Advanced Optimization

Lectures/Exercises 6 and 7: (Evolutionary) Multiobjective Optimmization

January 17, 2017 and January 31, 2017

Master AIC

Université Paris-Saclay, Orsay, France

Anne Auger
INRIA Saclay – Ile-de-France

Dimo Brockhoff
INRIA Saclay – Ile-de-France

INVENTORS FOR THE DIGITAL WORLD

Course Overview

	Date		Topic
1	Tue, 22.11.2016	Dimo	Randomized Algorithms for Discrete Problems
2	Tue, 29.11.2016	Dimo	Exercise: The Travelling Salesperson Problem
3	Tue, 6.12.2016	Anne	Continuous Optimization I
	vacation		
4	Tue, 3.1.2017	Anne	Continuous Optimization II
5	Tue, 10.1.2017	Anne	Continuous Optimization III
6	Tue, 17.1.2017	Dimo	Evolutionary Multiobjective Optimization I
7	Tue, 31.1.2017	Dimo	Evolutionary Multiobjective Optimization II
	???		oral presentations (individual time slots)

all from 14:00 till 17:15 in PUIO - E213

Organization Oral Exams

to be decided until last class (Jan 31), better today ☺

	Monday, Feb 20	Friday, Feb 24
10am		
10:30am		
11am	Anh Khoa Ngo Ho	
11:30am	Ahmed Mazari	
12am	Abdallah Benzine	
12:30pm	Jonathan Crouzet	
1:30pm	Mohamed Ali Fathallah	Amal Targhi
2:00pm	Antonin Raffin	Abdelhak Loukkal
2:30pm	Gabriel Quere	Yuxiang Wang
3pm	Laurent Cetinsoy	
3:30pm	Ghazi Felhi	
4pm	Clément Thierry	

Details on What We Expect from the Oral Exam

- 15min presentation about your paper
  - background
  - summary of content
  - critical discussion
  - organization of the slides is up to you
- 10-15min of discussion/exam questions
  - related to the paper
  - but potentially also related to the lecture

Don't forget to send us your slides by **Jan. 31, 2017** (via email)

Overview of the Remaining Two Lectures

Introduction to multiobjective optimization

(a bit more detailed than in the introductory lecture)

- difference to single-objective optimization, the basics
- algorithms and their design principles
- performance assessment and benchmarking
- integration of preferences
- a few aspects of visualization

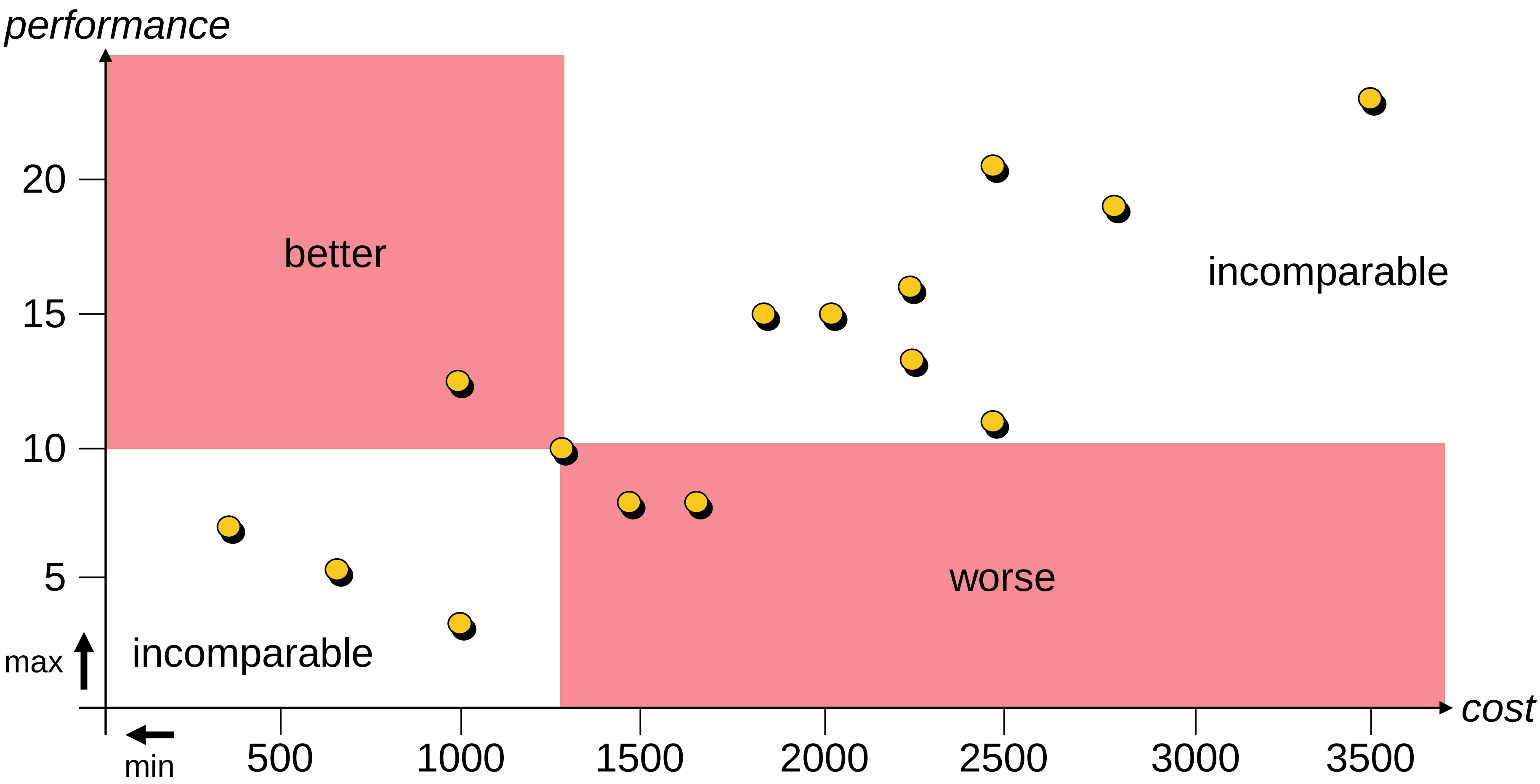
Exercise around COCO

- implement basic algorithm(s)
- benchmark on COCO
  - two goals: testing our new test suite and producing data for the upcoming BBOB-2017 workshop

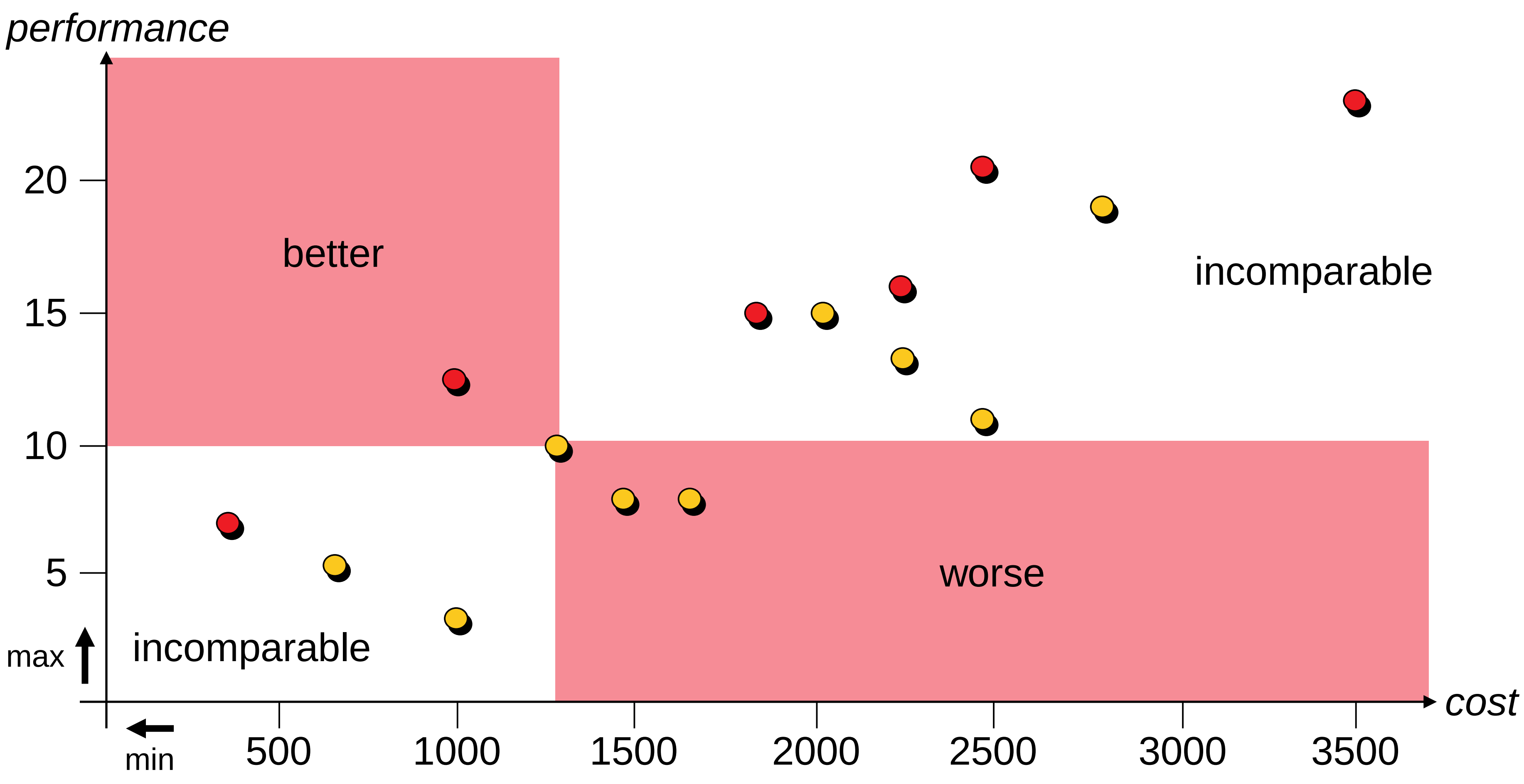
Multiobjective Optimization

Multiobjective Optimization

Multiple objectives that have to be optimized simultaneously

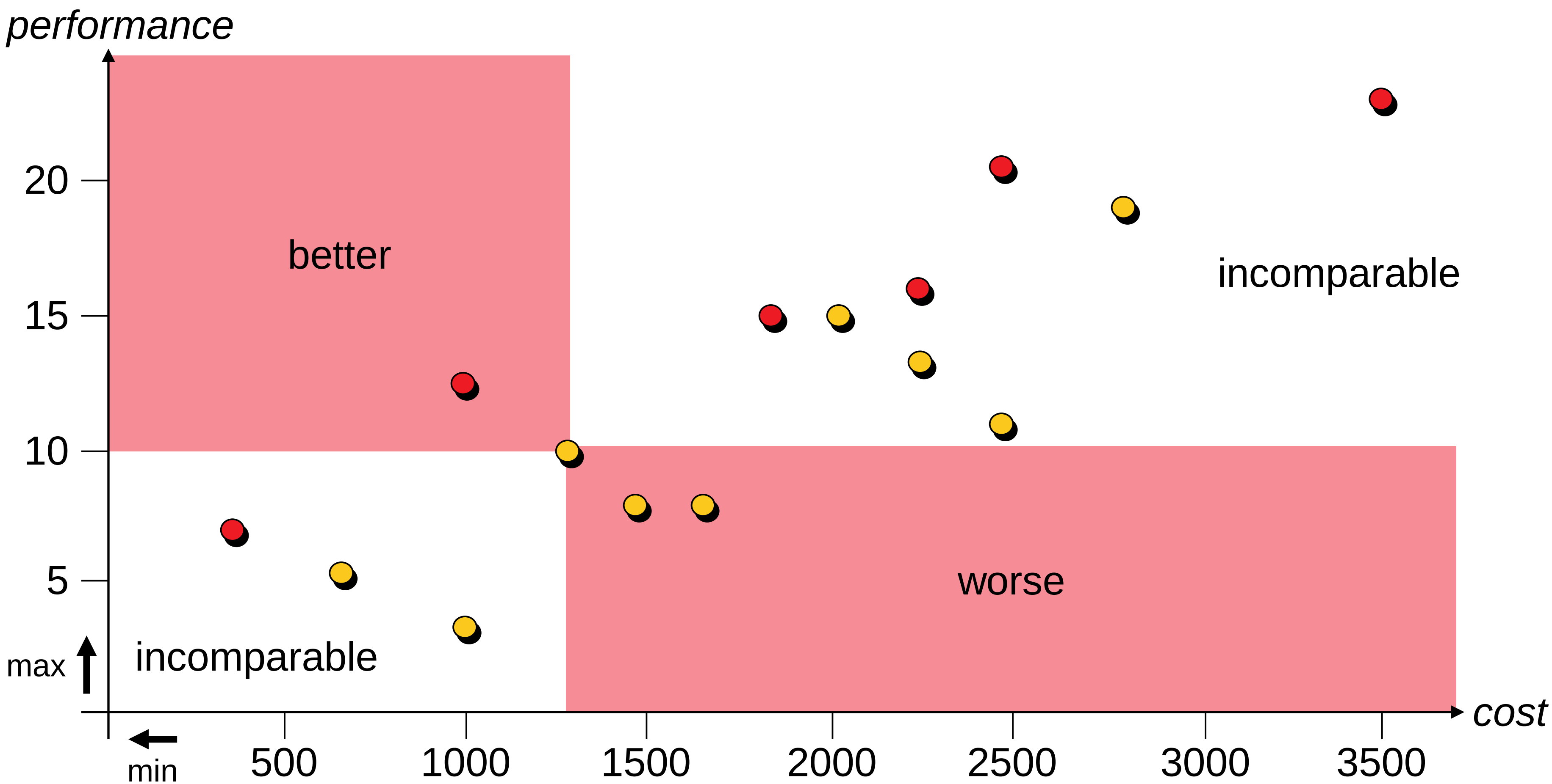


Observations: ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)



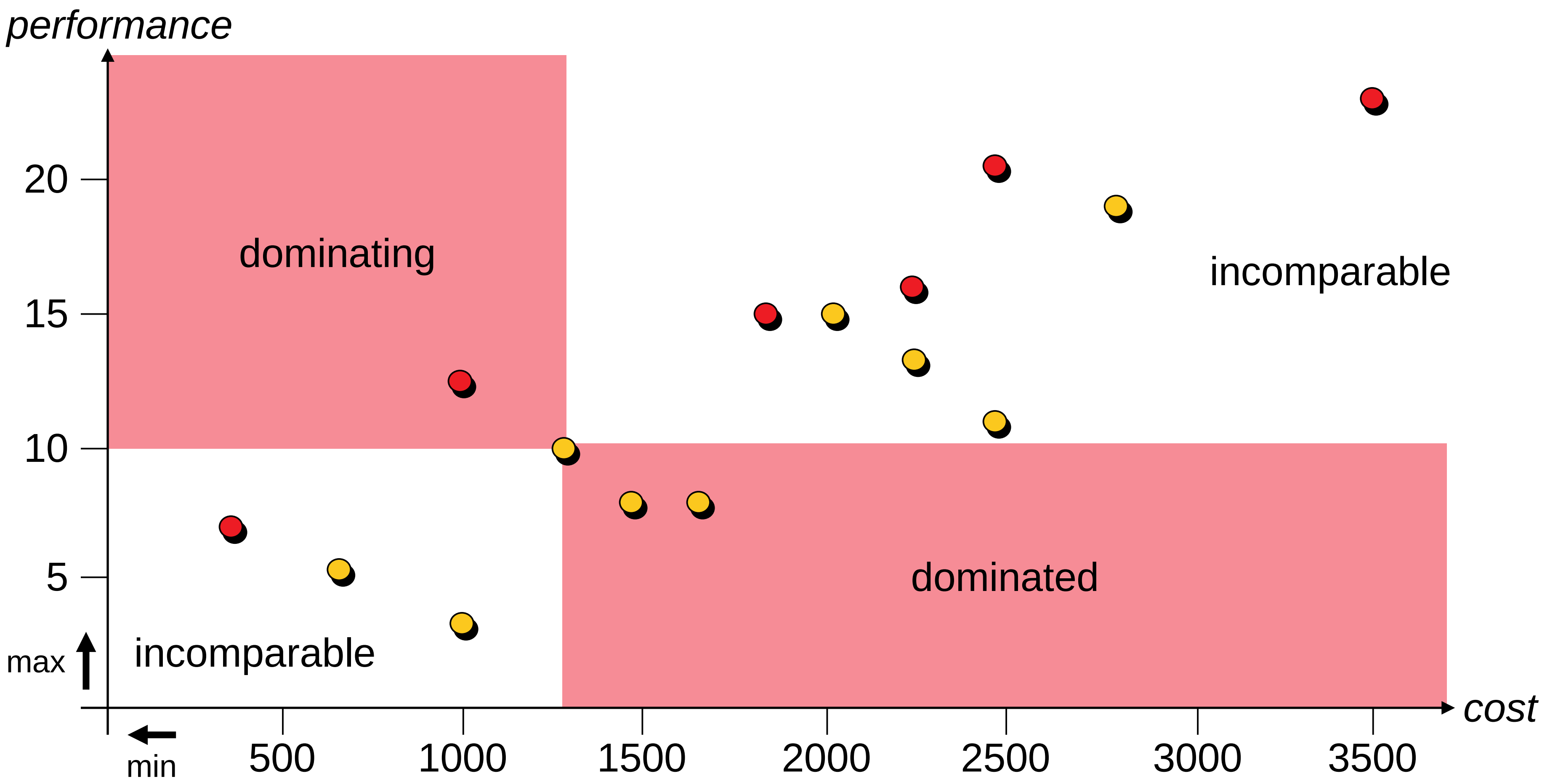
$$u \text{ weakly Pareto dominates } v \ (u \preccurlyeq_{par} v): \quad \forall 1 \le i \le k : f_i(u) \le f_i(v)$$

$$u \text{ Pareto dominates } v \ (u \prec_{par} v): \quad u \preccurlyeq_{par} v \ \wedge \ v \not\preccurlyeq_{par} u$$



$$u \text{ weakly Pareto dominates } v \; (u \preccurlyeq_{par} v): \quad \forall 1 \leq i \leq k : f_i(u) \leq f_i(v)$$

$$u \text{ Pareto dominates } v \; (u \prec_{par} v): \quad u \preccurlyeq_{par} v \;\wedge\; v \not\preccurlyeq_{par} u$$



Exercise 1

Show the equivalence between

$$u <_{par} v: \quad u \leq_{par} v \wedge v \nleq_{par} u$$

and

$$\forall 1 \leq i \leq k: f_i(u) \leq f_i(v) \text{ and } \exists 1 \leq j \leq k: f_i(u) < f_i(v)$$

Exercise 1: Solution

Proof:

$$u <_{par} v: \quad u \leq_{par} v \wedge v \not\leq_{par} u$$

$$\Leftrightarrow \forall 1 \leq i \leq k: f_i(u) \leq f_i(v) \text{ and not } (\forall 1 \leq i \leq k: f_i(v) \leq f_i(u))$$

$$\Leftrightarrow \forall 1 \leq i \leq k: f_i(u) \leq f_i(v) \text{ and not } \big(\forall 1 \leq i \leq k: f_i(u) \geq f_i(v)\big)$$

$$\forall 1 \leq i \leq k: f_i(u) \leq f_i(v) \text{ and } \exists 1 \leq j \leq k: f_i(u) < f_i(v)$$

Exercise 2: Understanding Pareto Dominance

Given the following solutions, tell which ones dominate each other and which don't for the double sphere problem

