Consider $u$ well-posed when $\epsilon \in O$ such that: $-\text{div}(\epsilon \nabla u) = f$ in $\Omega$.

For Radiation condition in $\Sigma$ to select the good singularity.

We use PMLs in a neighborhood of $\Sigma$ to approach the solution which is not of finite energy.

A black hole phenomena

Helmholtz equation in the bounded sector $\Omega$

$$-\text{div}(\epsilon \nabla u) = -r^{-2}(\epsilon(r \partial_r)^2 + \partial_\psi \partial_\psi) u = f.$$ 

For $\kappa_\epsilon \in (-1; -1/3)$, propagative singularities appear: 

$$s_\epsilon^2(r, \theta) = \varphi_\epsilon(\theta) e^{2\pi \eta r \text{ln} r}$$ 

where $\eta$ is a real number which depends on $\kappa_\epsilon$. 

Radiation condition in $\Omega$ to select the good singularity.

We use PMLs in a neighborhood of $\Omega$ to approach the solution which is not of finite energy.

The T-coercivity approach

This technique can be used to justify the classical finite element methods and to study the Maxwell’s problem.

Difficulties:

- Loss of coercivity: there is no constant $C$ such that 
  $$\int_{\Omega} \epsilon |\nabla u|^2 d\Omega > C \int_{\Omega} |\nabla u|^2 d\Omega, \forall u \in H_0^1(\Omega).$$ 

- Add some dissipation (modeled by $\eta$) is not sufficient: 
  $$\int_{\Omega} \epsilon^\eta |\nabla u|^2 d\Omega > C \eta \int_{\Omega} |\nabla u|^2 d\Omega.$$ 

Questions:

- Is problem $(P)$ well-posed?
- How to compute a numerical approximation of the solution?
- New model when $(P)$ is ill-posed?

Spurious reflections appear in the corner $\Sigma$.

Fichera’s corner $R_1$ and $R_2$ obtained from the symmetries $S_{Ox}^r, S_{Oy}^r, S_{Oz}^r$. 

$\mathcal{I}_C = [-\frac{2\pi - \alpha}{\alpha}, -\frac{\alpha}{2\pi - \alpha}]$.