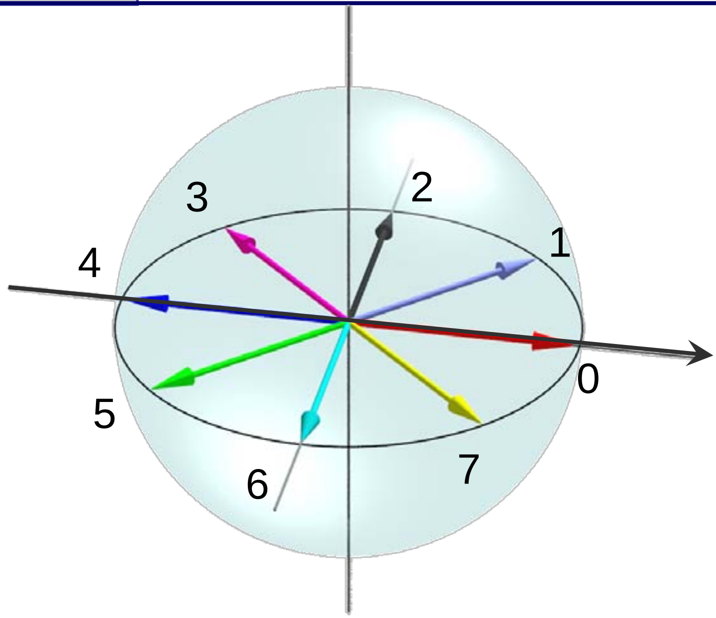


- Quantum feedback proposal:



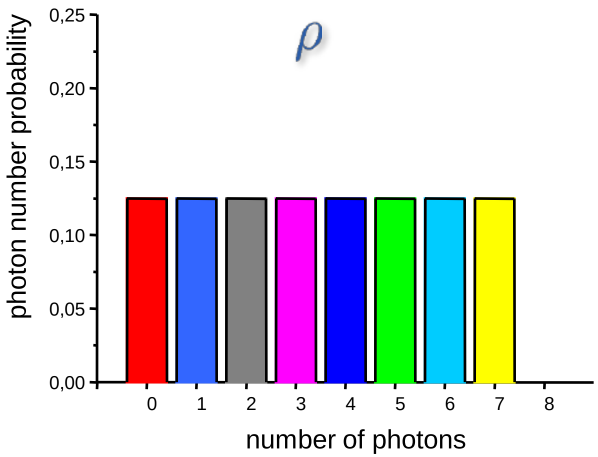
generation of photon-number states on demand

Single atom measurement



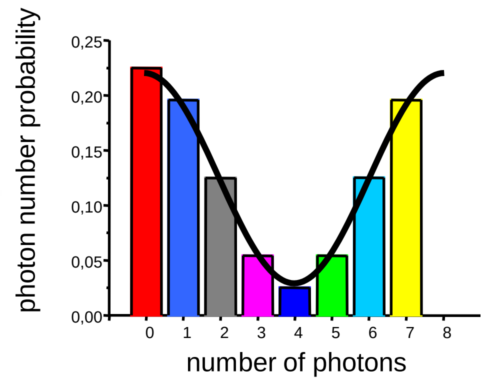
initial field

ρ

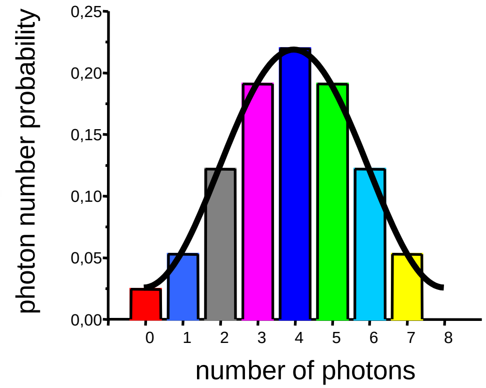


atom detection changes
photon-number distribution

ρ_{proj}



atom in $|e\rangle$



atom in $|g\rangle$

Back-action of weak measurement

initial state

$$\rho_{\text{proj}} = \frac{M_i\,\rho\,M_i^\dagger}{\mathrm{Tr}(M_i\,\rho\,M_i^\dagger)}$$

projected state

detection direction

$$M_e = \sin\left(\frac{\phi_{\mathrm{R}} + \phi(N)}{2}\right)$$

photon number operator

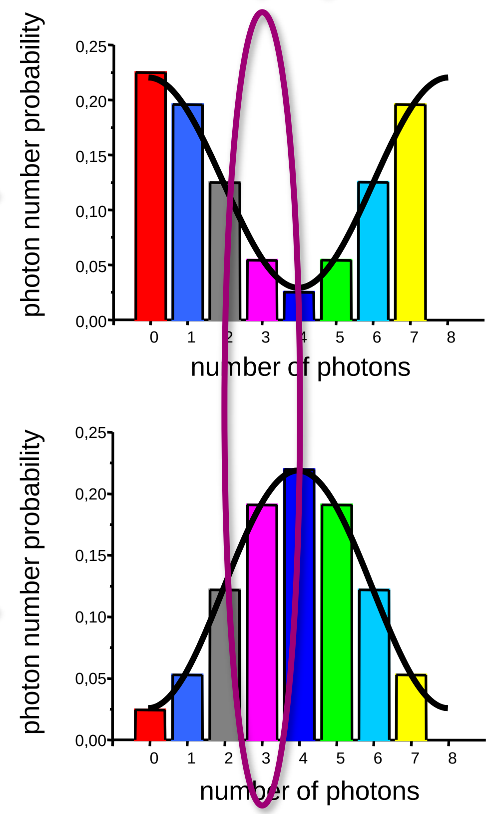
Two POVMs correspond to two possible experimental outcomes

phase shift

$$M_g = \cos\left(\frac{\phi_{\mathrm{R}} + \phi(N)}{2}\right)$$

atom detection changes photon-number distribution

ρ_{proj}



atom in |***e***⟩

⇒ bad

atom in |***g***⟩

⇒ good

on average, good/bad outcomes are equally probable

Back-action of weak measurement

initial state

$$\rho_{\text{proj}} = \frac{M_i\,\rho\,M_i^\dagger}{\text{Tr}(M_i\,\rho\,M_i^\dagger)}$$

projected state

detection direction

$$M_e = \sin\left(\frac{\phi_{\text{R}} + \phi(N)}{2}\right)$$

photon number operator

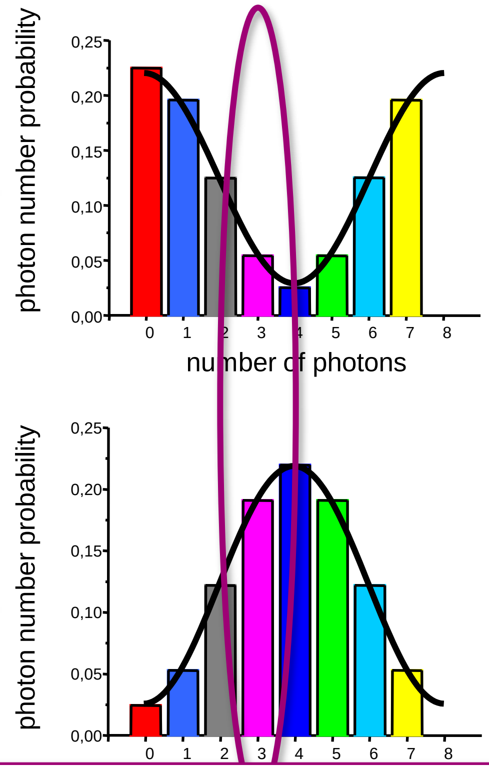
Two POVMs correspond to two possible experimental outcomes

phase shift

$$M_g = \cos\left(\frac{\phi_{\text{R}} + \phi(N)}{2}\right)$$

Deterministically prepare state |n⟩=3 ?