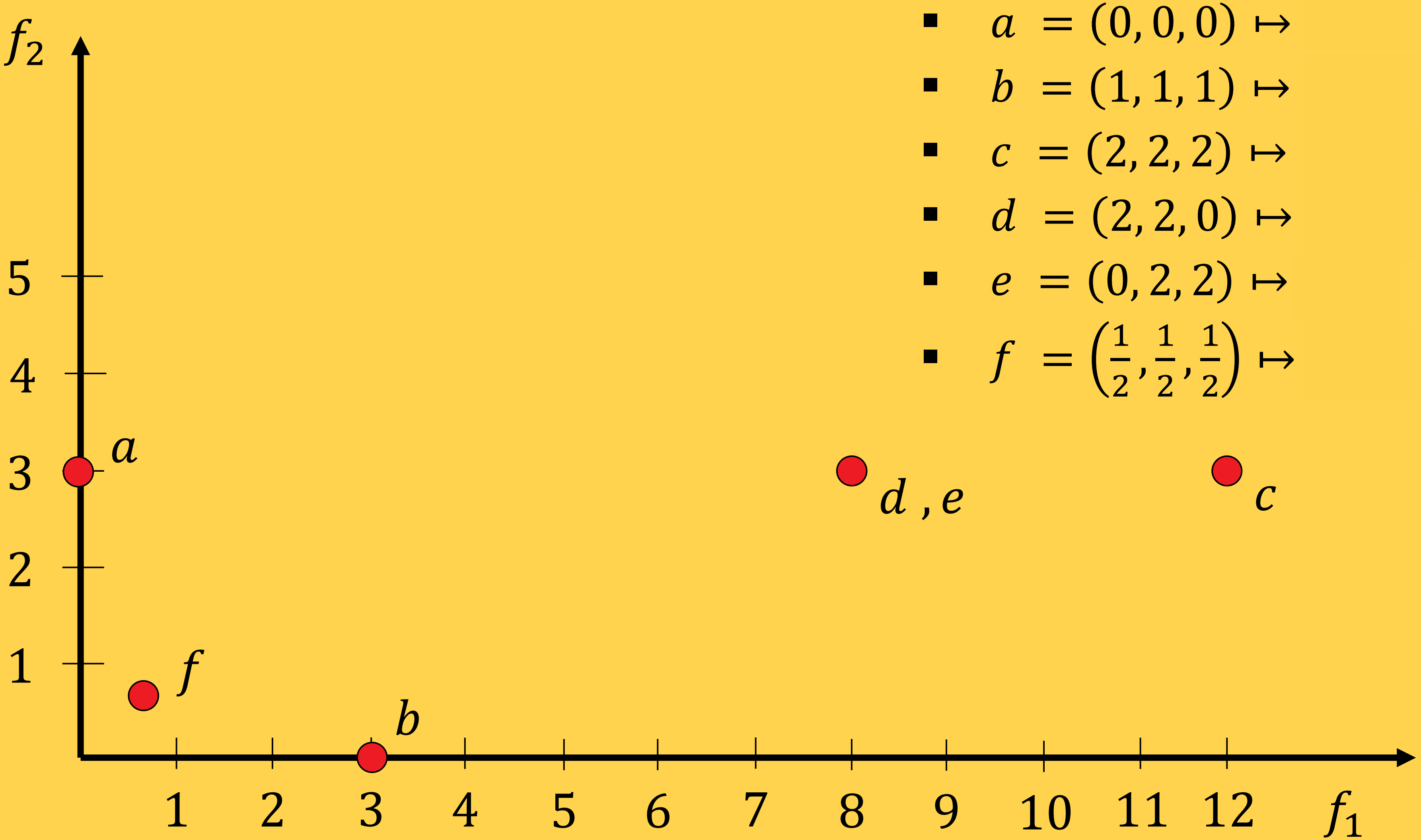
$$f_{\text{doublesphere}}: x \mapsto \left(\sum_{i=1}^{n} x_i^2 \,,\, \sum_{i=1}^{n} (x_i - 1)^2\right).$$

- $a = (0, 0, 0)$
- $b = (1, 1, 1)$
- $c = (2, 2, 2)$
- $d = (2, 2, 0)$
- $e = (0, 2, 2)$
- $f = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

Exercise 2: Understanding Pareto Dominance

$f_{\text{doublesphere}}: x \mapsto$
$\left(\sum_{i=1}^{n} x_i^2\,,\ \sum_{i=1}^{n}(x_i-1)^2\right).$

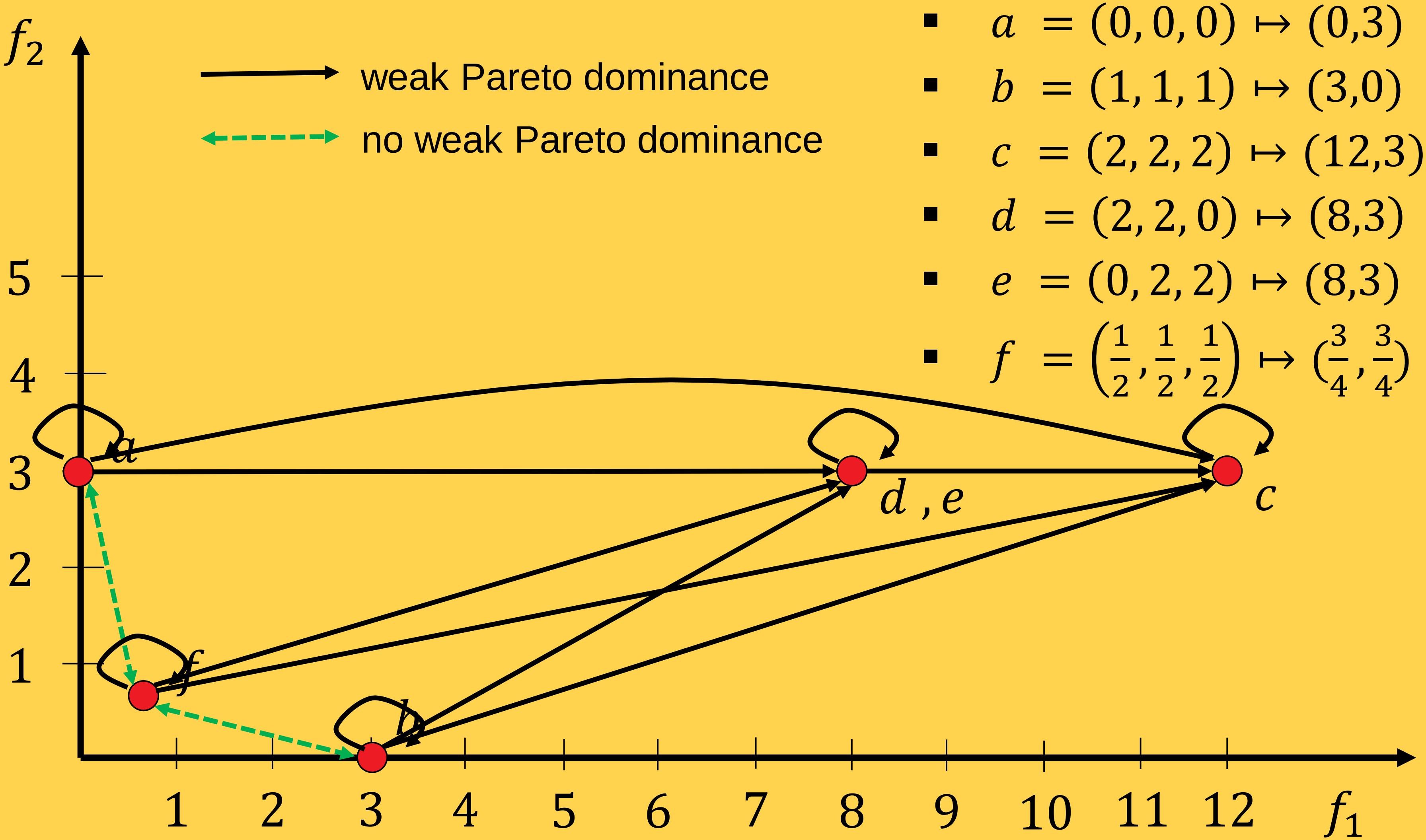
- $a = (0, 0, 0) \mapsto$
- $b = (1, 1, 1) \mapsto$
- $c = (2, 2, 2) \mapsto$
- $d = (2, 2, 0) \mapsto$
- $e = (0, 2, 2) \mapsto$
- $f = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \mapsto$



Exercise 2: Understanding Pareto Dominance

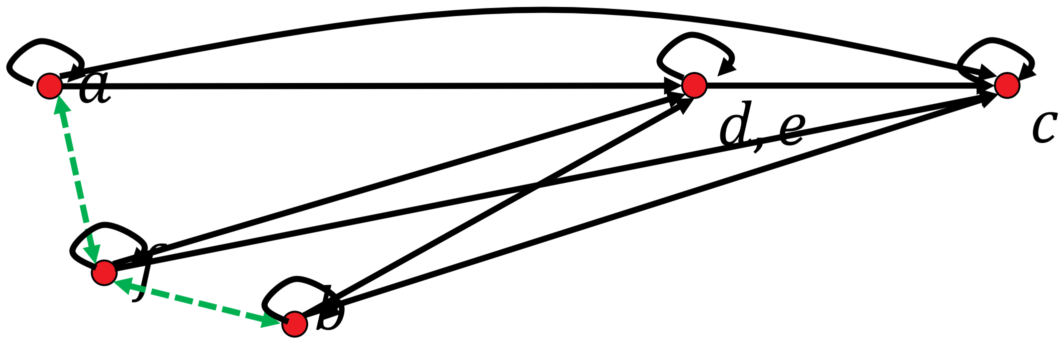
$f_{\text{doublesphere}}: x \mapsto \left(\sum_{i=1}^{n} x_i^2\,,\ \sum_{i=1}^{n}(x_i - 1)^2\right).$

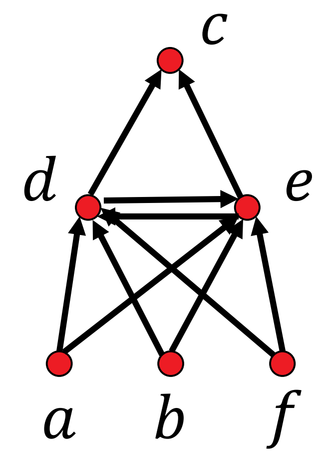
- $a\ = (0, 0, 0) \mapsto (0,3)$
- $b\ = (1, 1, 1) \mapsto (3,0)$
- $c\ = (2, 2, 2) \mapsto (12,3)$
- $d\ = (2, 2, 0) \mapsto (8,3)$
- $e\ = (0, 2, 2) \mapsto (8,3)$
- $f\ = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \mapsto \left(\frac{3}{4}, \frac{3}{4}\right)$



Visualizing Dominance Relations as Graphs

We can simplify the visualization of the (weak) Pareto dominance relation by *transitive reduction:*



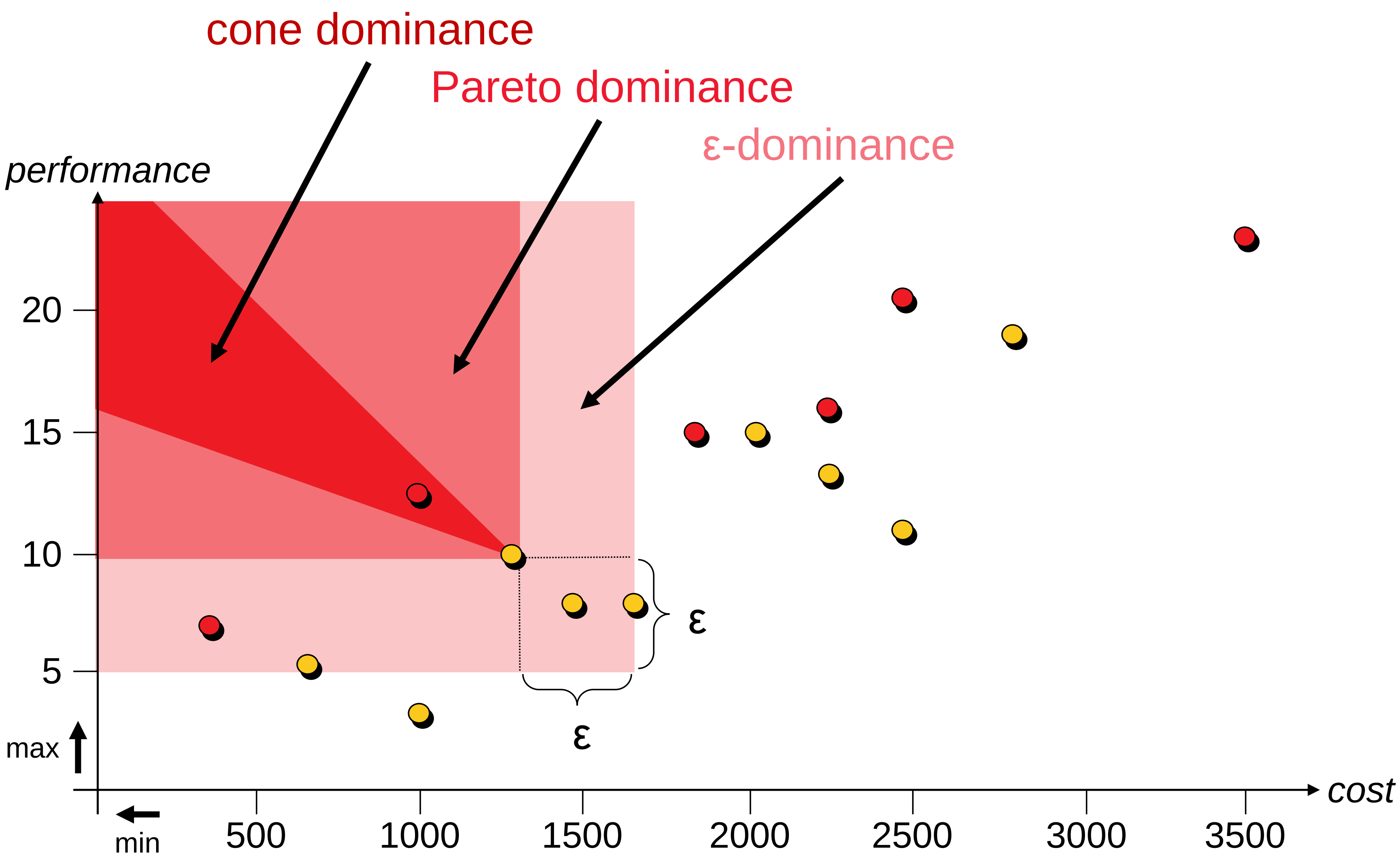


The weak Pareto dominance is a preorder, i.e. a relation that is

- reflexive and transitive
- minimal elements = Pareto-optimal solutions

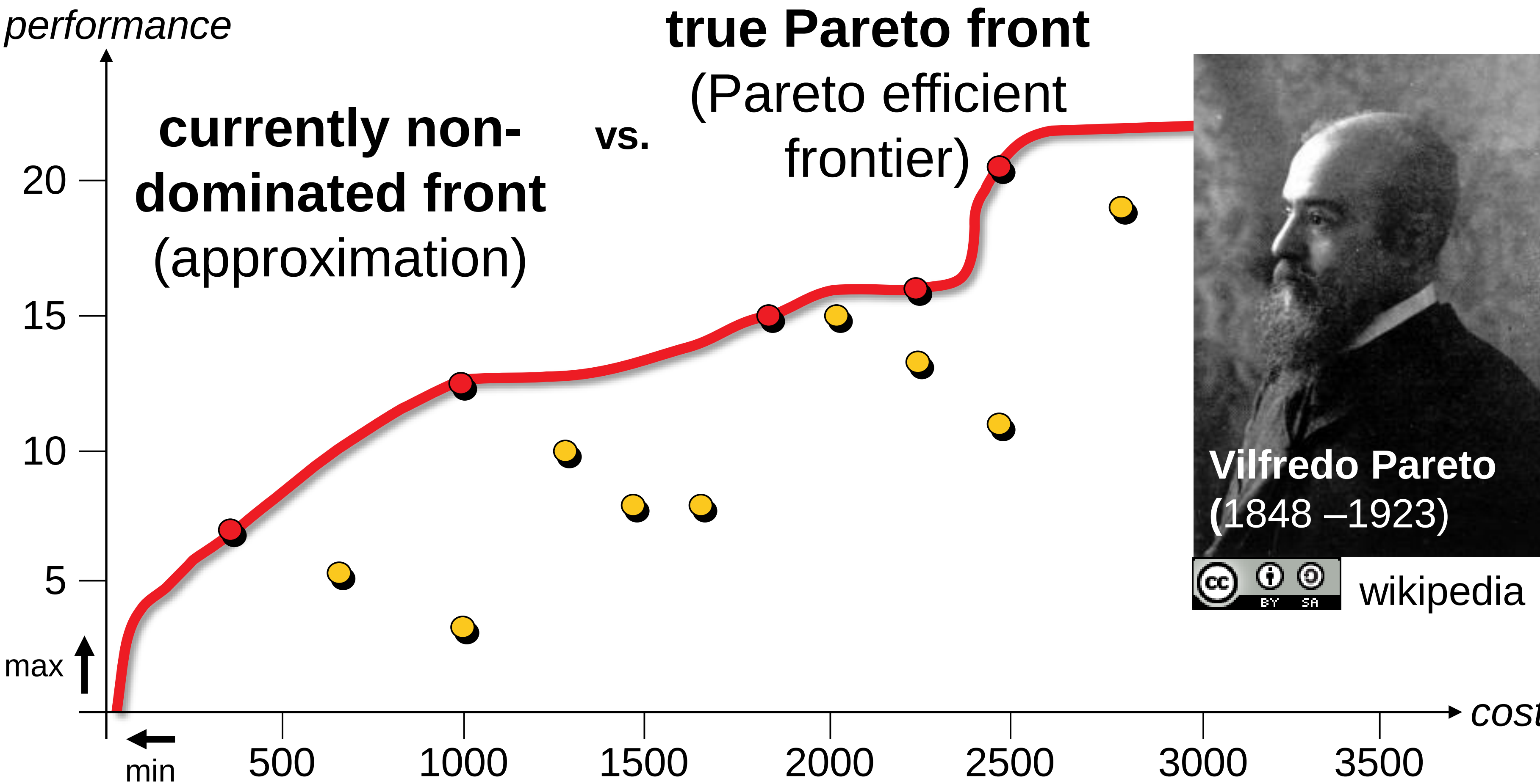
If no *indifferent* solutions $x \neq y$ with $f(x) = f(y)$ exist, we have antisymmetry and a partial order ("poset")---visualizable as Hasse diagram.

! The Pareto dominance itself is not reflexive and thus, never a poset!

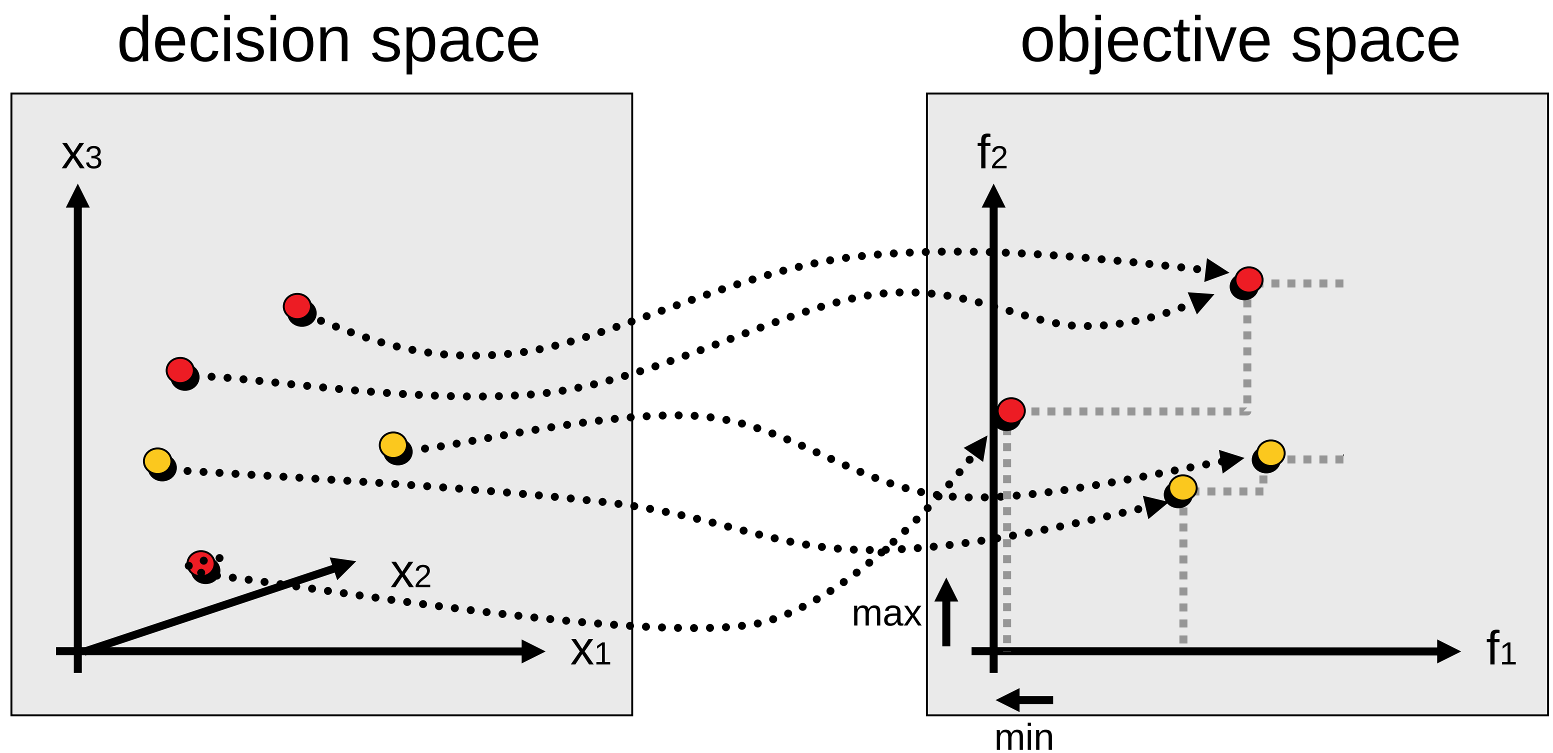


cone dominance
Pareto dominance
ε-dominance
performance
20
15
10
5
max
min
cost
500
1000
1500
2000
2500
3000
3500
ε
ε

Pareto set: set of all non-dominated solutions (decision space)
Pareto front: its image in the objective space



wikipedia



solution of Pareto-optimal set — vector of Pareto-optimal front

non-optimal decision vector — non-optimal objective vector

Exercise 3: Pareto Front of Double Sphere

What is the Pareto set/front of the double sphere problem
$f_{\mathrm{doublesphere}}: x \mapsto \left(\sum_{i=1}^{n} x_i^2 \,,\, \sum_{i=1}^{n} (x_i - 1)^2\right)$?

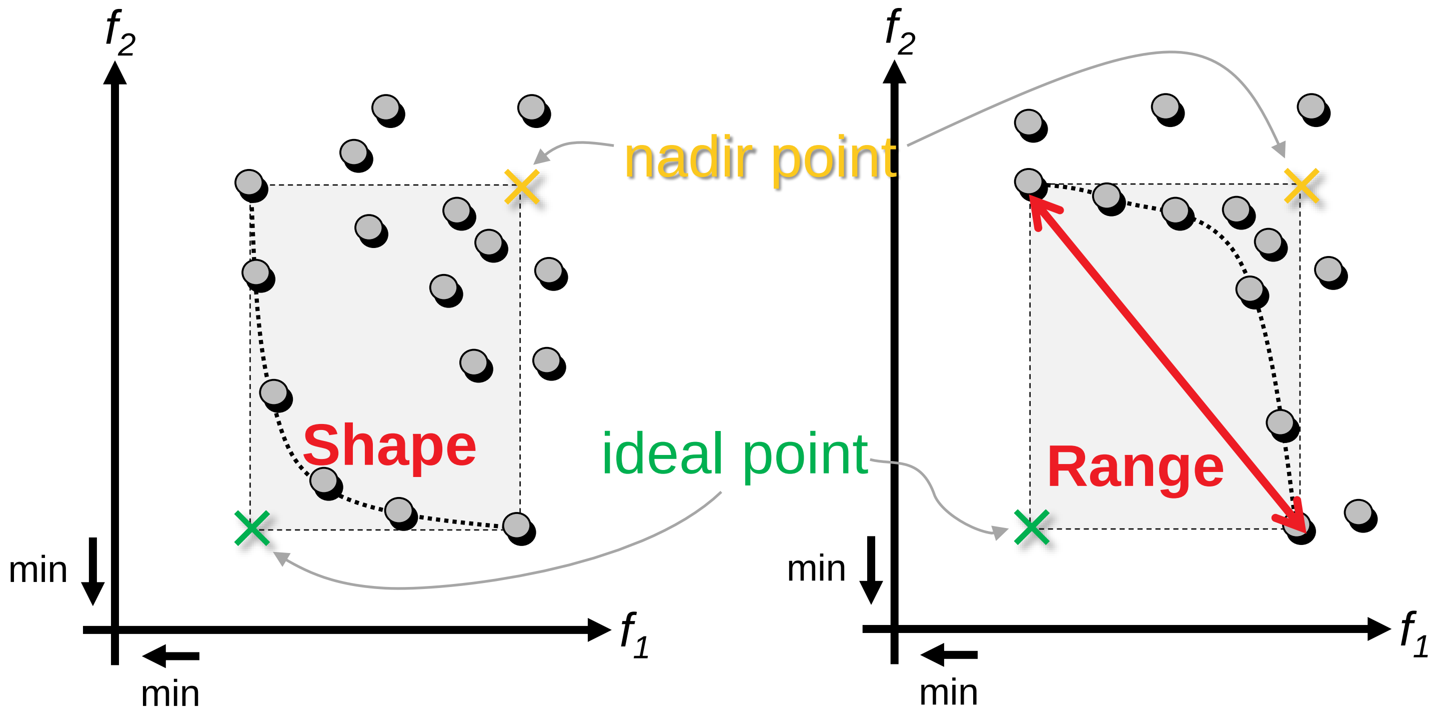
a) what is the Pareto set?

b) what is the associated Pareto front?

Tips for a)

- display some solutions in the search space (let's say in 2-D)
- investigate where dominating solutions lie
- investigate where dominated solutions lie
- finally, show graphically that what you think is the Pareto set is actually the Pareto set (take a point anywhere within your guessed set and show in which direction you can improve and where you cannot improve anymore)

Ideal and Nadir Point

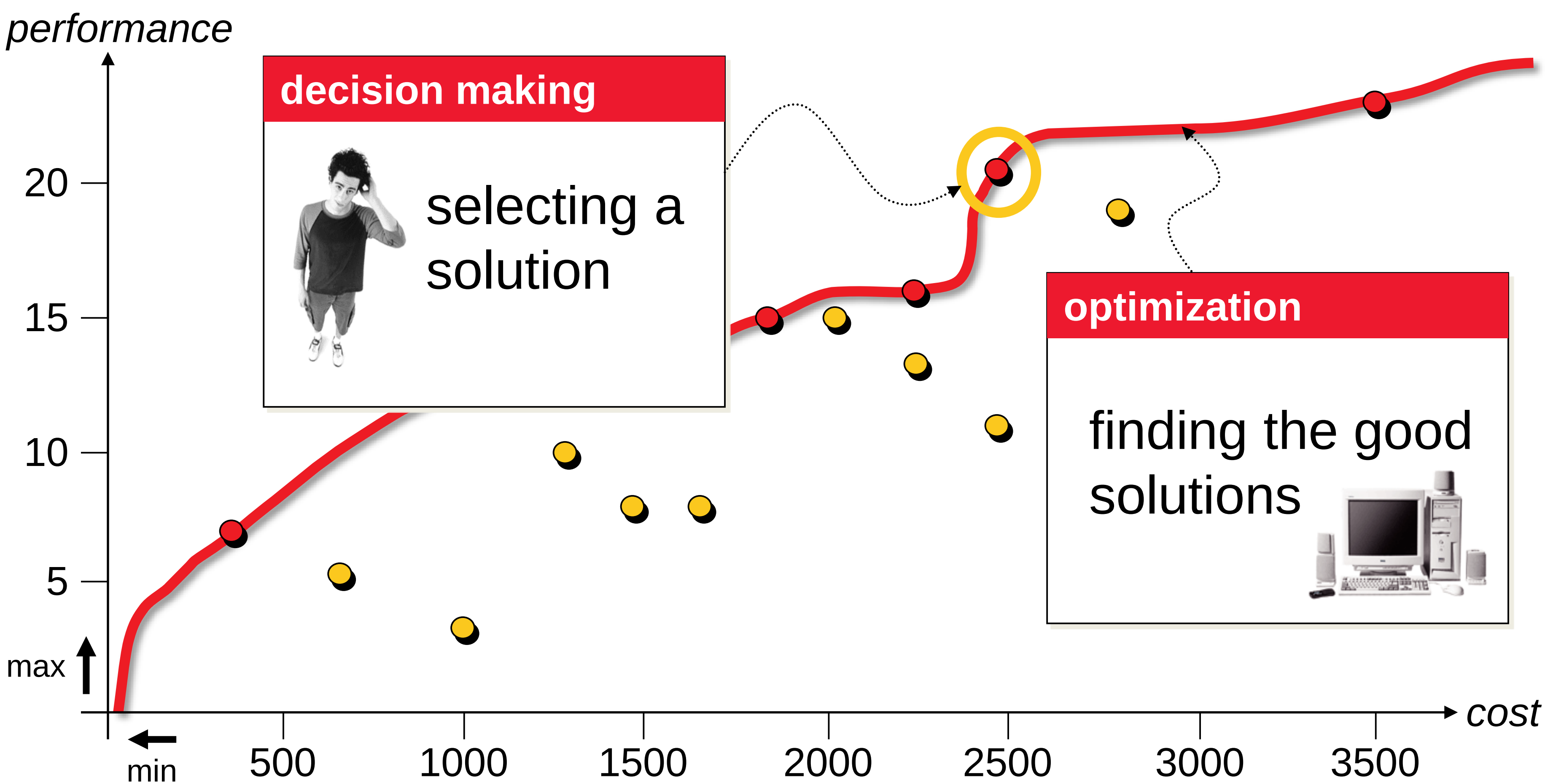


ideal point: best values
nadir point: worst values
} obtained for *Pareto-optimal* points

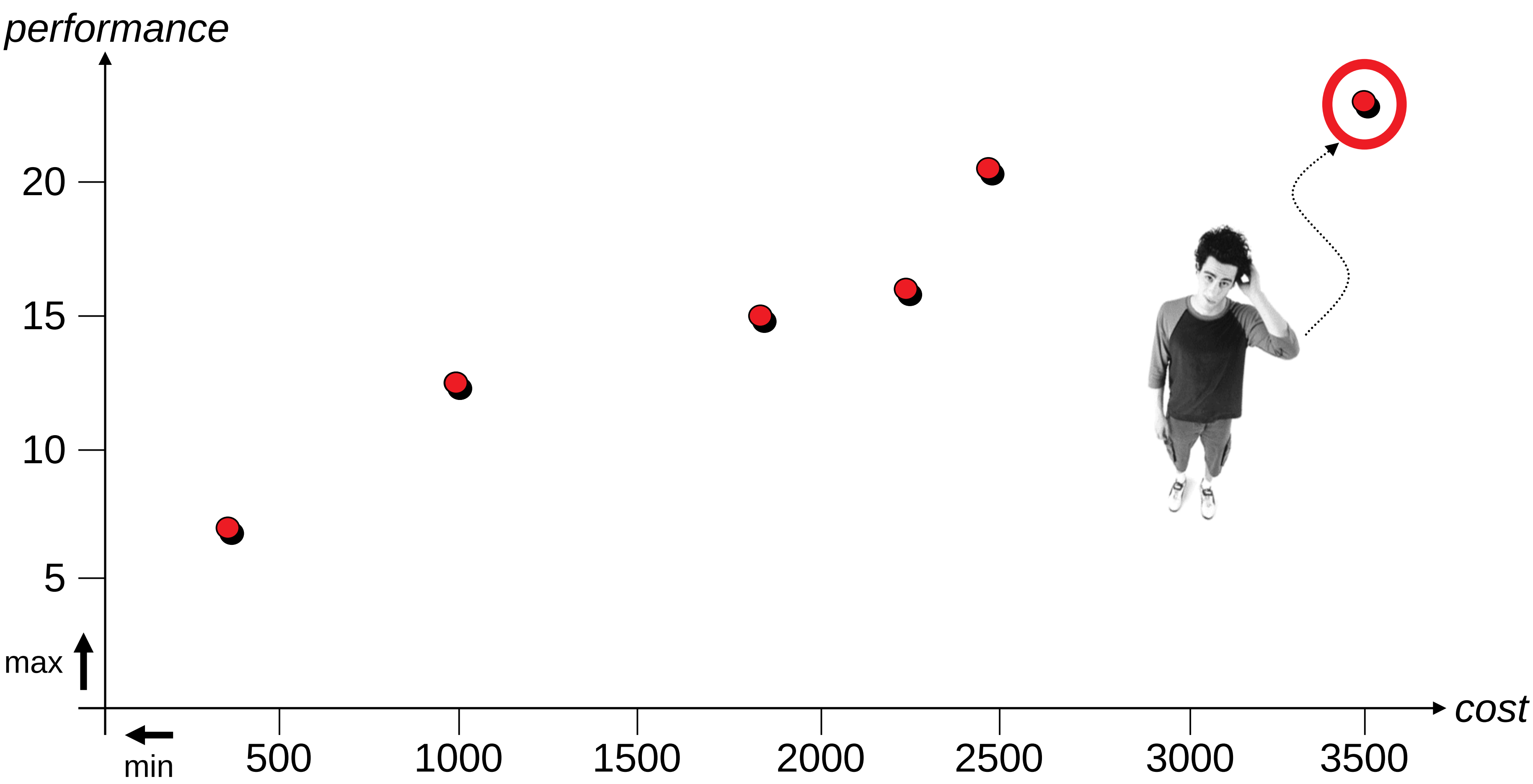
Optimization vs. Decision Making

Multiobjective Optimization

combination of optimization of a set and a decision for a solution



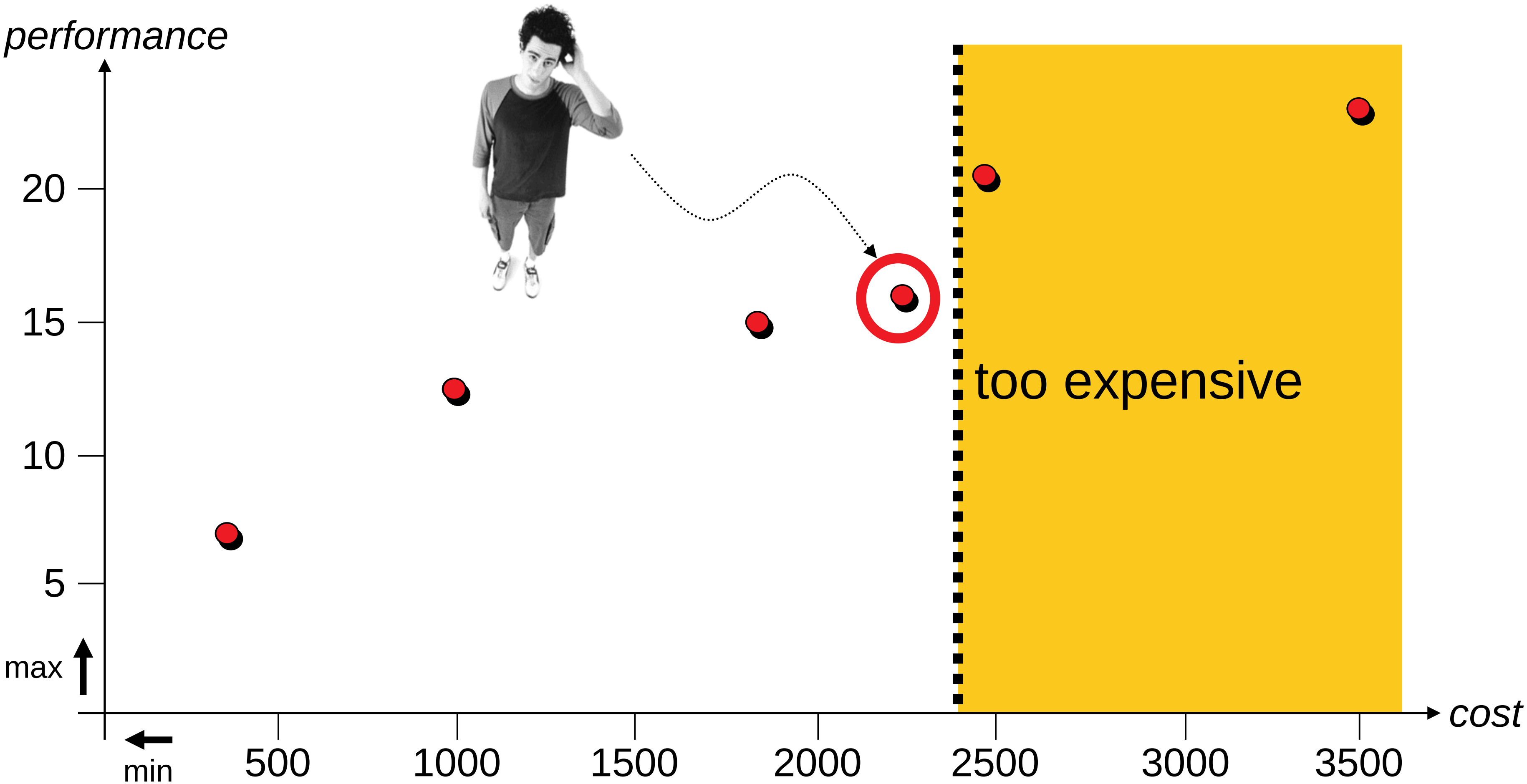
Possible Approaches: ❶ **ranking:** performance more important than cost



Possible Approaches:

❶ ranking: performance more important than cost

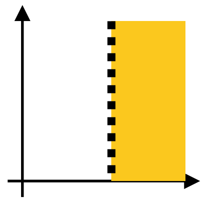
❷ **constraints:** cost must not exceed 2400



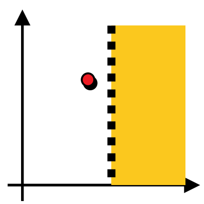
When to Make the Decision

Before Optimization:

rank objectives,
define constraints,…



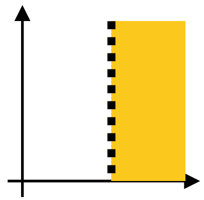
search for one
(good) solution



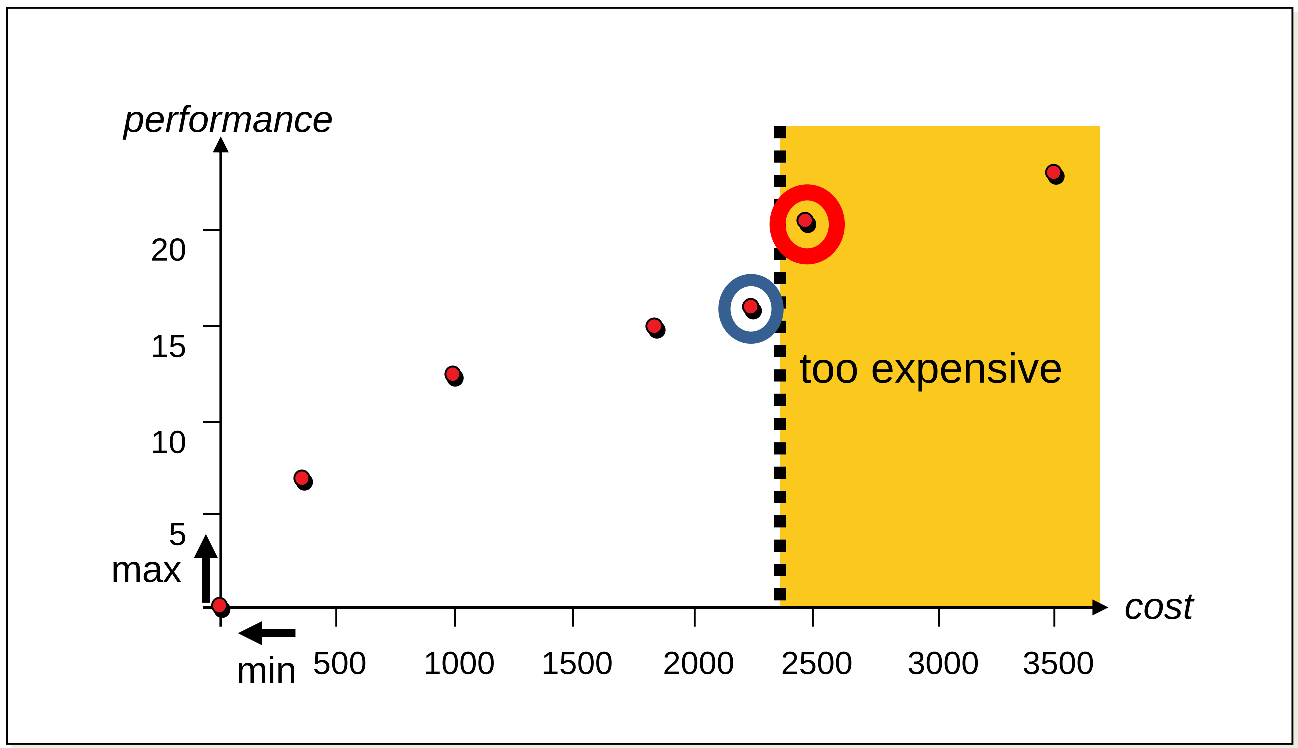
When to Make the Decision

Before Optimization:

rank objectives,
define constraints,…



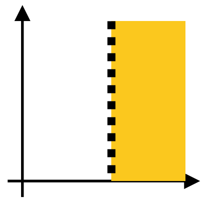
search for one
(good) solution



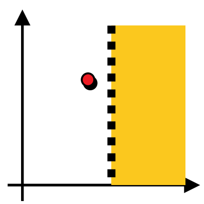
When to Make the Decision

Before Optimization:

rank objectives,
define constraints,…

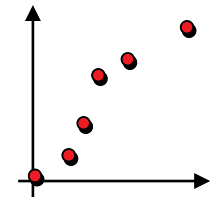


search for one
(good) solution

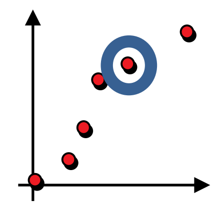


After Optimization:

search for a set of
(good) solutions



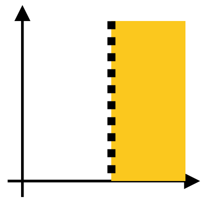
select one solution
considering
constraints, etc.



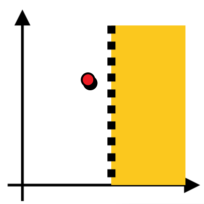
When to Make the Decision

Before Optimization:

rank objectives,
define constraints,…

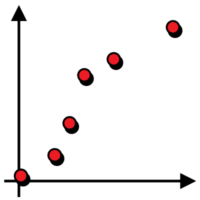


search for one
(good) solution

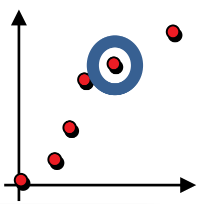


After Optimization:

search for a set of
(good) solutions



select one solution
considering
constraints, etc.



Focus: learning about a problem

- trade-off surface
- interactions among criteria
- structural information
- also: interactive optimization

Two Communities...