ρ_{proj}



atom in |***e***⟩

⇒ bad

atom in |***g***⟩

⇒ good

Idea: Let us alter the distribution, *i.e.* increase P(n=3), depending on measurement outcome before the next measurement

Field displacement as feedback control

We modify the photon-number distribution by displacing the field's state:

displacement operator $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$: *injection of a coherent pulse into the cavity*

$$\rho_{\text{disp}} = D(\alpha)\,\rho_{\text{proj}}\,D(-\alpha)$$

displacement amplitude: complex amplitude of the injection pulse

Displacement amplitude is chosen to maximize the fidelity to the desired photon number (*i.e.* population of this state):

$$F = \mathrm{Tr}(\rho_{\text{disp}}\,\rho_{\text{target}})$$

desired photon number state

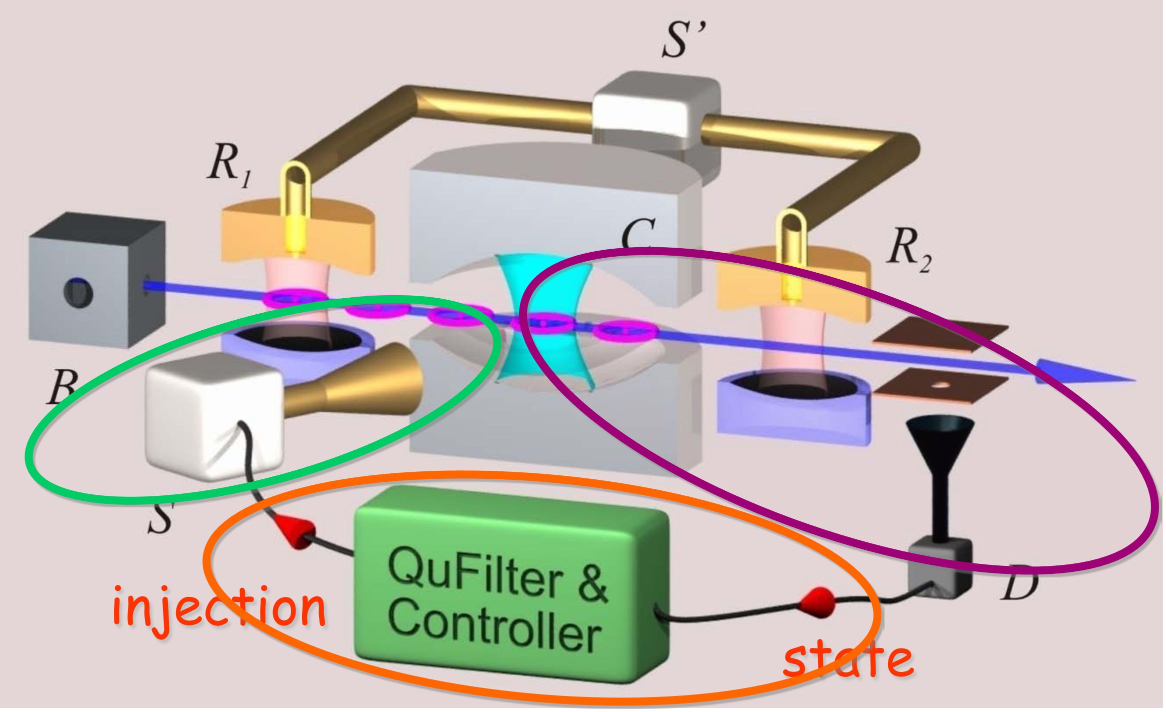
Efficient feedback law (using Lyapunov function approach) reads:

$$\alpha = c\,\mathrm{Tr}\big([\rho_{\text{target}}, a^\dagger - a]\rho_{\text{proj}}\big)$$

optimal gain

$$c = \mathrm{Tr}\big([\rho_{\text{target}}, a^\dagger - a]^2\big)^{-1} = (4n_{\text{target}} + 2)^{-1}$$

Proposal: Quantum feedback loop



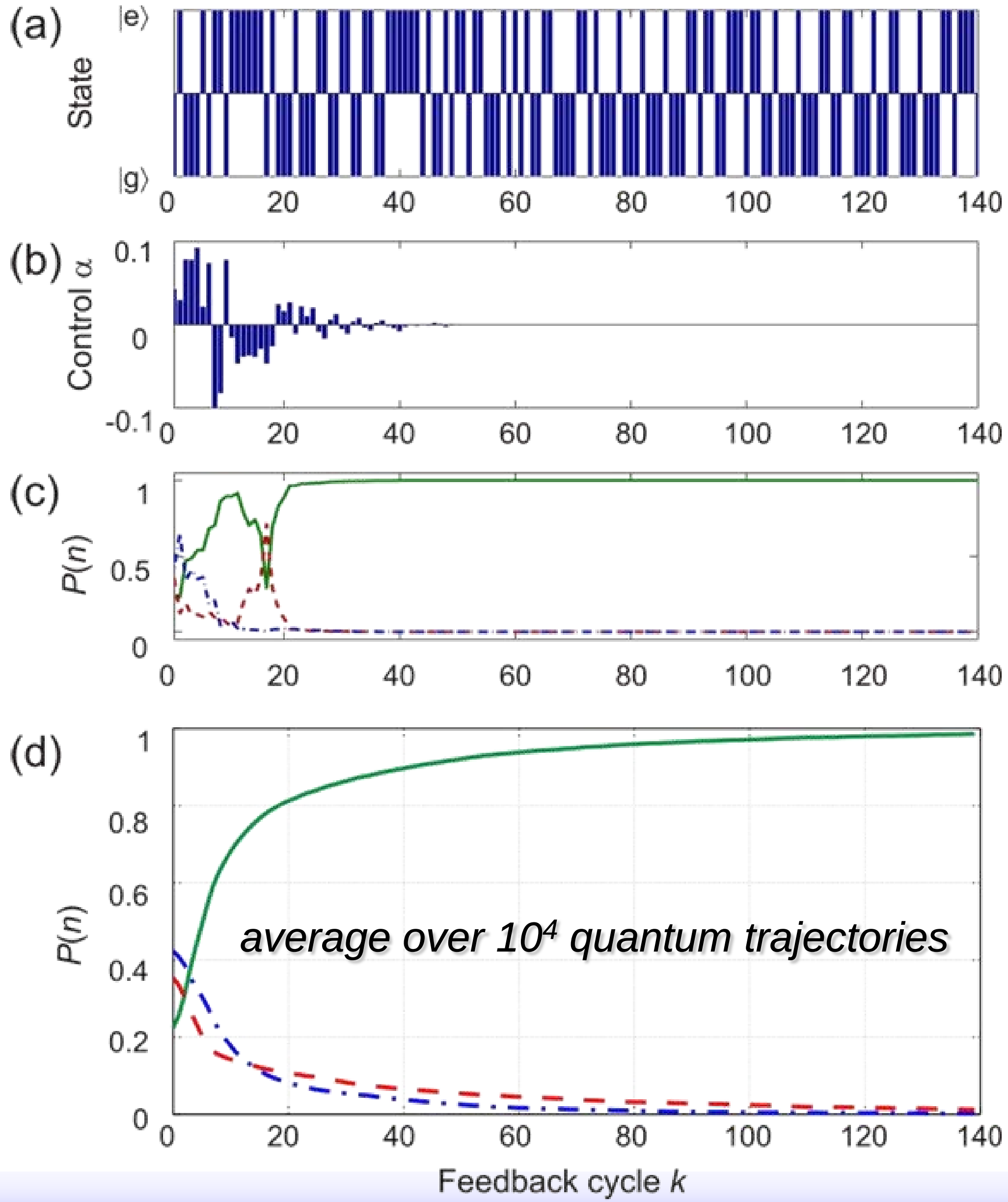
Standard closed-loop components:

- **Sensor** (quantum):
  atoms and QND measurement
- **Controller** (classical):
  classical computer
- **Actuator** (classical):
  microwave injection

Feedback protocol:

- Inject initial coherent field into the cavity
- Send one-by-one atoms in a Ramsey configuration
- Detection of each atom projects cavity field ρ into a new state ρ_{proj}
- Calculate displacement α, which maximizes overlap F between ρ_{target} and ρ_{disp}
- Close feedback loop by injecting a control coherent field $|\alpha\rangle$
- Repeat feedback cycles until success when $F \approx 1$

Feedback performance: ideal case



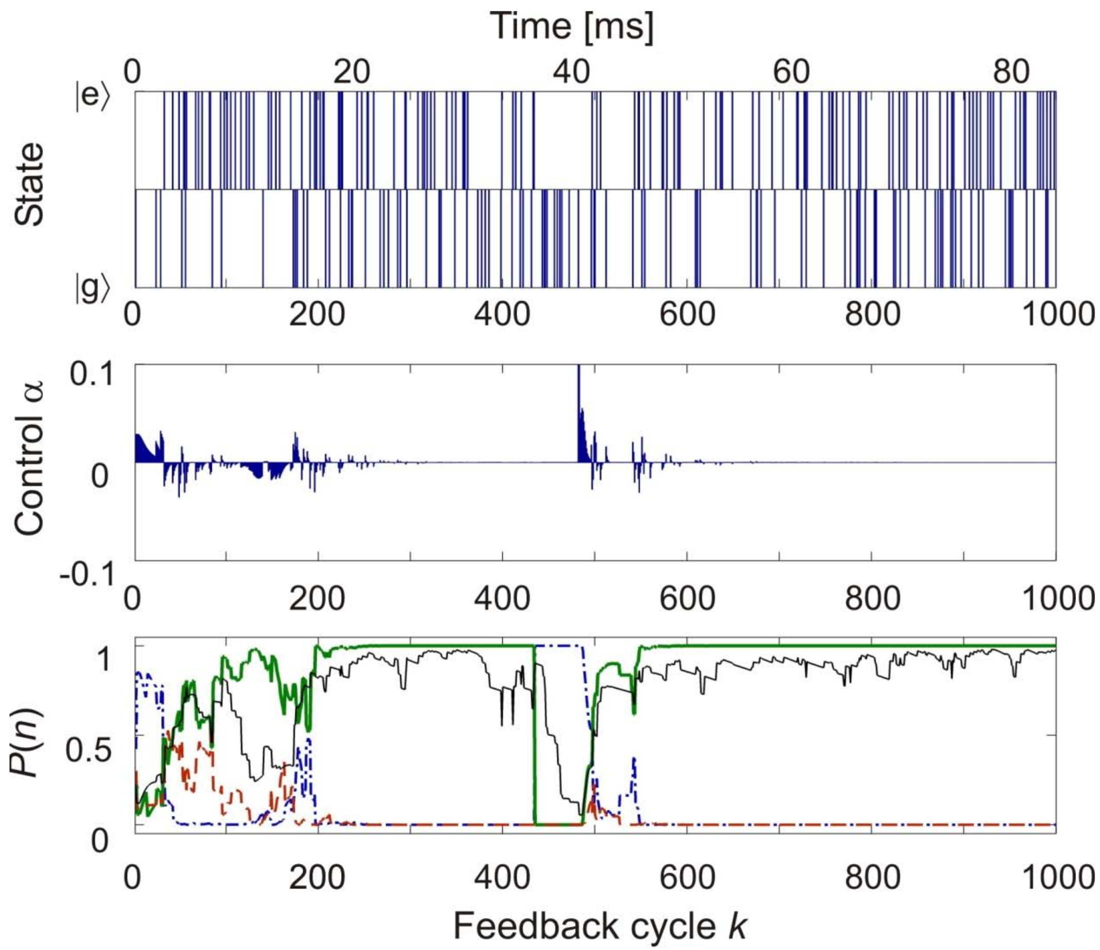
Monte-Carlo simulation with n_{target} = 3 photons

$n > n_{target}$

$n = n_{target}$

$n < n_{target}$

Feedback performance: realistic case

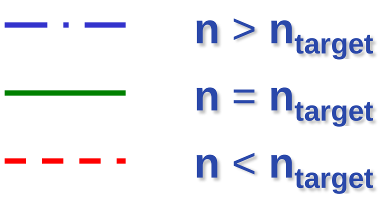


Simulation parameters:

- average number of atoms per pulse: 0.4
- detection efficiency: 80%
- false state detection: 10%