MCDM International Society on Multiple Criteria Decision Making

- established field (beginning in 1950s/1960s)
- bi-annual conferences since 1975
- background in economics, math, management and social sciences
- focus on optimization and decision making

emo

- quite young field (first papers in mid 1980s)
- bi-annual conference since 2001
- background in computer science, applied math and engineering
- focus on optimization algorithms

...Slowly Merge Into One

MCDM International Society on Multiple Criteria Decision Making

emo

- MCDM track at EMO conference since 2009
- special sessions on EMO at the MCDM conference since 2008
- joint Dagstuhl seminars since 2004

Blackbox optimization

$$x \in X \longrightarrow \boxed{f} \longrightarrow (f_1(x), \ldots, f_k(x))$$

only mild assumptions

→ EMO therefore well-suited for real-world engineering problems

objectives

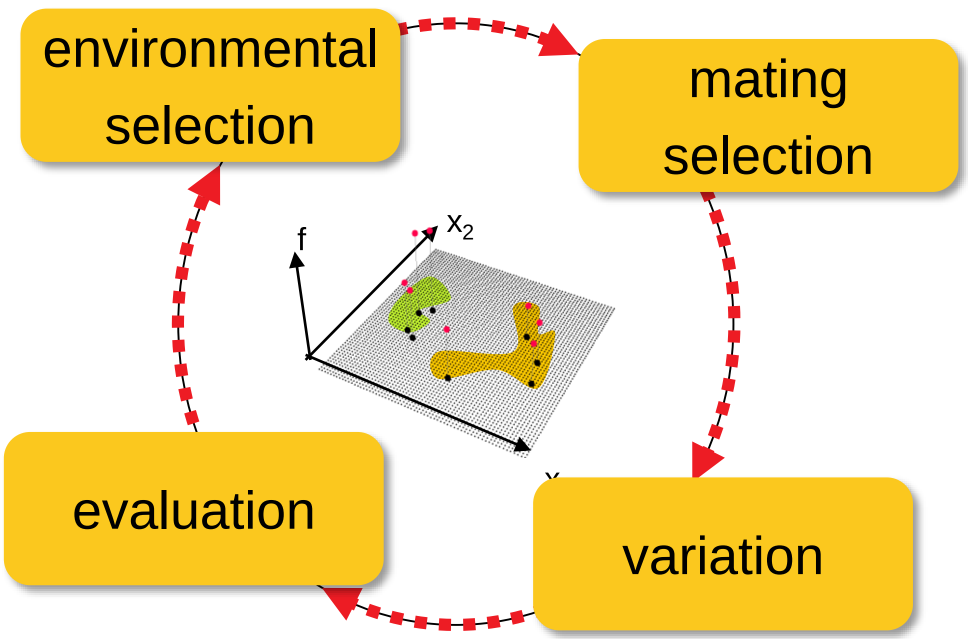
- non-linear
- noisy
- uncertain
- expensive (integrated simulations, real experiments)
- non-differentiable

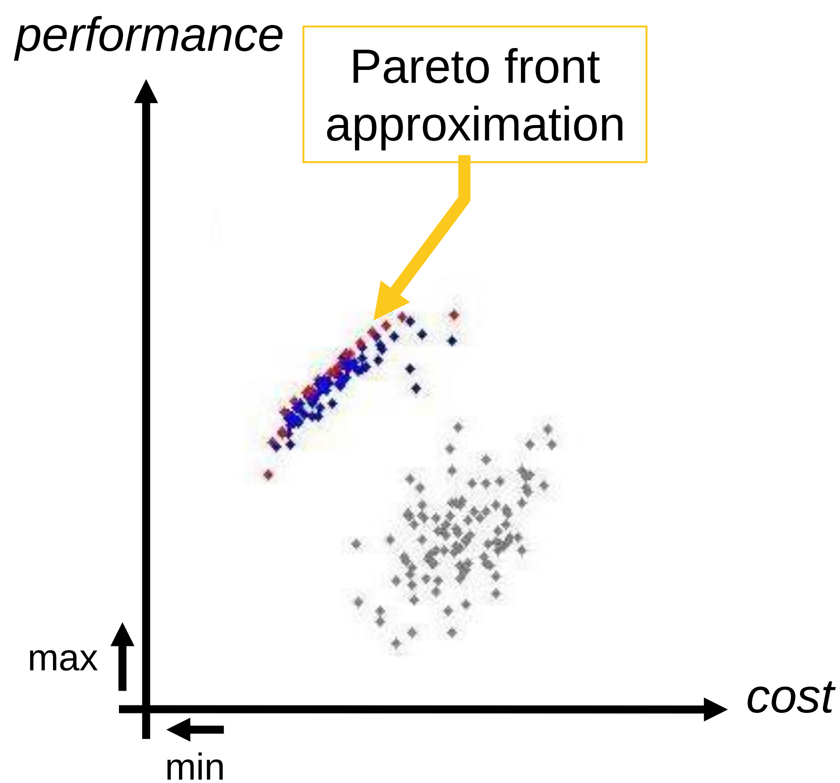
problem

- many objectives
- huge search spaces
- many constraints

Evolutionary Multiobjective Optimization

- set-based algorithms
- therefore possible to approximate the Pareto front in one run



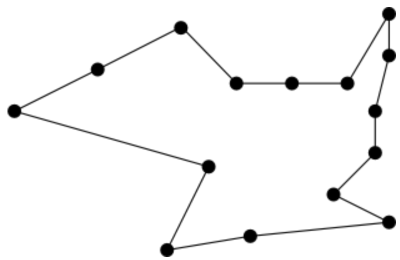


Multiobjectivization

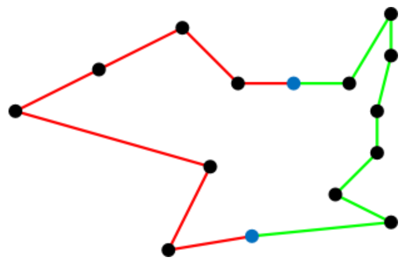
Some problems are easier to solve in a multiobjective scenario

example: TSP

[Knowles et al. 2001]



$\pi \in S_n \to f(\pi)$



$\pi \in S_n \to (f_1(\pi, a, b), f_2(\pi, a, b))$

Multiobjectivization

- by addition of new "helper objectives" [Jensen 2004]
  - job-shop scheduling [Jensen 2004], frame structural design [Greiner et al. 2007], VRP [Watanabe and Sakakibara 2007], ...
- by decomposition of the single objective
  - TSP [Knowles et al. 2001], minimum spanning trees [Neumann and Wegener 2006], protein structure prediction [Handl et al. 2008a], ...

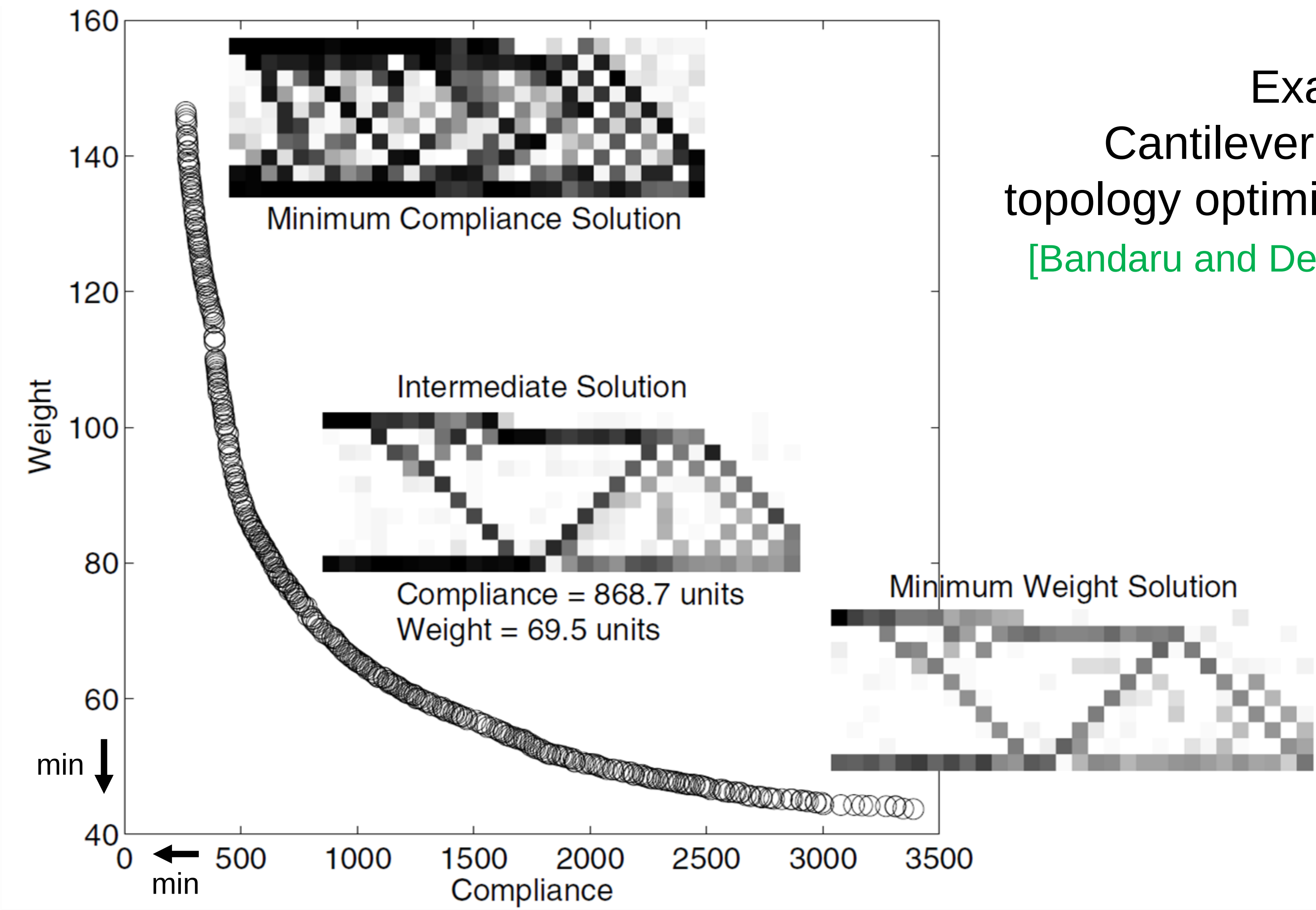
also backed up by theory e.g. [Brockhoff et al. 2009, Handl et al. 2008b]

related to *constrained* and *multimodal* single-objective optimization

see also this recent overview: [Segura et al. 2013]

Innovization

Often innovative design principles among solutions are found



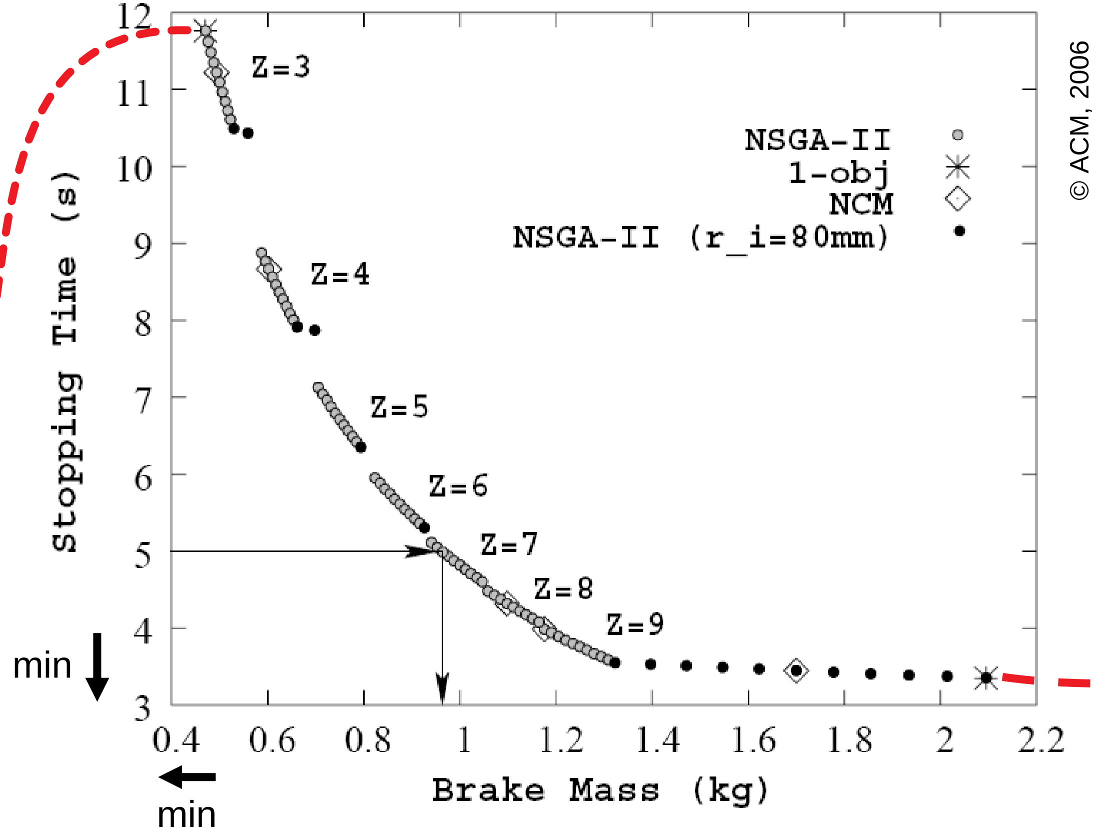
Example:
Cantilever beam
topology optimization
[Bandaru and Deb 2015]

Often innovative design principles among solutions are found

Example:

Clutch brake design

[Deb and Srinivasan 2006]



Solution	x_1	x_2	x_3	x_4	x_5	f_1	f_2
Min. f_1	70	90	1.5	1000	3	0.4704	11.7617
Min. f_2	80	110	1.5	1000	9	2.0948	3.3505

Innovization

Often innovative design principles among solutions are found

Innovization [Deb and Srinivasan 2006]

= using machine learning techniques to find new and innovative design principles among solution sets

= learning from/about a multiobjective optimization problem

Other examples:

- Self-Organizing Maps for supersonic wing design [Obayashi and Sasaki 2003]
- Biclustering for processor design and knapsack [Ulrich et al. 2007]
- Successful case studies in engineering (noise barrier design, polymer extrusion, friction stir welding) [Deb et al. 2014]

Course Overview

	Date		Topic
1	Tue, 22.11.2016	Dimo	Randomized Algorithms for Discrete Problems
2	Tue, 29.11.2016	Dimo	Exercise: The Travelling Salesperson Problem
3	Tue, 6.12.2016	Anne	Continuous Optimization I
	vacation		
4	Tue, 3.1.2017	Anne	Continuous Optimization II
5	Tue, 10.1.2017	Anne	Continuous Optimization III
6	Tue, 17.1.2017	Dimo	Evolutionary Multiobjective Optimization I
7	Tue, 31.1.2017	Dimo	Evolutionary Multiobjective Optimization II
	Feb 20 & Feb 24		oral presentations (individual time slots)

all from 14:00 till 17:15 in PUIO - E213

Organization Oral Exams

to be decided until last class (Jan 31), better today ☺

	Monday, Feb 20	Friday, Feb 24
10am		
10:30am		
11am	Anh Khoa Ngo Ho	
11:30am	Ahmed Mazari	
12am	Abdallah Benzine	
12:30pm	Jonathan Crouzet	
1:30pm	Mohamed Ali Fathallah	Amal Targhi
2:00pm	Antonin Raffin	Abdelhak Loukkal
2:30pm	Gabriel Quere	Yuxiang Wang
3pm	Laurent Cetinsoy	
3:30pm	Ghazi Felhi	
4pm	Clément Thierry	

Overview

The Big Picture

Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

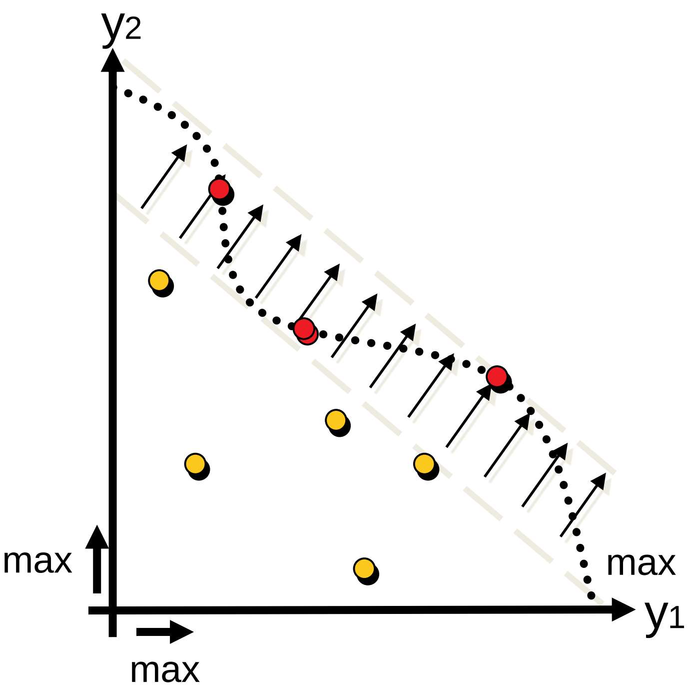
Selected Advanced Concepts

- preference articulation
- visualization aspects

Approaches to Multiobjective Optimization

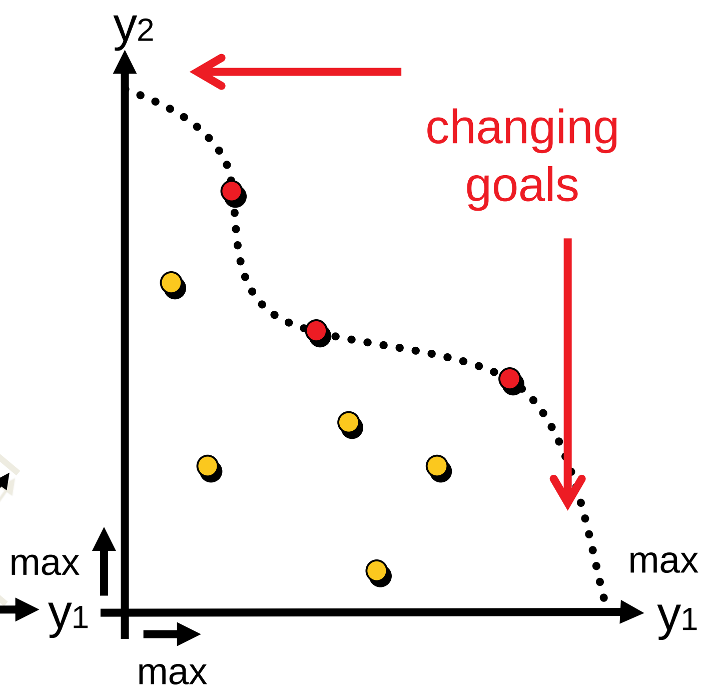
aggregation-based

problem decomposition (multiple single-objective optimization problems)