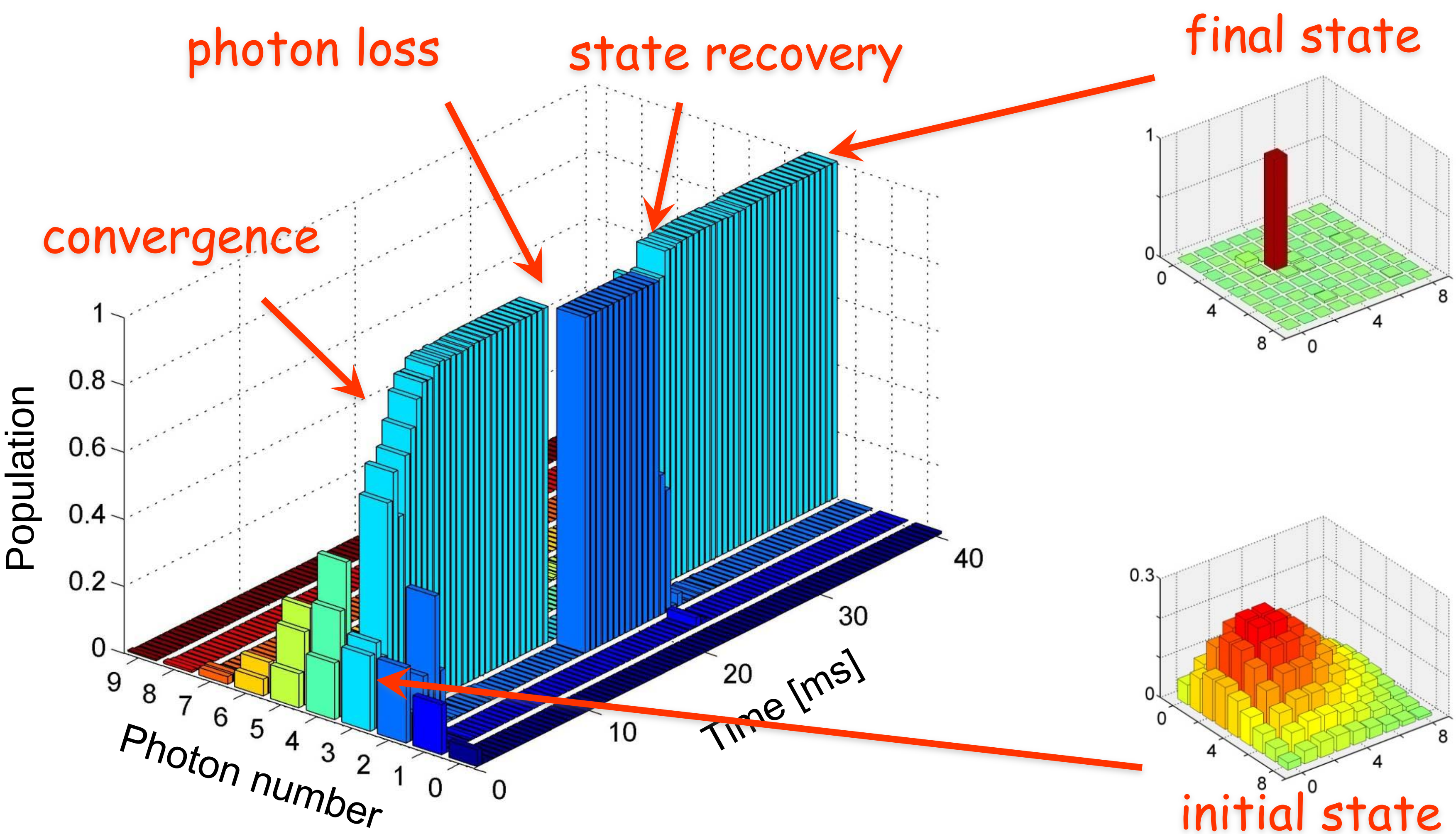
- cavity decay time: 130 ms
- separation of atomic pulses: 100 μs
- delay in atom detection of 4 atoms
- black-body thermal field: 0.05 photons

- target Fock state: |3⟩
- Hilbert space size: 10



Feedback performance

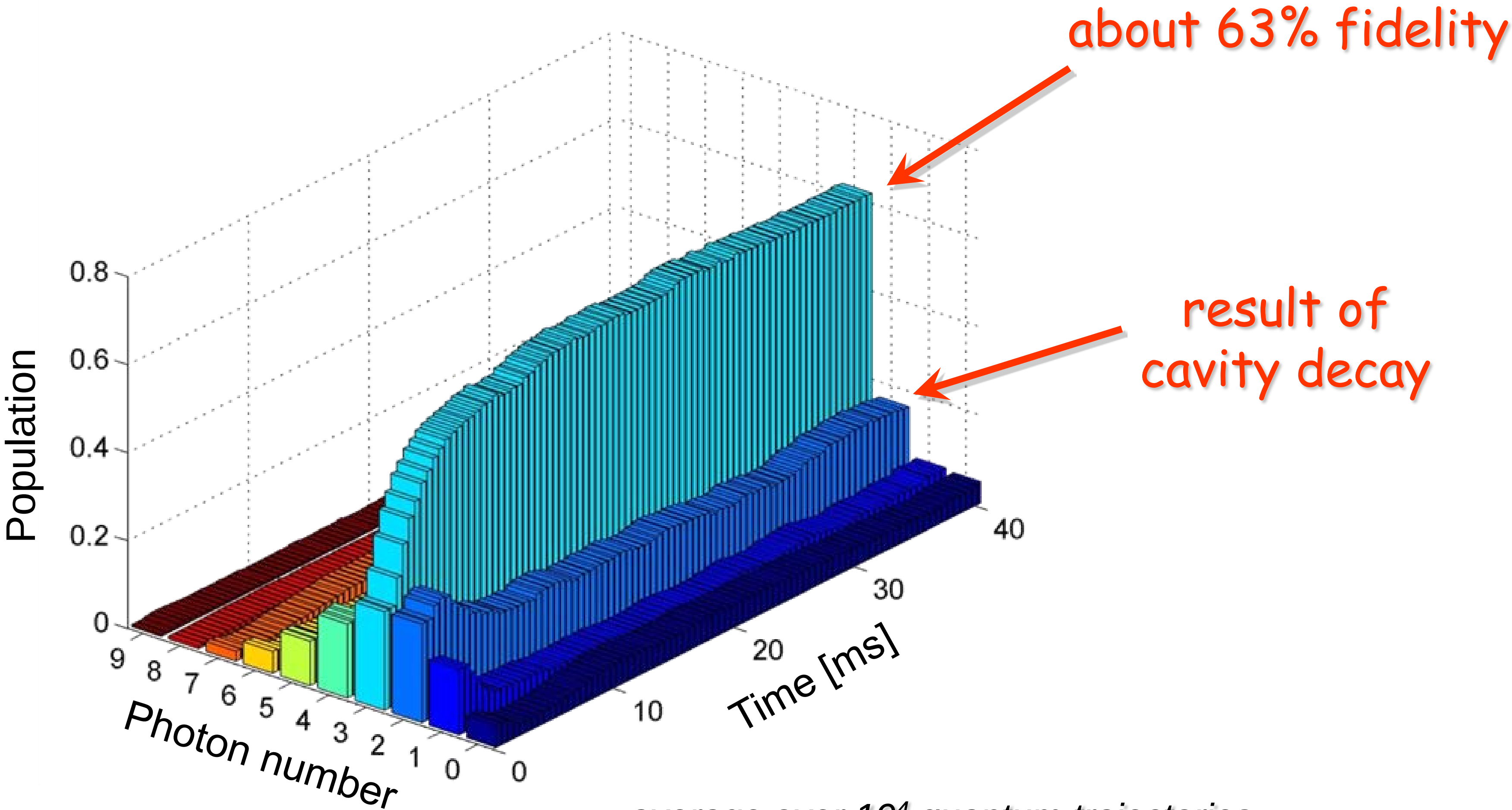
Simulation results for a target Fock state $\rho_{target} = |3\rangle$