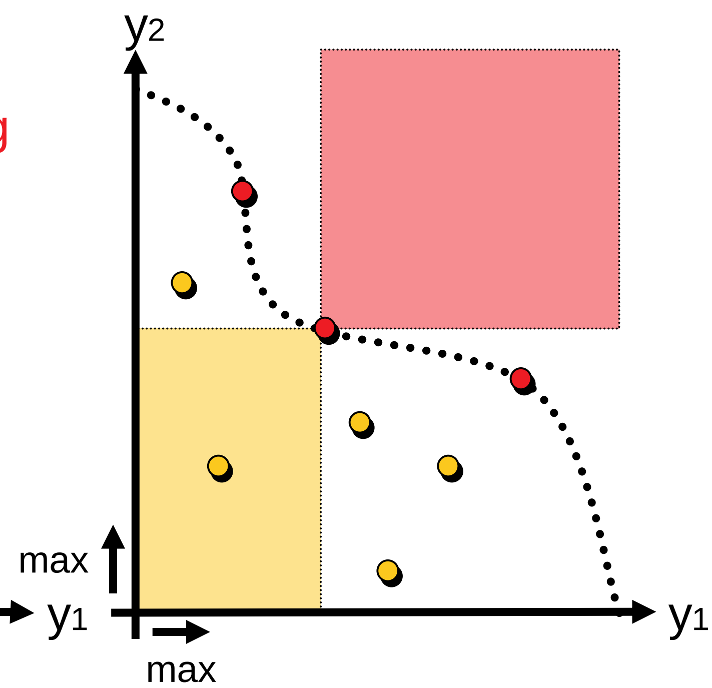
criterion-based

VEGA



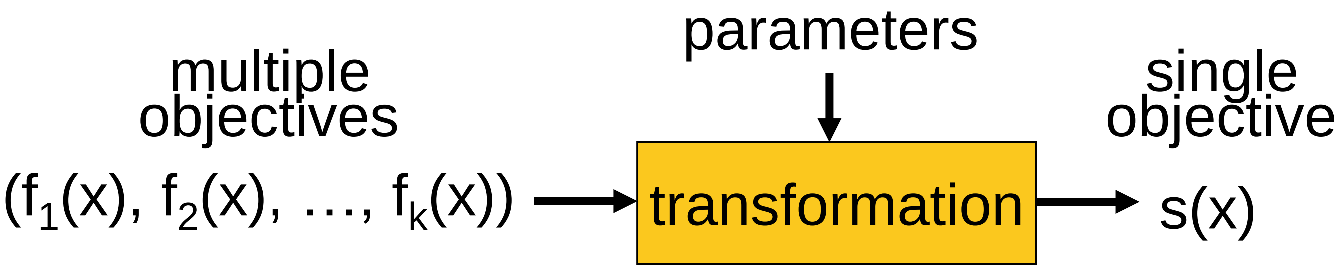
dominance-based

SPEA2, NSGA-II
“modern” EMOA

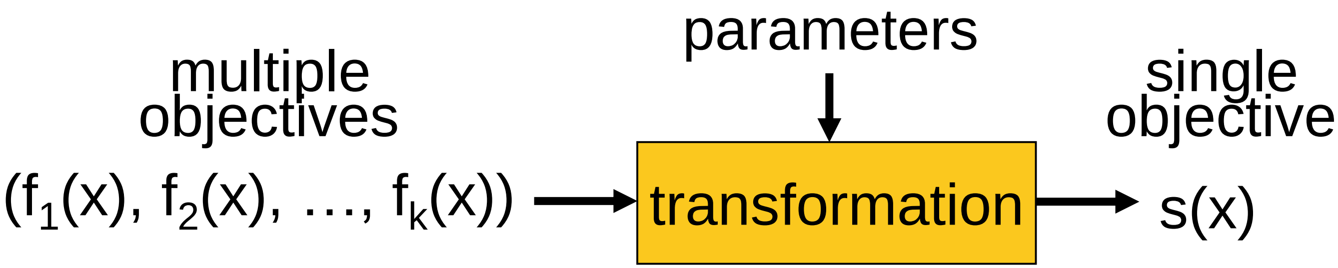


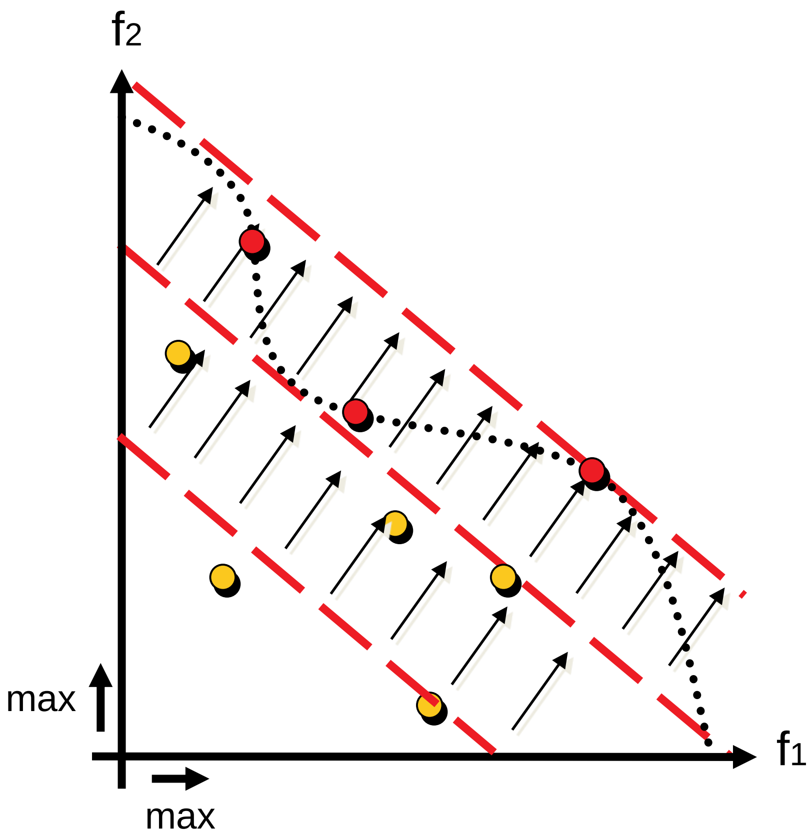
solution-oriented
scaling-dependent

set-oriented
less scaling-independent

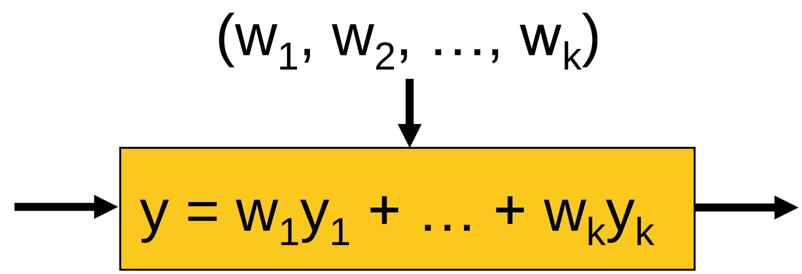


A scalarizing function s is a function $s : Z \to \mathbb{R}$ that maps each objective vector $u = (u_1, \ldots, u_n) \in Z$ to a real value $s(u) \in \mathbb{R}$

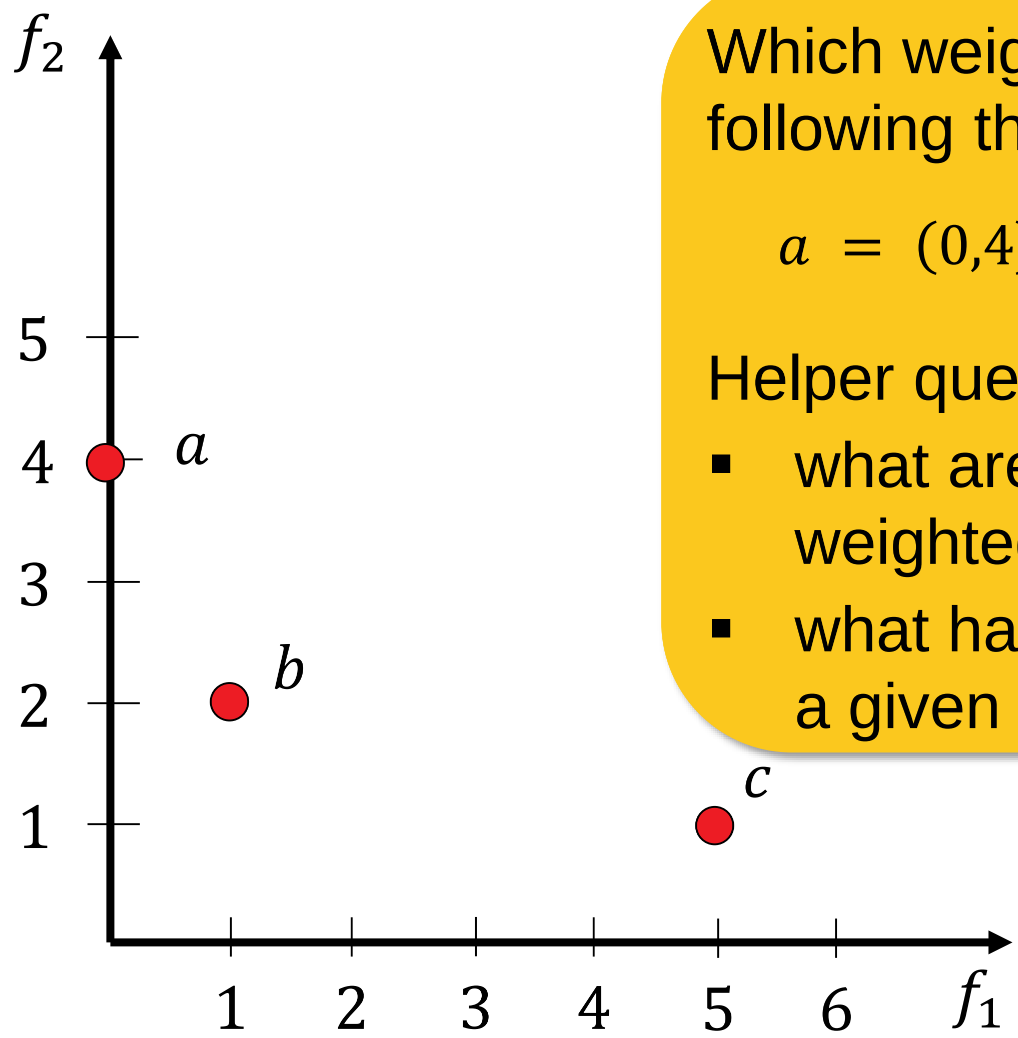




Example 1: weighted sum approach



Exercise 4: Weighted Sum



Which weights are optimal for the following three points?

$$a = (0,4) \quad b = (1,2) \quad c = (5,1)$$

Helper questions:

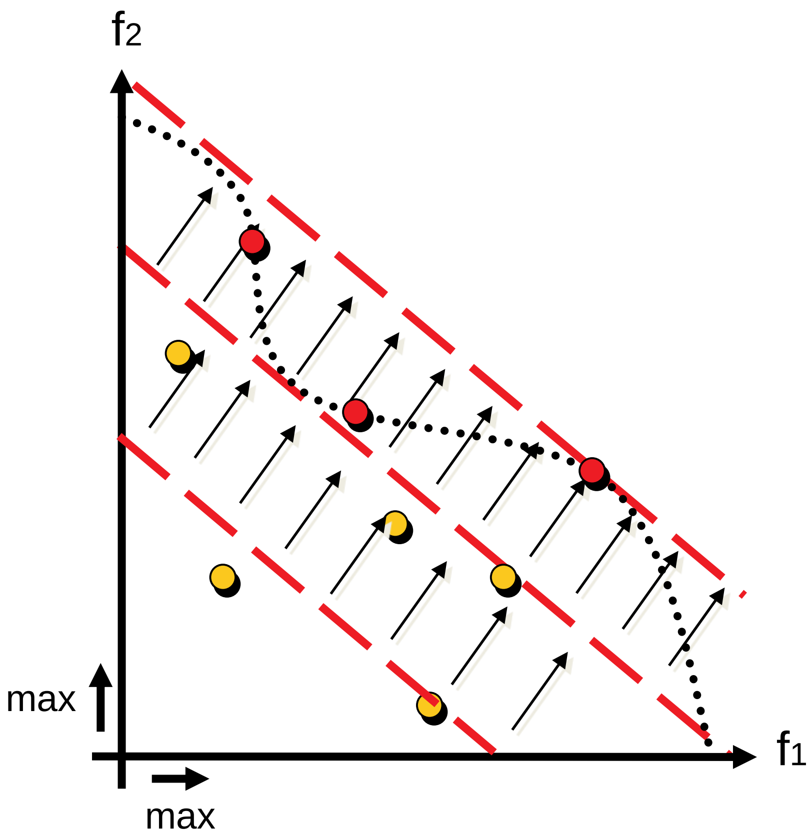
- what are the lines of equal weighted sum for a given weight?
- what happens if you optimize wrt. a given weighted sum?

Solution-Oriented Problem Transformations

multiple objectives

$(f_1(x), f_2(x), \ldots, f_k(x))$ → **transformation** (parameters) → $s(x)$

single objective



Example 1: weighted sum approach

$(w_1, w_2, \ldots, w_k)$

→ $y = w_1 y_1 + \ldots + w_k y_k$ →

Disadvantage: not all Pareto-optimal solutions can be found if the front is not concave (for maximization)

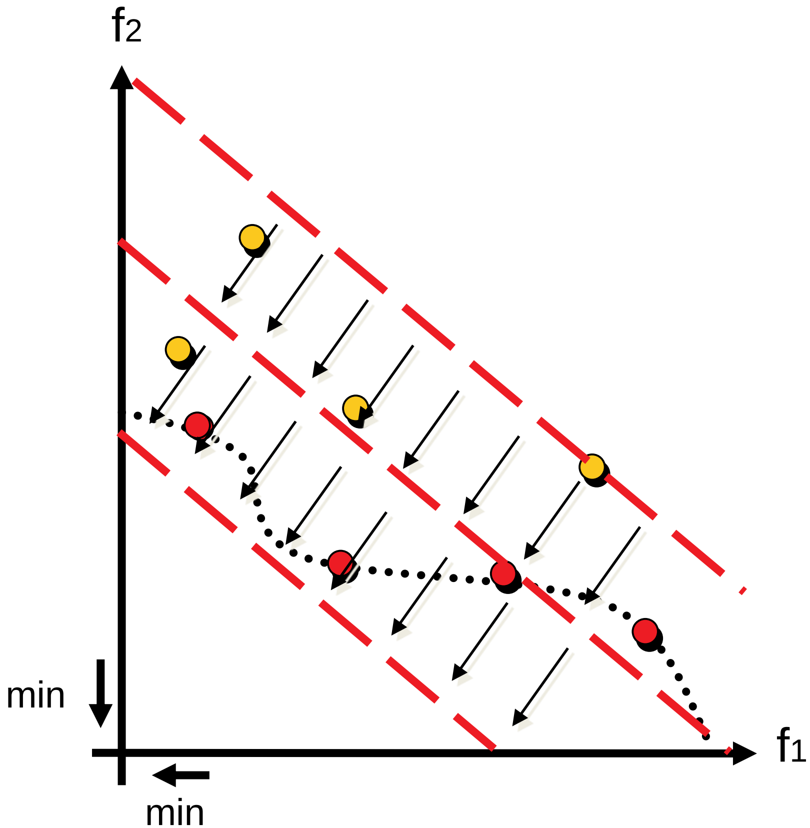
Solution-Oriented Problem Transformations

multiple objectives

$(f_1(x), f_2(x), \ldots, f_k(x))$ → transformation → $s(x)$

parameters

single objective



Example 1: weighted sum approach

$(w_1, w_2, \ldots, w_k)$

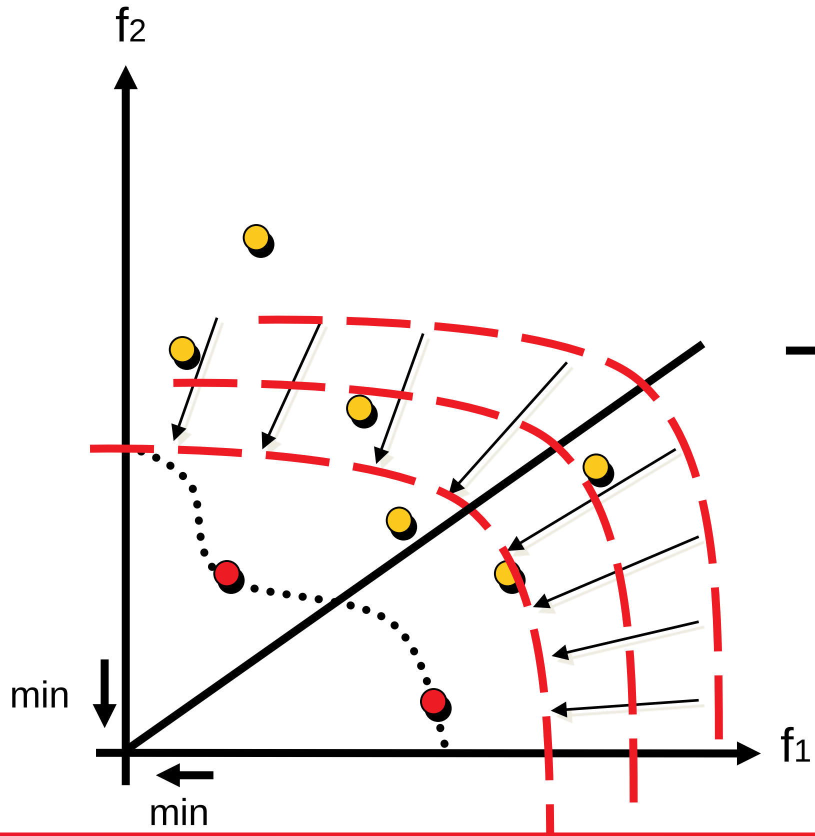
$y = w_1 y_1 + \ldots + w_k y_k$

Disadvantage: not all Pareto-optimal solutions can be found if the front is not convex (for minimization)

Solution-Oriented Problem Transformations

parameters

multiple objectives $(f_1(x), f_2(x), \ldots, f_k(x))$ → **transformation** → single objective $s(x)$



Example 2: weighted p-norm

$(w_1, w_2, \ldots, w_k)$

$$y = \sqrt[p]{(w_1 y_1)^p + \ldots + (w_k y_k)^p}$$

$p = 1$: weighted sum

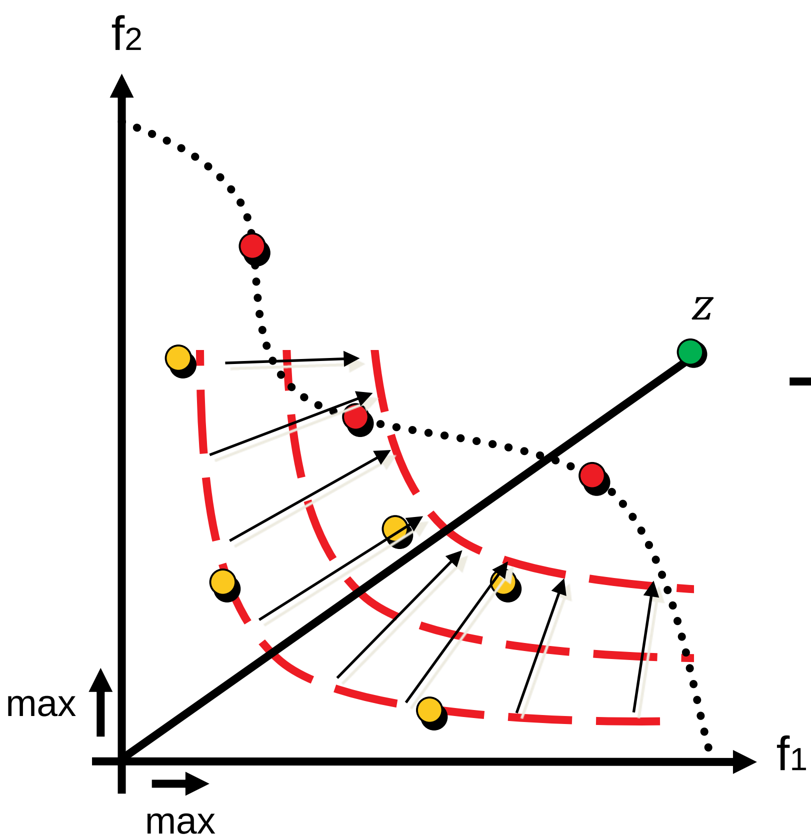
$p = \infty$: weighted Tchebycheff

Solution-Oriented Problem Transformations

multiple objectives

$(f_1(x), f_2(x), \dots, f_k(x))$ → transformation → single objective $s(x)$

parameters ↓ (into transformation)