about 3 atoms in 1 ms

Average over many trajectories

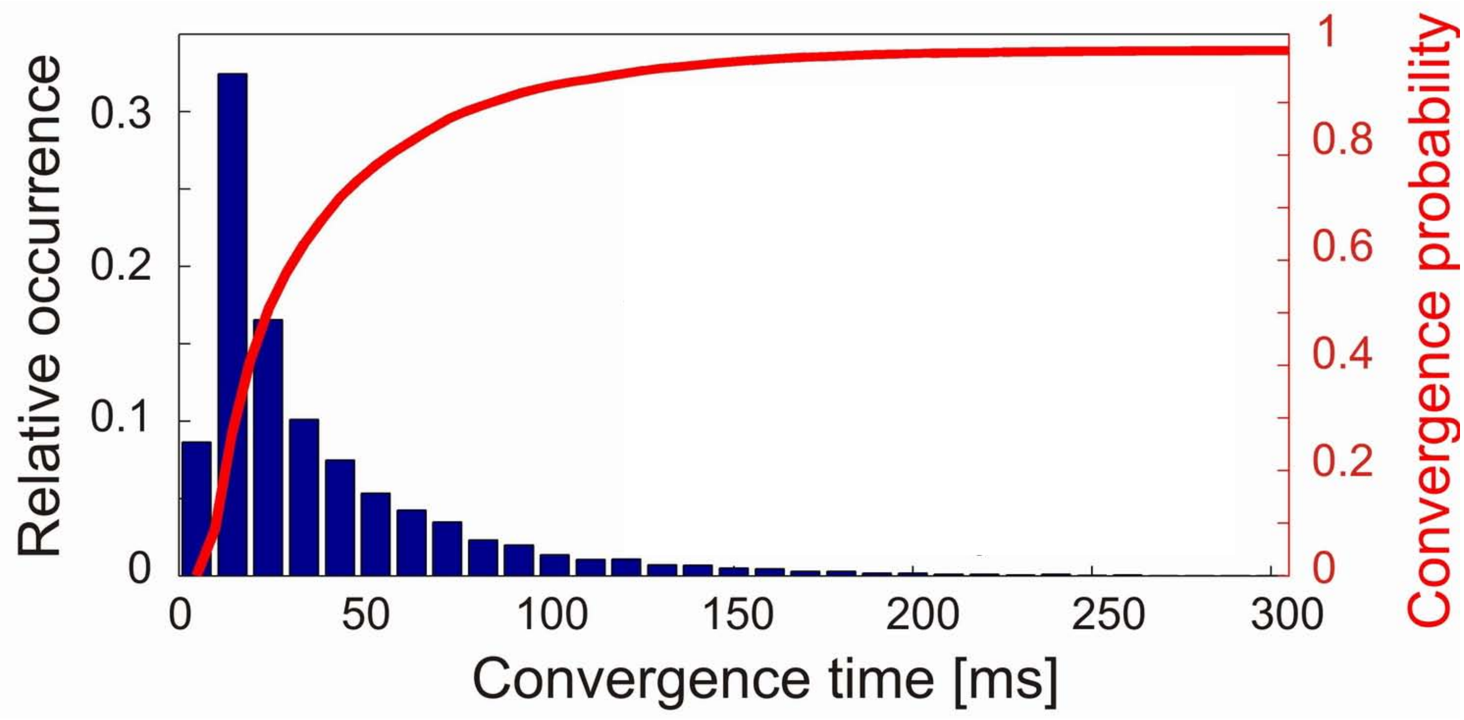
What is fidelity of the state production at arbitrary time?



average over 10^4 quantum trajectories

Convergence rate

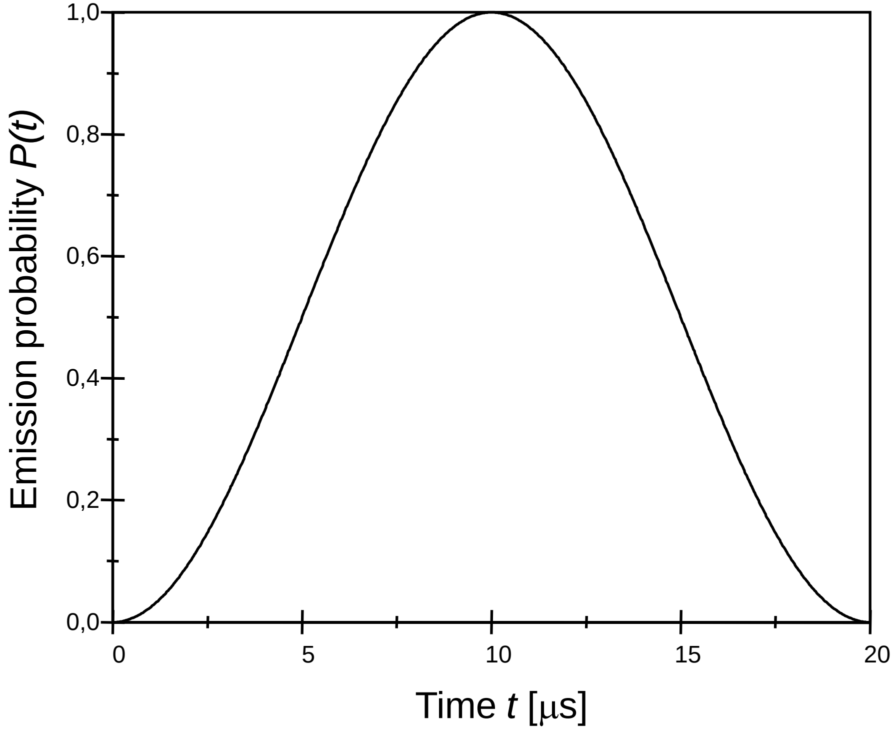
How fast the target state is prepared ?



- We chose the feedback to converge if **F > 95%**
- Convergence probability of about **50% after 20 ms**
- Inevitably, **all trajectories** converge

Resonant atoms as field injectors

Rabi oscillation in a photon number state

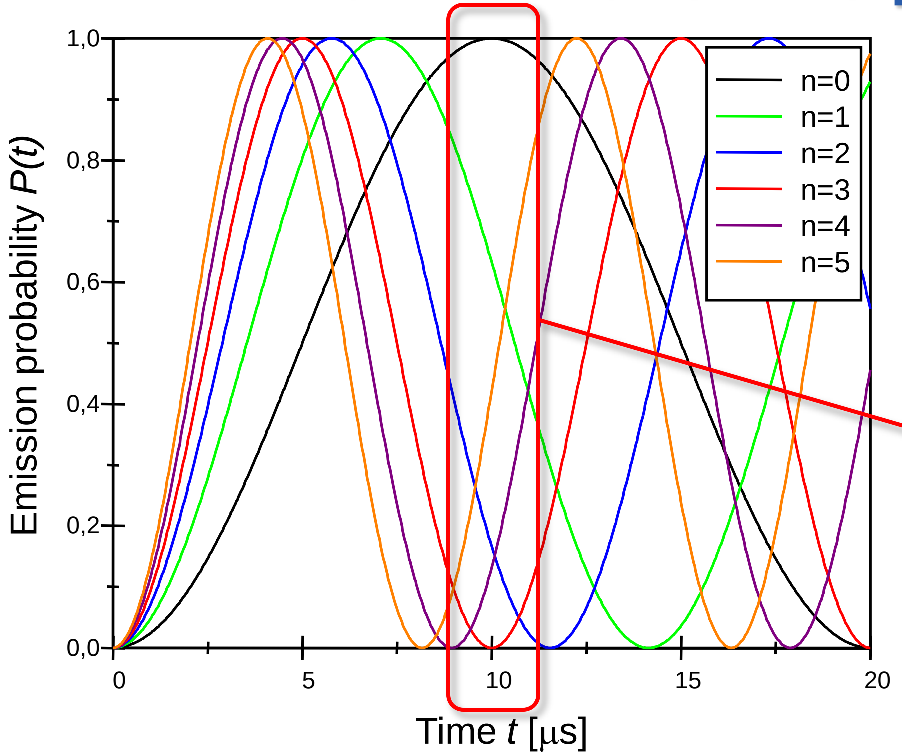


Probability to inject a photon for an atom initially in |e⟩:

$$P(t) = \frac{1}{2}\left[1 - \cos\left(\Omega_0 \sqrt{n+1}\, t\right)\right]$$

Resonant atoms as field injectors

Rabi oscillation in a photon number state



Probability to inject a photon for an atom initially in $|e\rangle$:

$$P(t) = \frac{1}{2}\left[1 - \cos\left(\Omega_0\sqrt{n+1}\,t\right)\right]$$

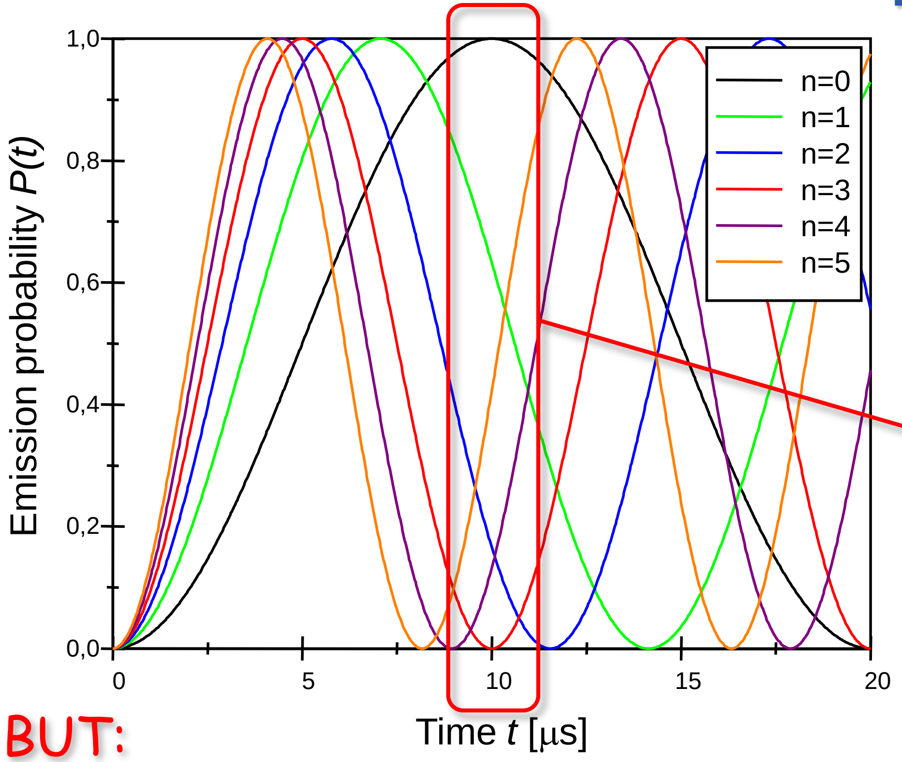
At t =10 μs, atoms perform 2π pulse in 3 photons: "trapping state" situation

"Photon pumping" of the cavity:

1. Start from empty cavity (vacuum state with n=0)
2. Send atoms in $|e\rangle$ and set interaction time to 2π for $|3\rangle$ state
3. After several atoms, the cavity will be "pumped" and "trapped" in $|3\rangle$

Resonant atoms as field injectors

Rabi oscillation in a photon number state



Probability to inject a photon for an atom initially in $|e\rangle$:

$$P(t) = \frac{1}{2}\left[1 - \cos\left(\Omega_0\sqrt{n+1}\,t\right)\right]$$

At t =10 µs, atoms perform 2π pulse in 3 photons: "trapping state" situation

BUT:

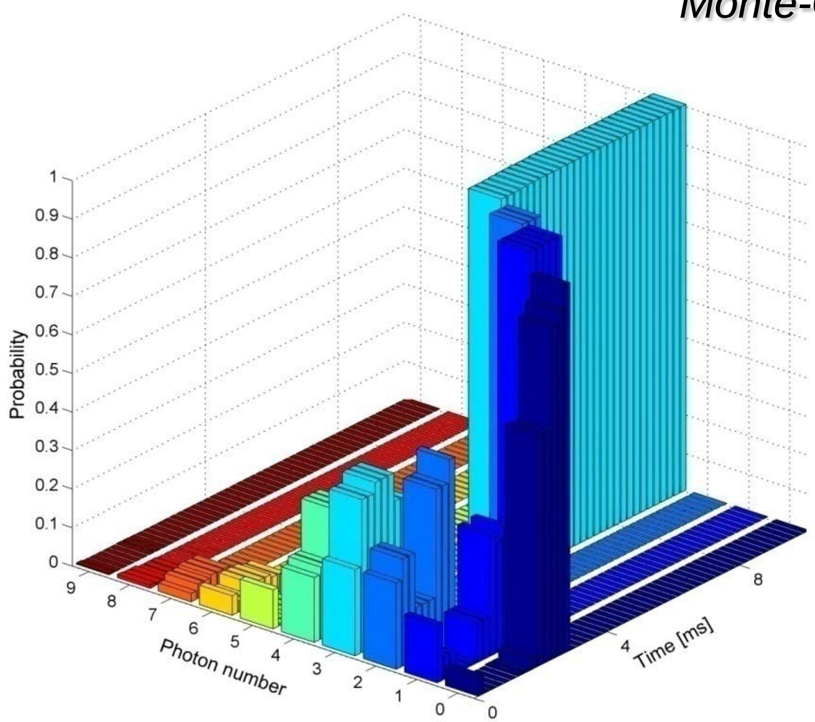
As soon as n>3 (e.g. due to thermal field excitation), field will continue to uncontrollably increase and run away from the desired state !

SOLUTION:

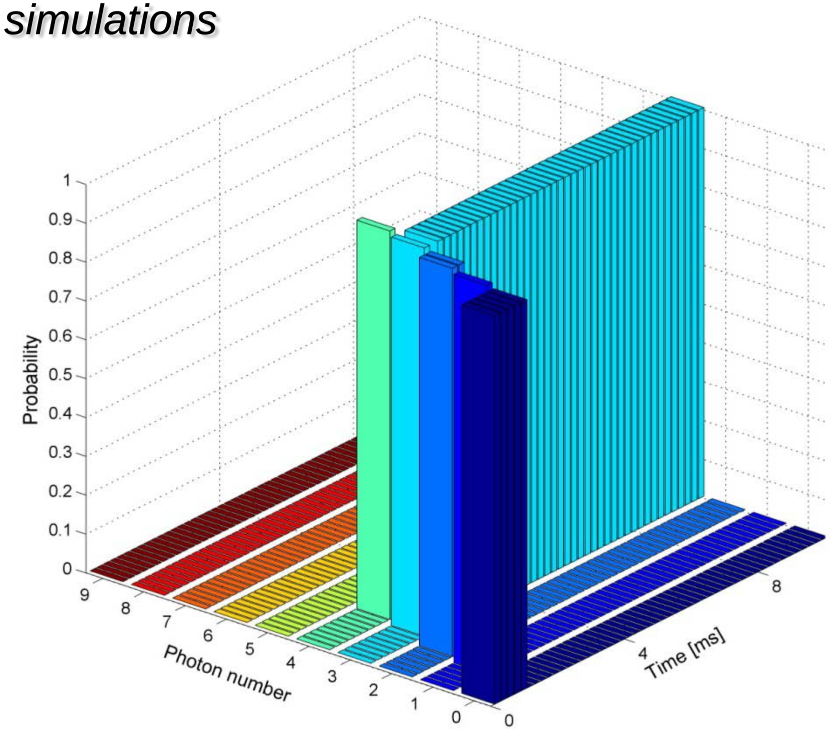
Probe the field with QND atoms and start to send resonant atoms in state $|g\rangle$ if n>3 in order to absorb the excess field !

Atomic feedback convergence

Monte-Carlo simulations



initial state:
coherent with ⟨n⟩=3



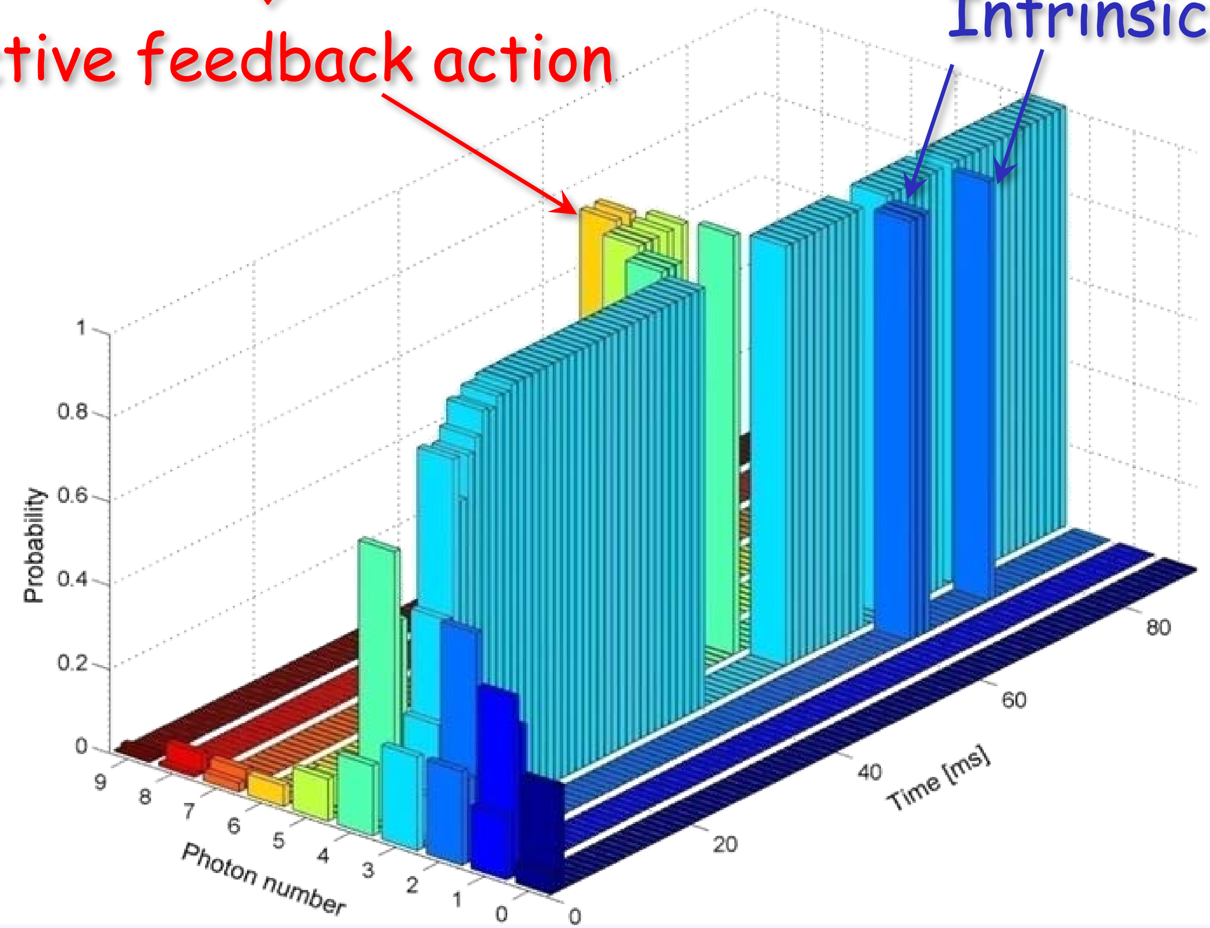
initial state:
vacuum

Very fast convergence toward the target

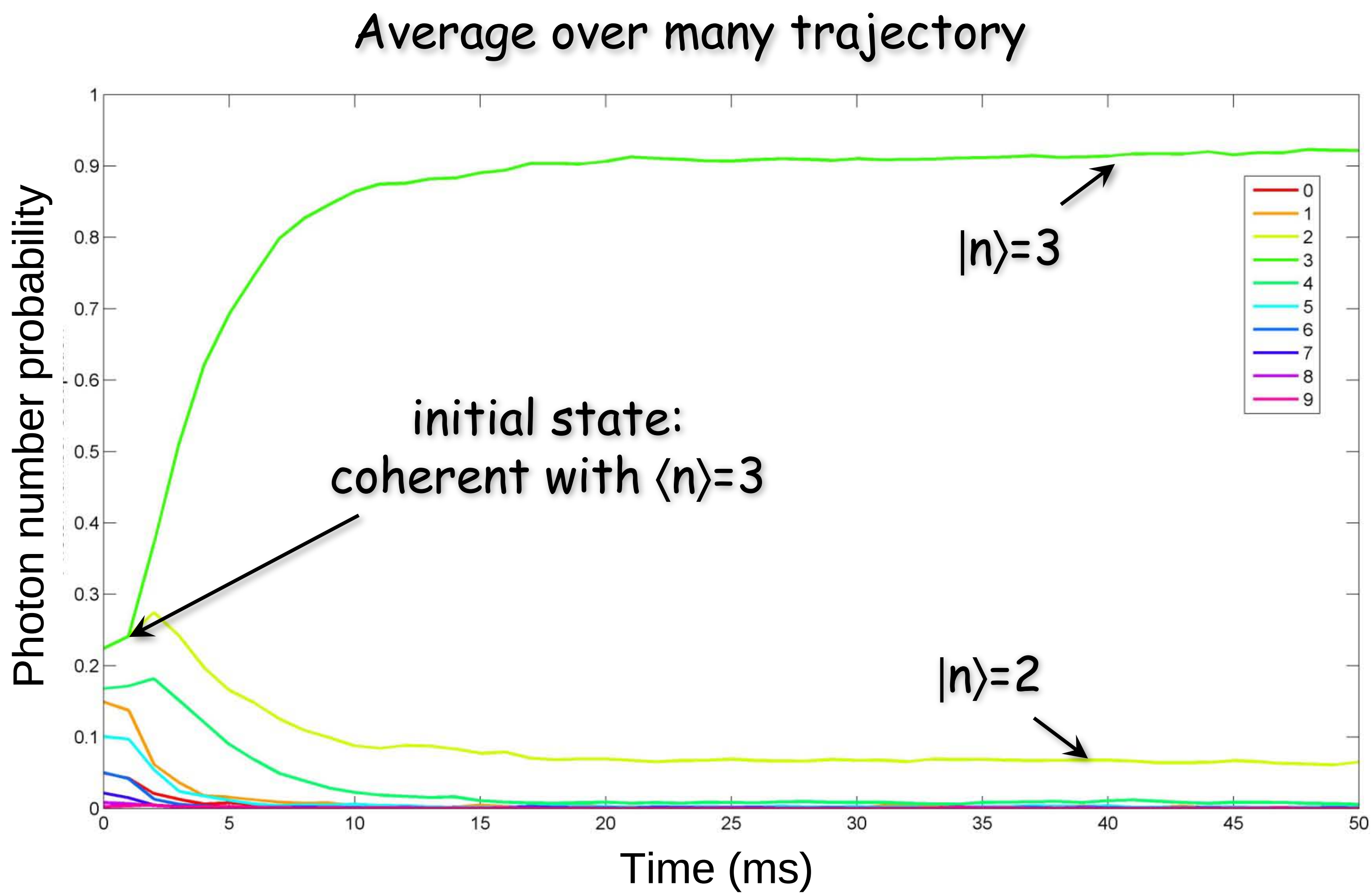
Stabilization of decoherence

Black-body field emission:
|3⟩ → |4⟩ quantum jump
⇓
Active feedback action

Cavity decay:
|3⟩ → |2⟩ quantum jump
⇓
Intrinsic passive stability

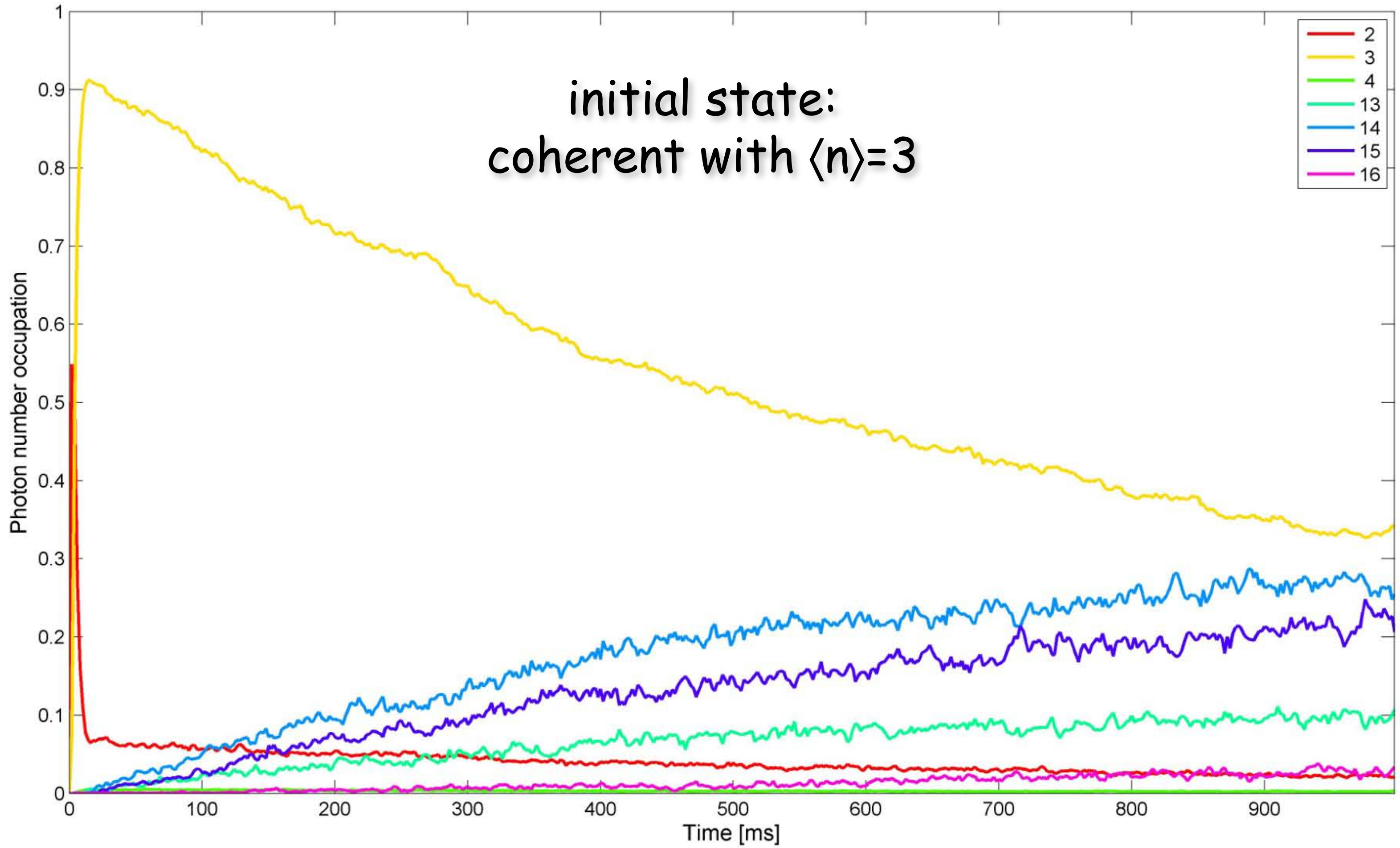


Atomic feedback convergence



Convergence to |n⟩=3 with 90% fidelity in 15 ms

Trapping state instability



Photon number evolution in trapping state condition without feedback: instability due to blackbody radiation