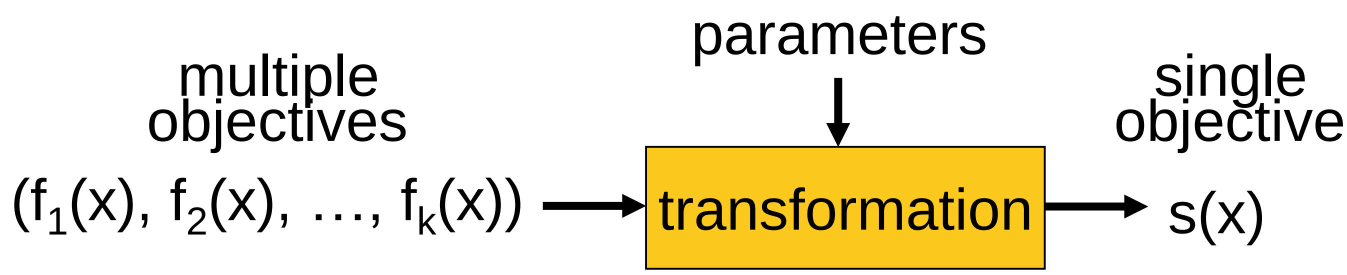
Example 2: weighted p-norm

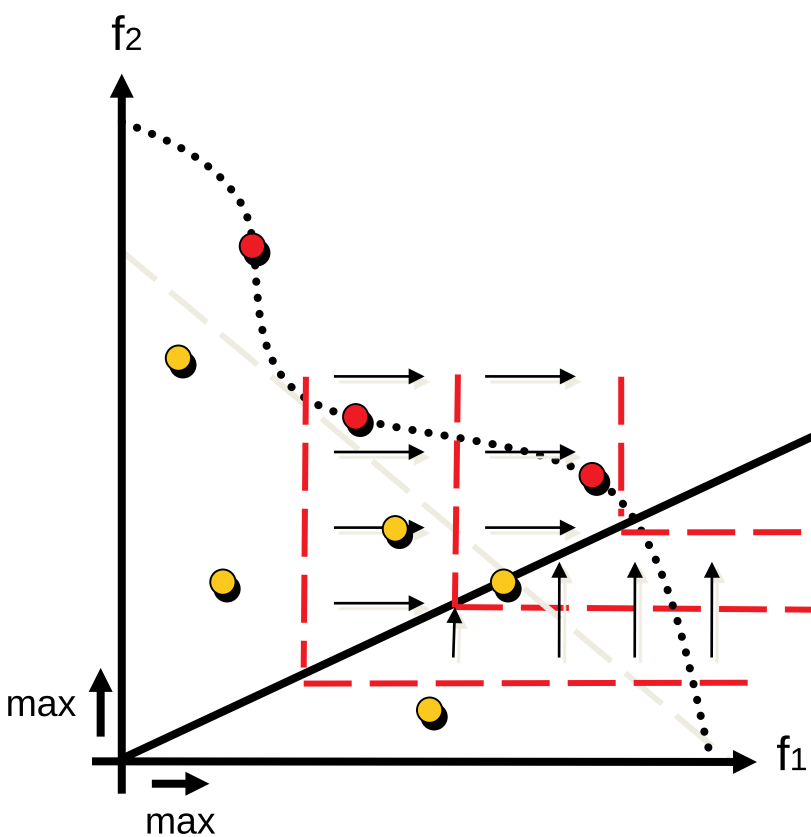
$(w_1, w_2, \dots, w_k)$

$$y = \sqrt[p]{\sum_{i=1}^{k} (|w_1(y_1 - z_i)|)^p}$$

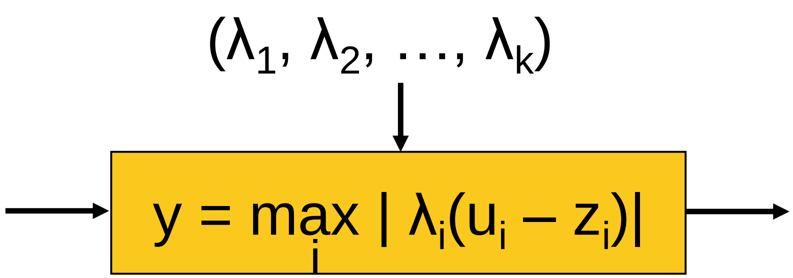
$p = 1$: weighted sum

$p = \infty$: weighted Tchebycheff



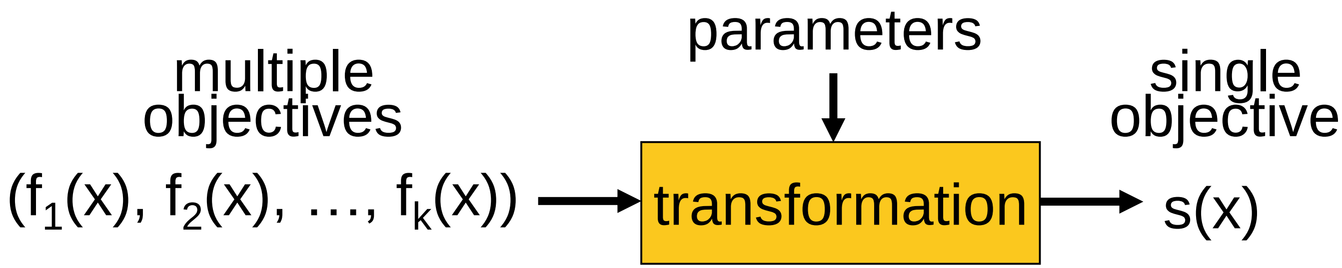


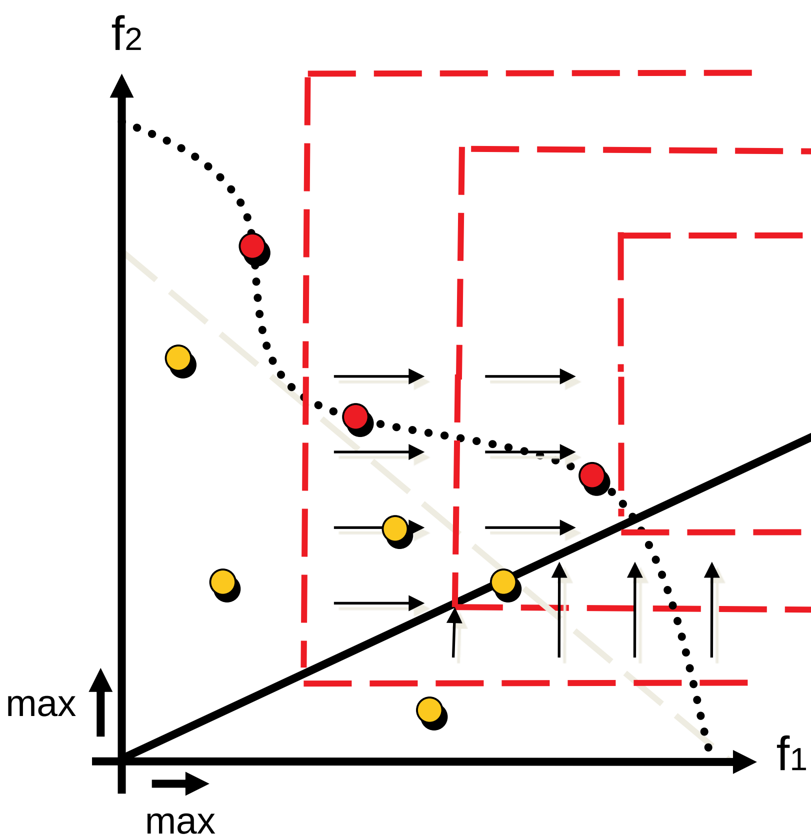
Example 2: weighted Tchebycheff



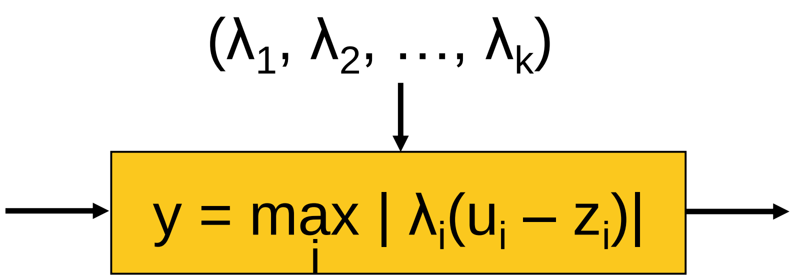
Several other scalarizing functions are known, see e.g. [Miettinen 1999]

Solution-Oriented Problem Transformations





Example 2: weighted Tchebycheff



Several other scalarizing functions are known, see e.g. [Miettinen 1999]

Exercise: Benchmarking a Weighted Sum Approach on COCO

Exercise

Goal: Implement a Simple Weighted Sum Approach:

- N scalarizing functions, optimized with CMA-ES
- Python: use CMA-ES after **`pip install cma`** (more details here: **`https://pypi.python.org/pypi/cma`**)
- use ask and tell interface (next slide)
- CMA-ES parameters as default (with $\sigma_{init} = 3$ and initialized in [-5,5]
- no details given about:
  - how to normalize the objectives and estimate z
  - the order in which the N scalarizing functions are optimized
  - how to do restarts and how to distribute the budget

2nd Goal:

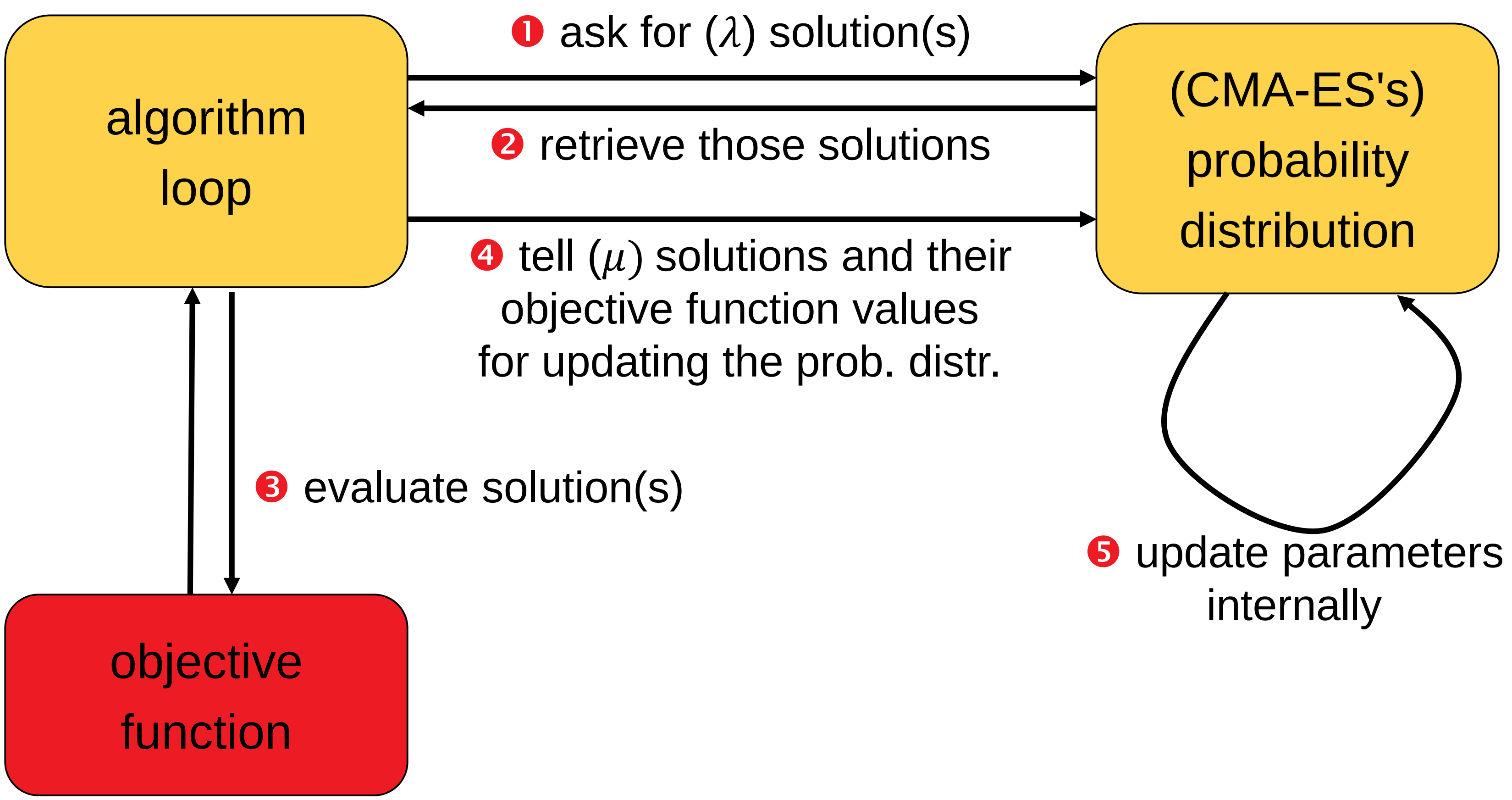
- produce data for the new **`bbob-biobj-ext`** suite
- hence, interested in your evaluations

The Idea of the Ask&Tell Interface to Optimization

example from the CMA-ES web page:

```
>>> import cma
>>> es = cma.CMAEvolutionStrategy(12 * [0], 0.5)
>>> while not es.stop():
...     solutions = es.ask()
...     es.tell(solutions,
                [cma.fcts.rosen(x) for x in solutions])
...     es.logger.add() # write data to disc
                          to be plotted
...     es.disp()
<output omitted>
>>> es.result_pretty()
<output omitted>
>>> cma.plot() # shortcut for es.logger.plot()
```

Ask&Tell with CMA-ES (Visually)



Exercise: concrete

a) download COCO (release 2.0) from **`https://github.com/numbbo/coco/`**

b) install and test it via **`python do.py run-python`**

c) run the previous example code of CMA-ES (e.g. in ipython shell) to get an idea how it works

d) start your implementation of a weighted sum optimizer from **`researchers.lille.inria.fr/~brockhof/advancedOptSaclay/2016/exercises/example_experiment-WS.py`** within the function **`def weighted_sum(fun, budget)`**

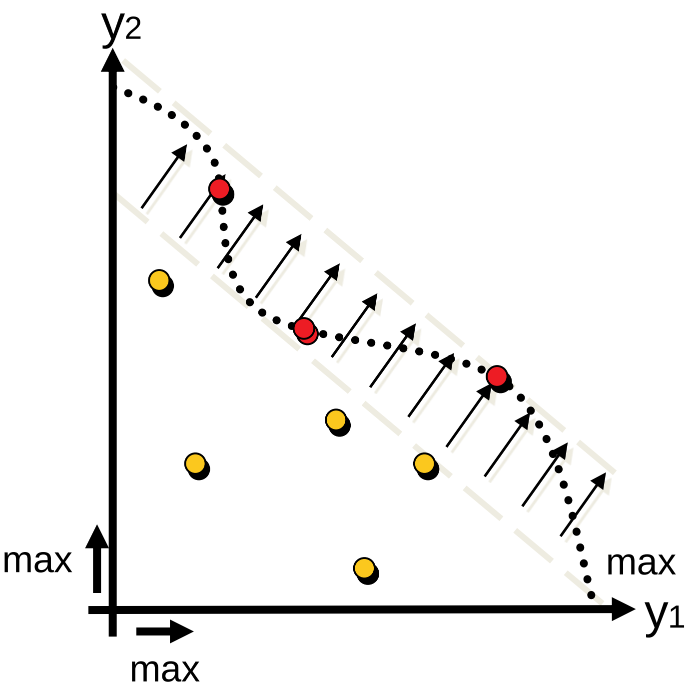
tip: start simple and extend!

e) run the experiments by typing **`python example_experiment-WS.py bbob-biobj-ext BUDGET`** with **`BUDGET`** any integer (start small and then increase) and send all data to me by email ☺

Approaches to Multiobjective Optimization

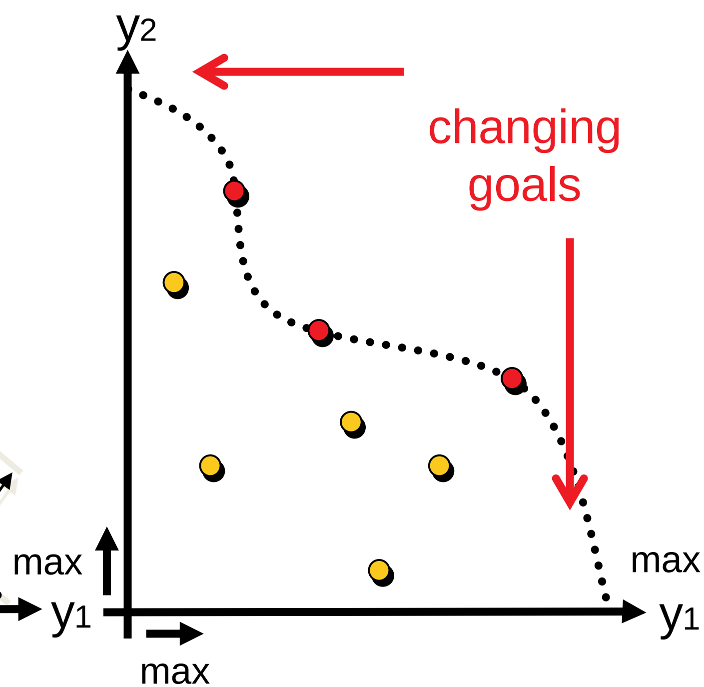
aggregation-based

problem decomposition (multiple single-objective optimization problems)



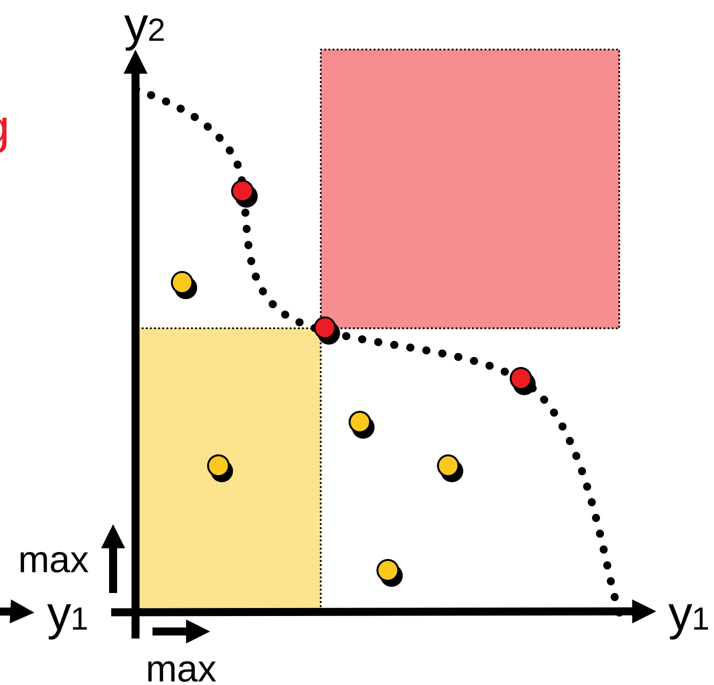
criterion-based

VEGA



dominance-based

SPEA2, NSGA-II
“modern” EMOA

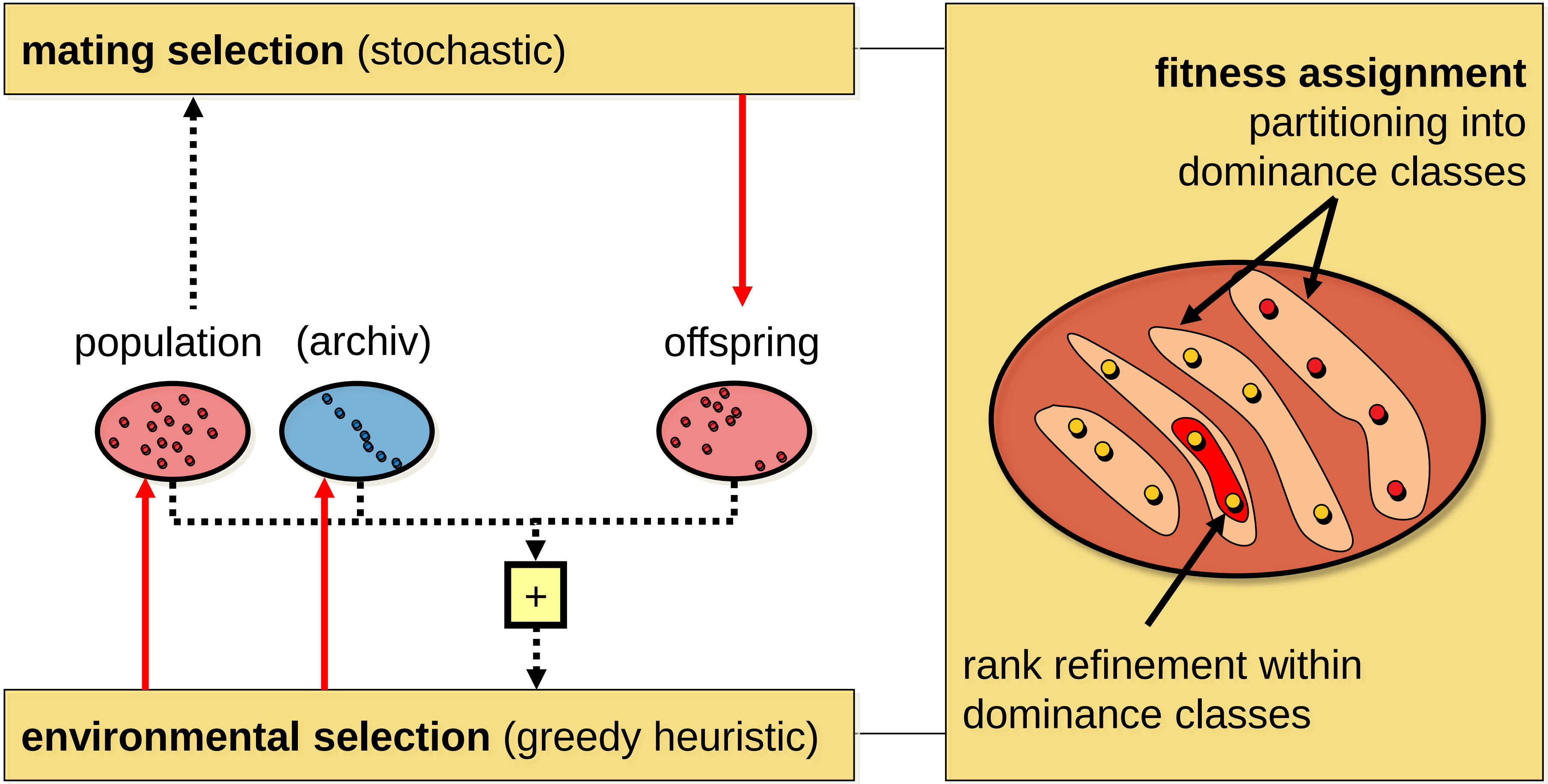


solution-oriented
scaling-dependent

set-oriented
less scaling-independent

Set-Oriented Approaches

General Scheme of Most Set-Oriented EMO

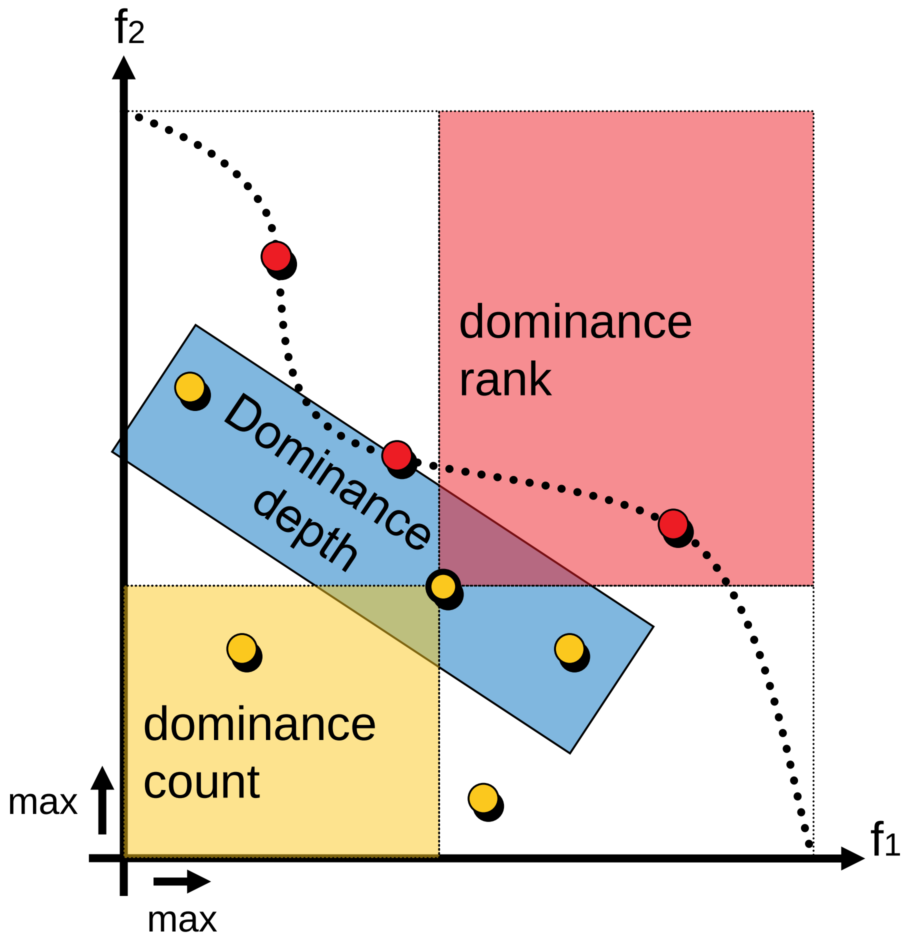


Ranking of the Population Using Dominance

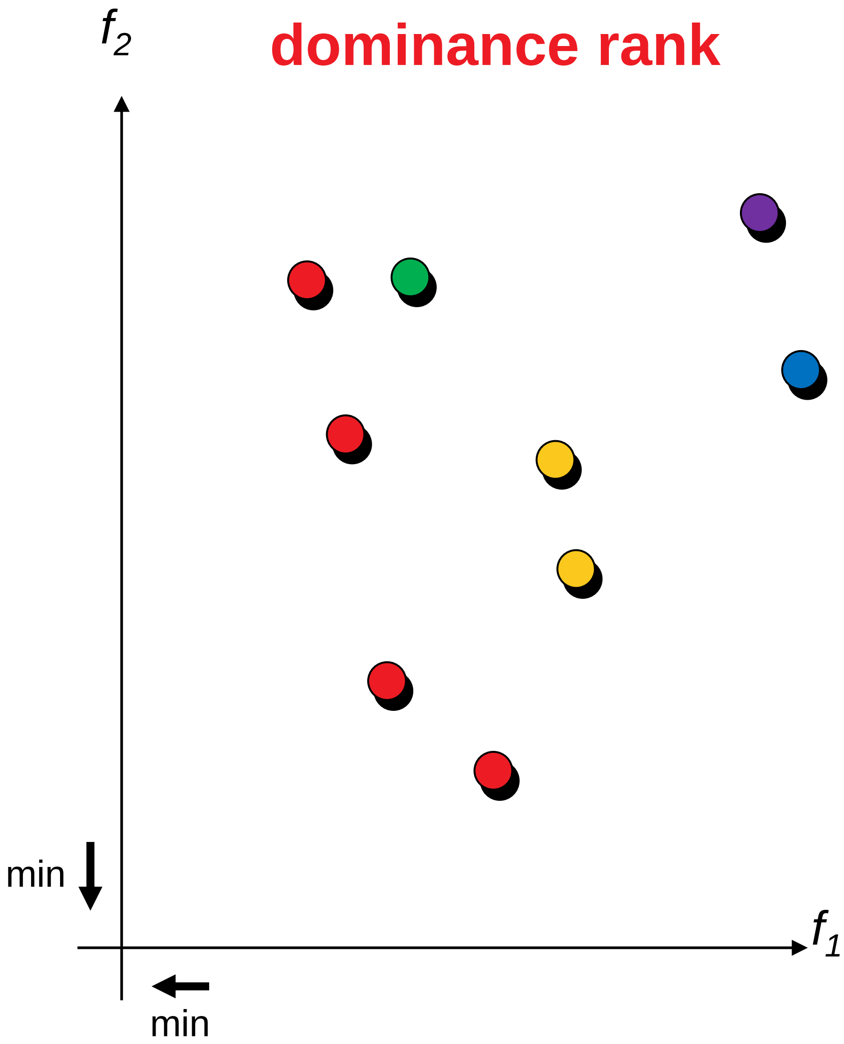
... goes back to a proposal by David Goldberg in 1989.

... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  MOGA, NPGA
- **dominance count:** how many individuals does an individual dominate?
  SPEA, SPEA2
- **dominance depth:** at which front is an individual located?
  NSGA, NSGA-II, most of the recently proposed algorithms



Exercise: Dominance-Based Partitioning



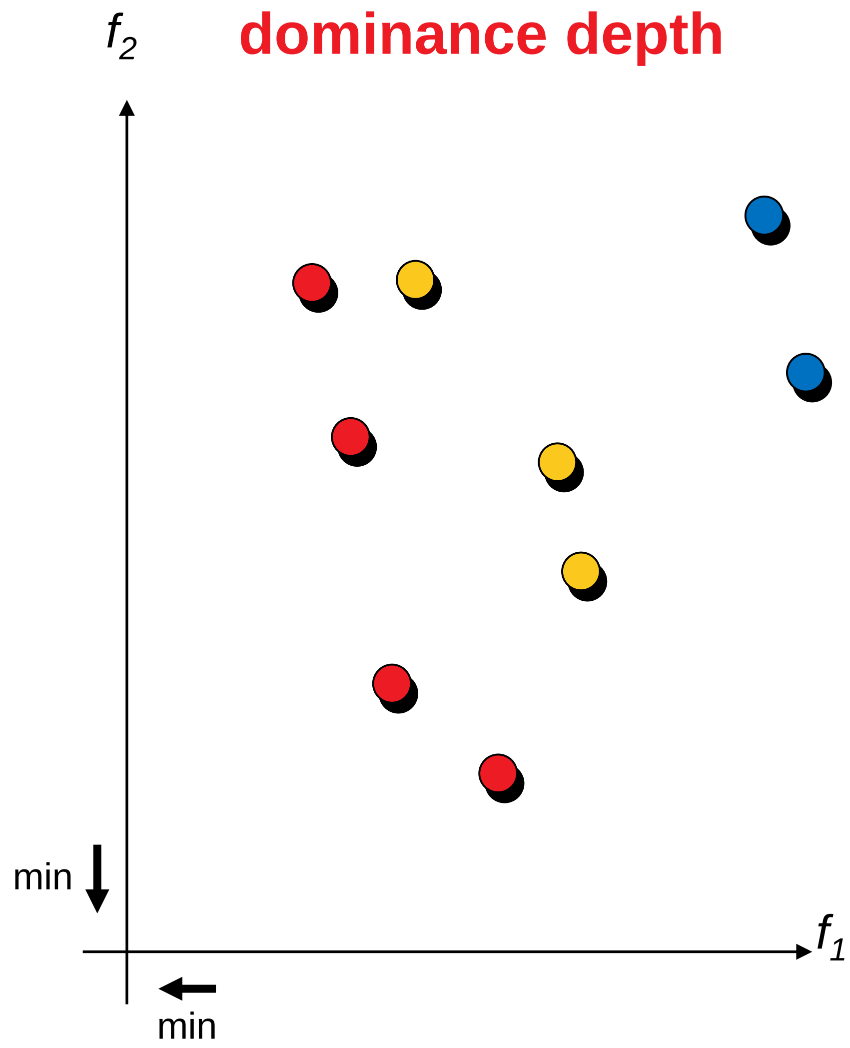
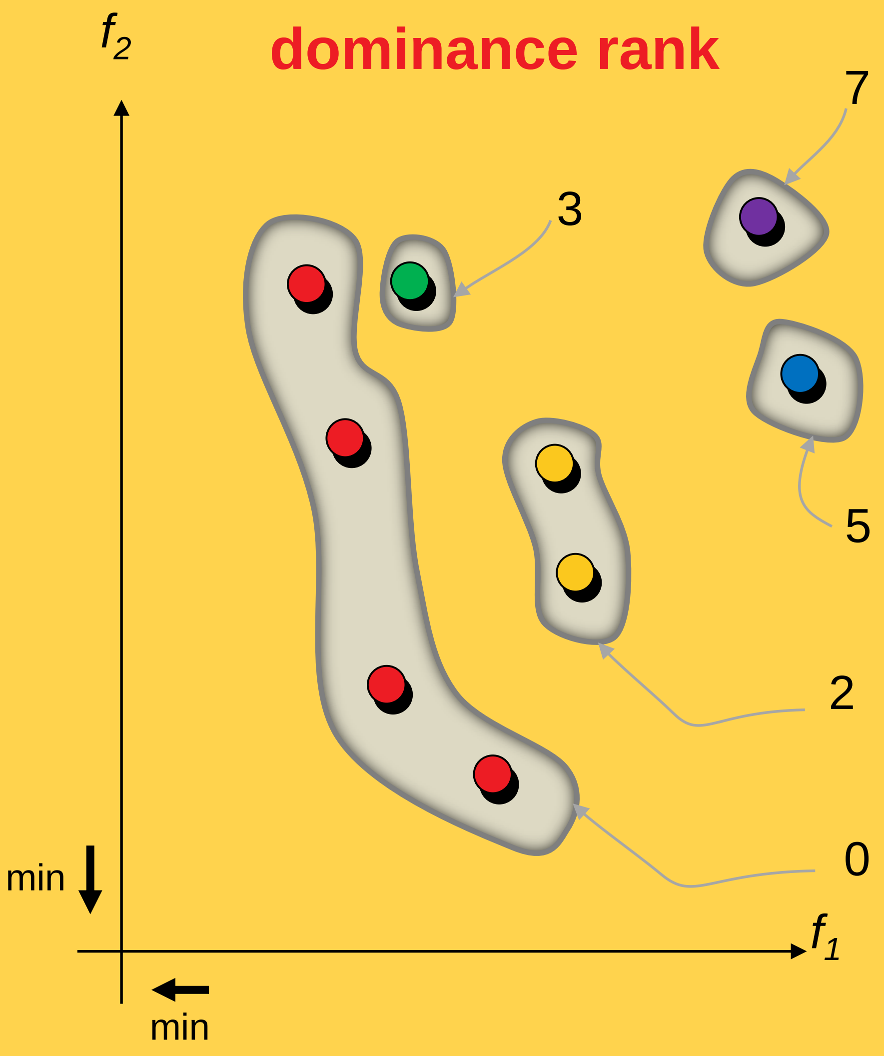
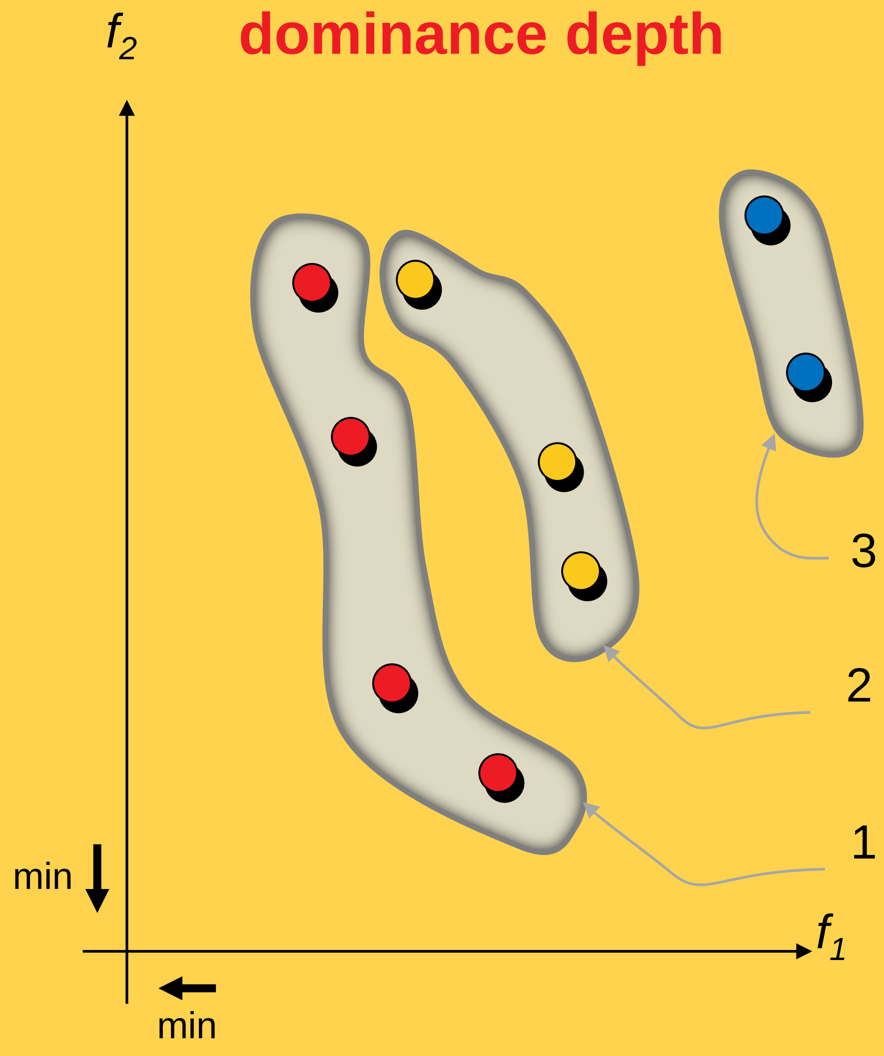


Illustration of Dominance-Based Partitioning



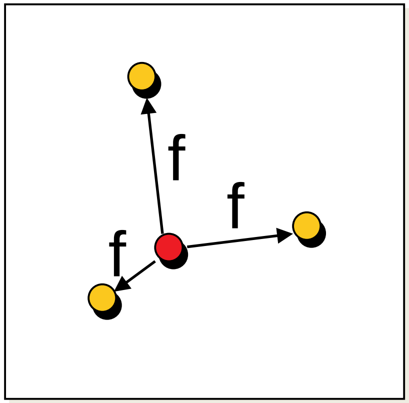


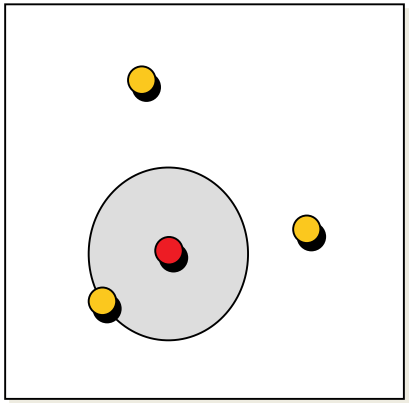
Refinement of Dominance Rankings

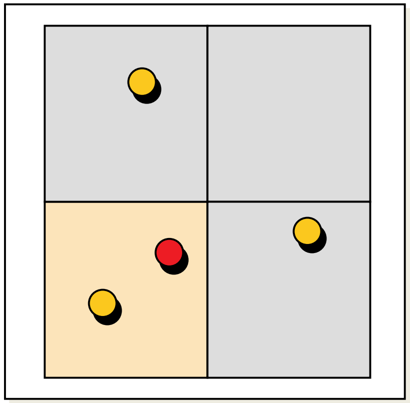
Goal: rank incomparable solutions within a dominance class

❶ Diversity information

Kernel method	k-th nearest neighbor	Histogram method
diversity = function of the distances	diversity = function of distance to k-th nearest neighbor	diversity = number of elements within box(es)



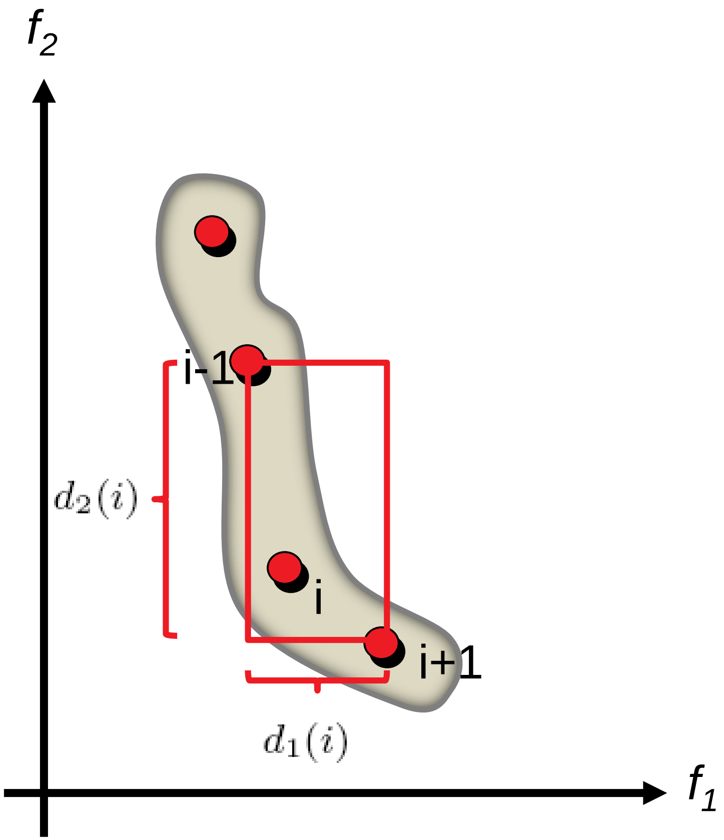




❷ (Contribution to a) quality indicator

Crowding Distance (CD)

- sort solutions with regard to each objective
- assign CD maximum value to extremal objective vectors
- compute CD based on the distance to the neighbors in each objective

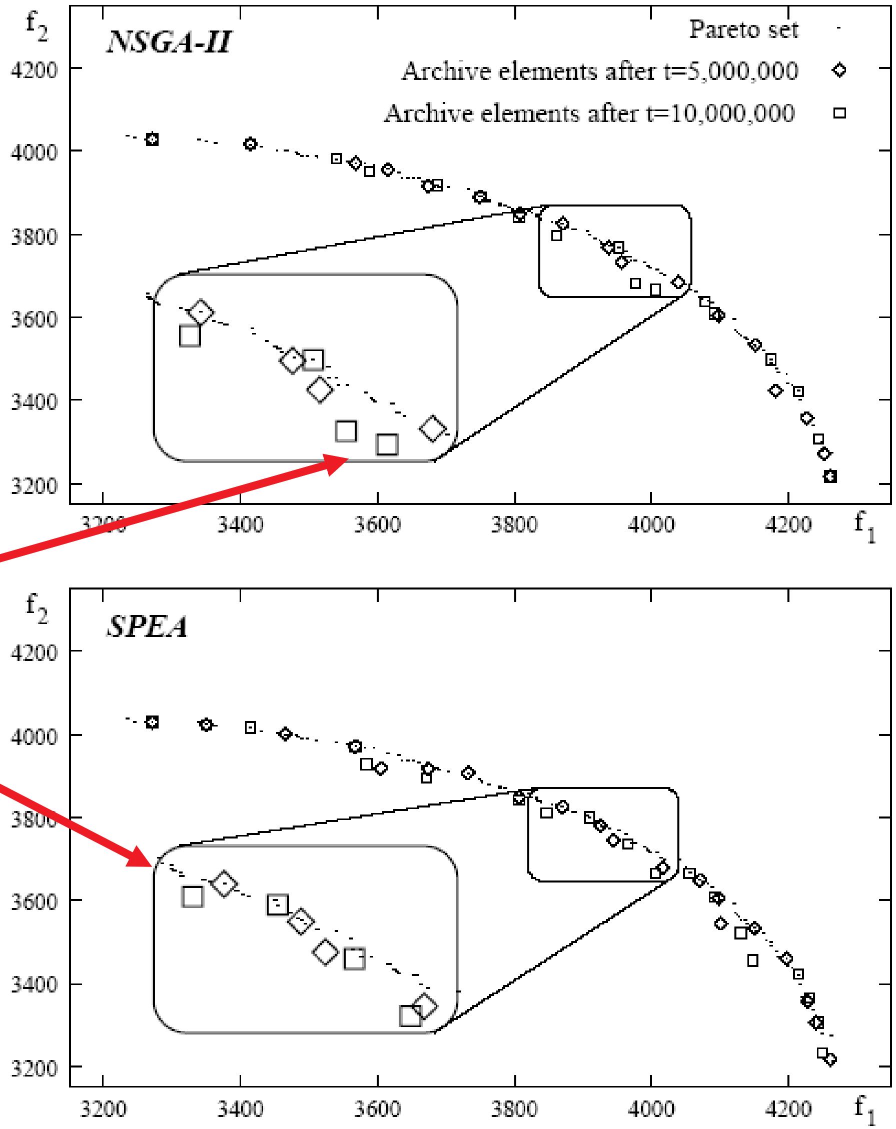


$$\mathrm{CD}(i) = \frac{d_1(i)}{f_{1,\max} - f_{1,\min}} + \cdots + \frac{d_m(i)}{f_{m,\max} - f_{m,\min}}$$