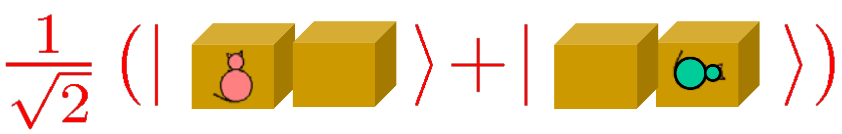
Conclusion / Perspectives

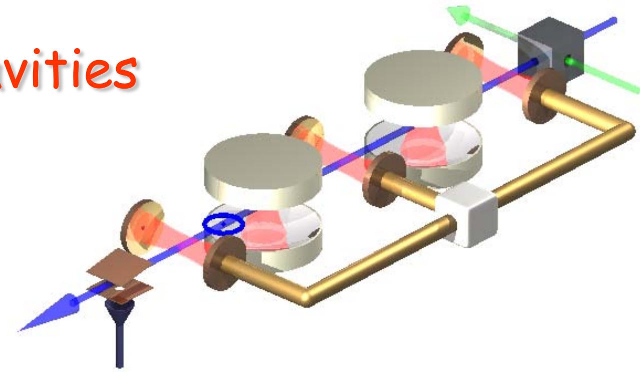
- Quantum Non-Demolition photon counting for quantum state preparation :
  - weak and projective measurement

- Quantum feedback proposals – coherent and atomic:
  - deterministic preparation of number states with high fidelity
  - protection of these states with respect to decoherence

Realization in progress

- Other work in progress:
  - non-local state preparation in two cavities
  - EPR pair of Schrödinger cats





Thank you

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