Selection in SPEA2 and NSGA-II can result in

deteriorative cycles

non-dominated solutions already found can be lost



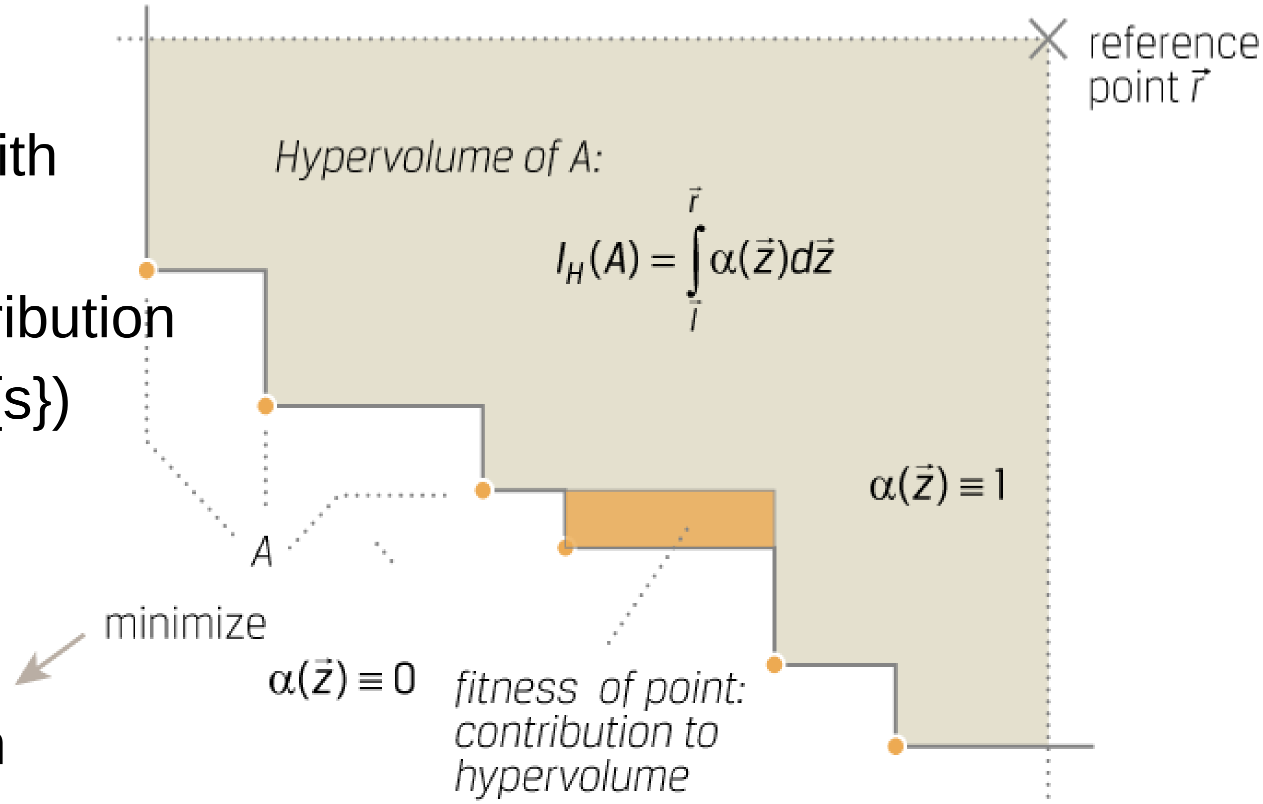
Hypervolume-Based Selection

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …)

use hypervolume indicator to guide the search: refines dominance

Main idea

Delete solutions with the smallest hypervolume contribution $d(s) = I_H(P)-I_H(P / \{s\})$ iteratively



But: can also result in cycles if reference point is not constant [Judt et al. 2011] and is expensive to compute exactly [Bringmann and Friedrich 2009]

Indicator-Based Selection

- Concept can be generalized to any quality indicator

A (unary) quality indicator I is a function $I : \Psi = 2^X \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.

$$\text{Multiobjective Problem} \xrightarrow{\text{Indicator}} \text{Single-objective Problem}$$

- for example: R2-indicator [Brockhoff et al. 2012], [Trautmann et al. 2013], [Díaz-Manríquez et al. 2013]

- Generalizable also to contribution to larger sets

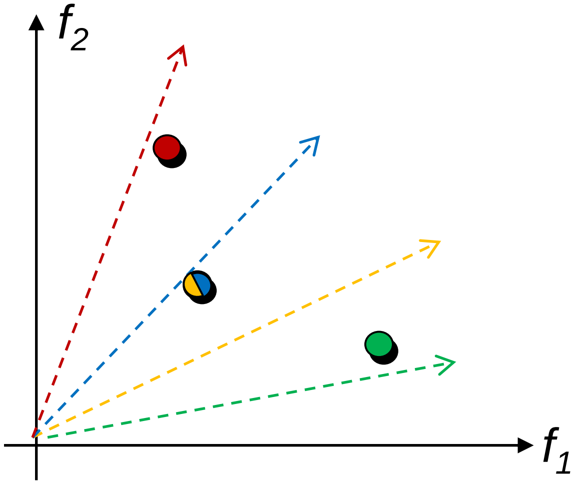
  HypE [Bader and Zitzler 2011]: Hypervolume sampling + contribution if more than 1 (random) solution deleted

Decomposition-Based Selection: MOEA/D

MOEA/D: Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

Ideas:

- optimize N scalarizing functions in parallel
- use best solutions of neighbor subproblems for mating
- keep the best solution for each scalarizing function
- update neighbors
- use external archive for non-dominated solutions
- several variants and enhancements



Remark: Variation in EMO

- at first sight not different from single-objective optimization
- most research on selection mechanisms (until now)
- but: convergence to a set ≠ convergence to a point

Open Question:

- how to achieve fast convergence to a *set*?

Related work:

- set-based gradient of the HV [Emmerich et al. 2007]
- multiobjective CMA-ES [Igel et al. 2007] [Voß et al. 2010]
- RM-MEDA [Zhang et al. 2008]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]
- offline and online configuration based on libraries of variation operators [Bezerra et al. 2015] [Hadka and Reed 2013]

Overview

The Big Picture

Basic Principles of Multiobjective Optimization

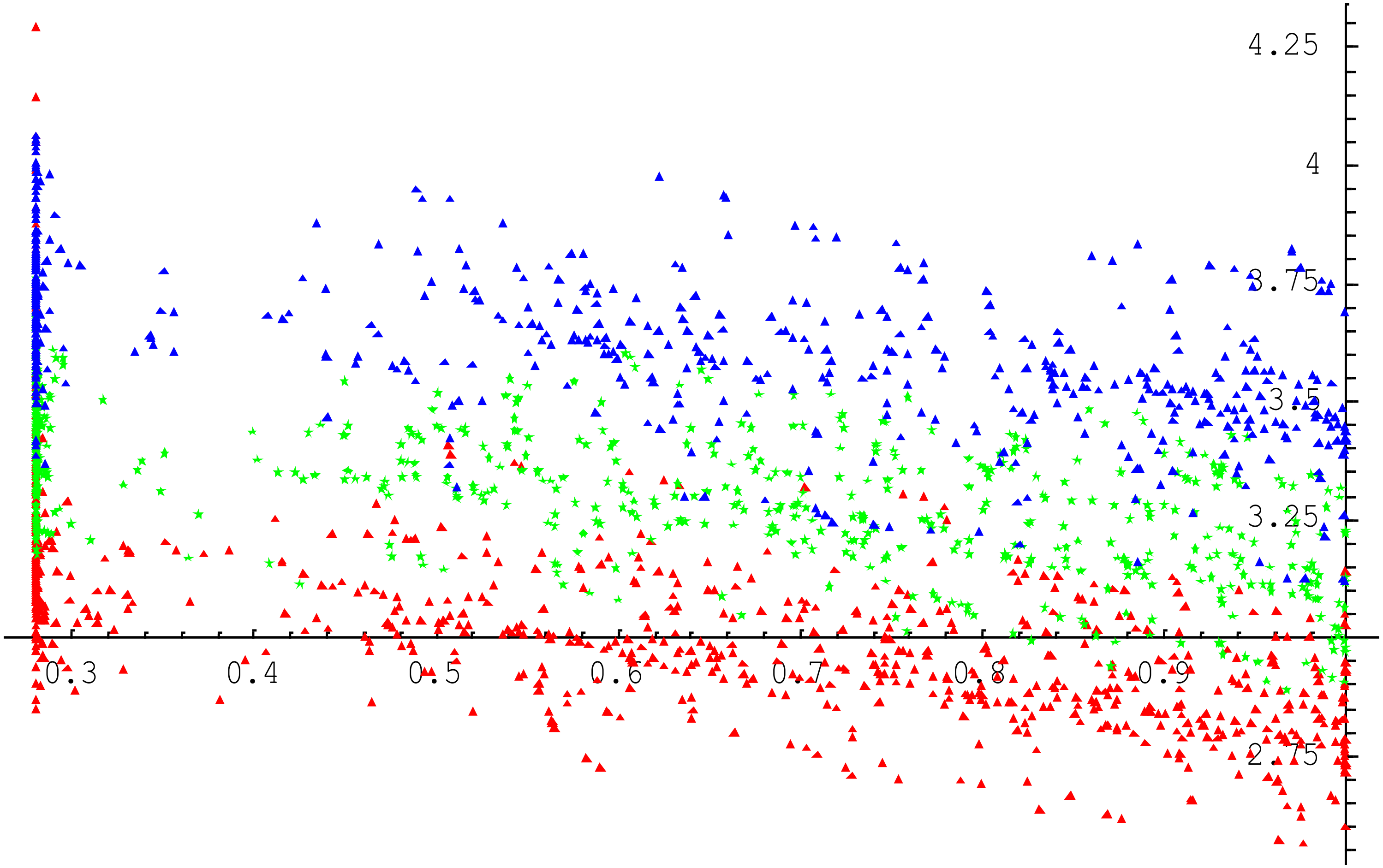
- algorithm design principles and concepts
- performance assessment

Selected Advanced Concepts

- preference articulation
- visualization aspects

Once Upon a Time...

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

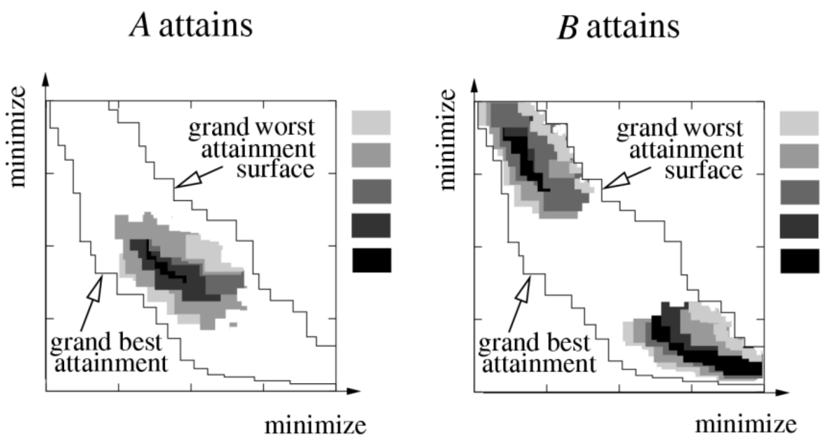
Two Main Approaches for Empirical Studies

Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur

Quality indicator approach

- reduces each approximation set to a single quality value
- applies statistical tests to the quality values

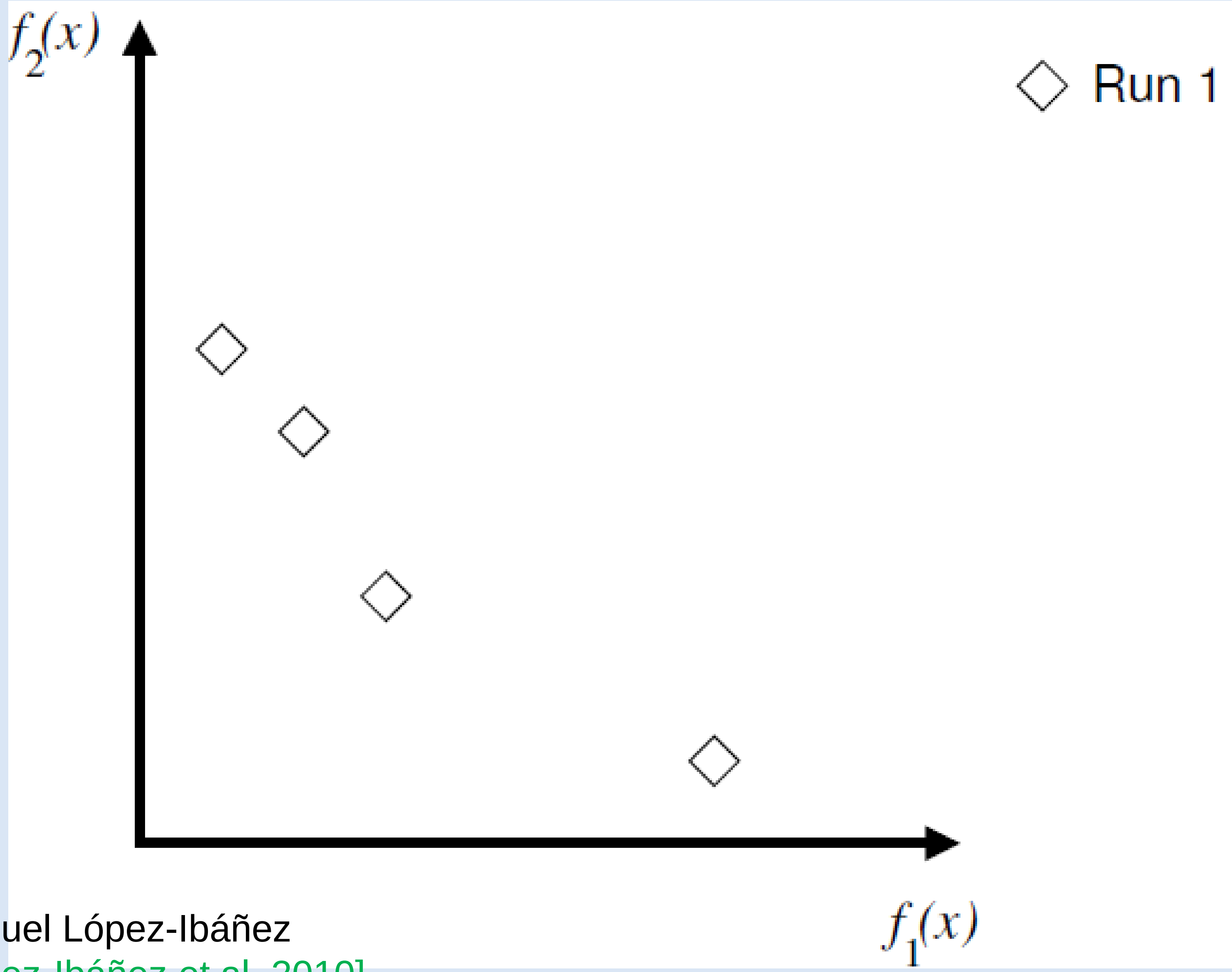


Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

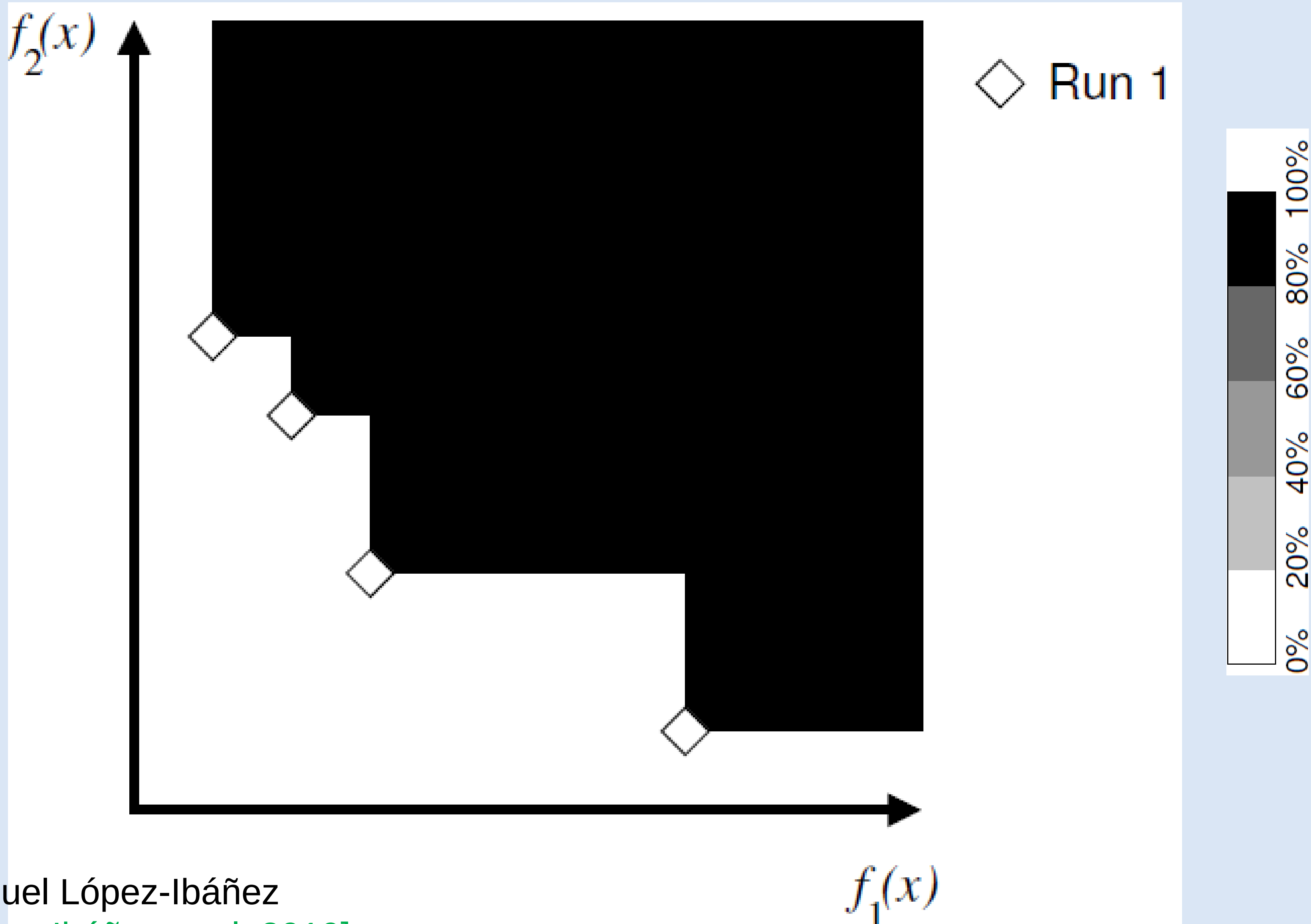
note that slides of this light blue color have not been discussed in depth during the lecture due to the restricted time

Empirical Attainment Functions: Idea



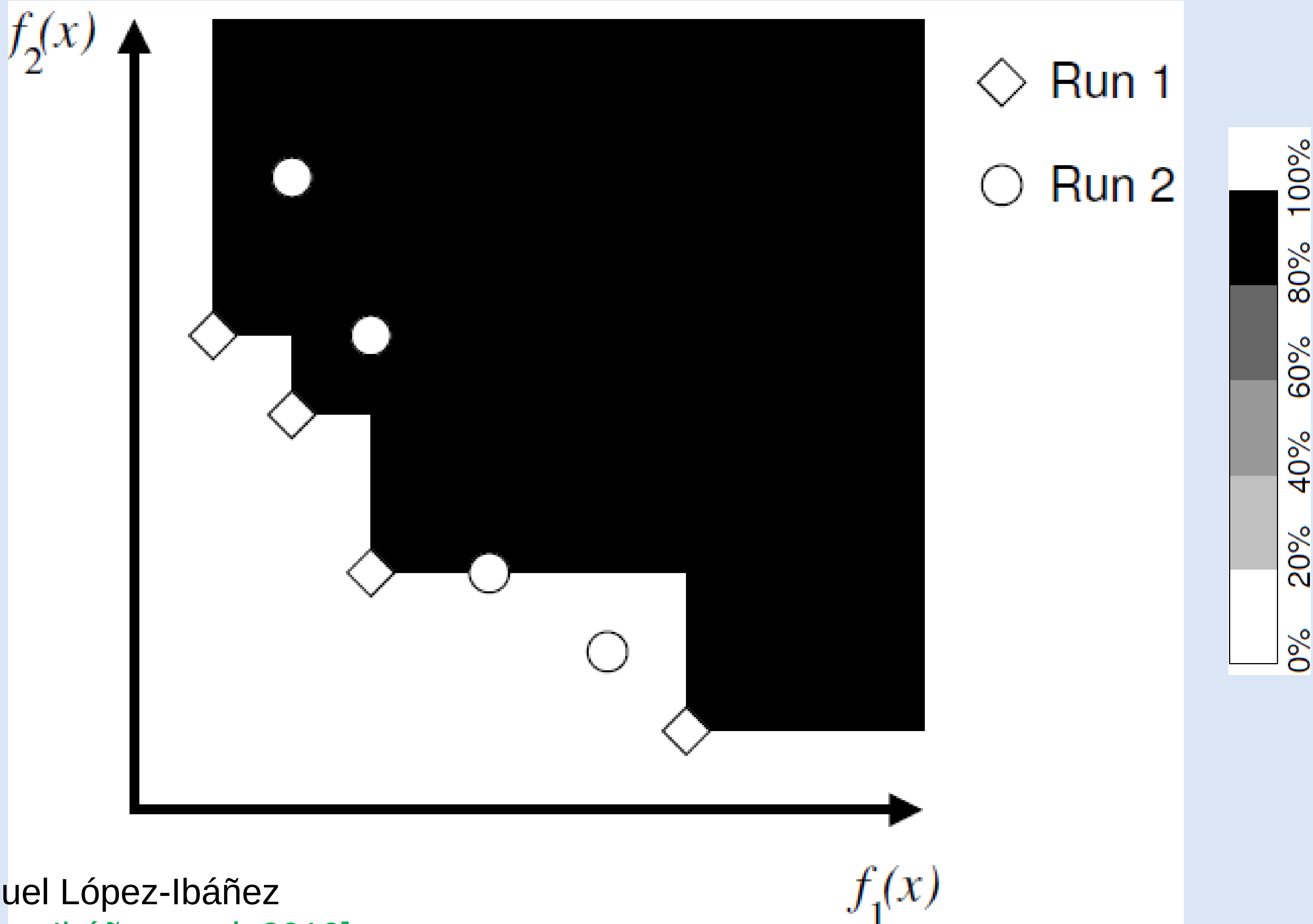
© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Idea



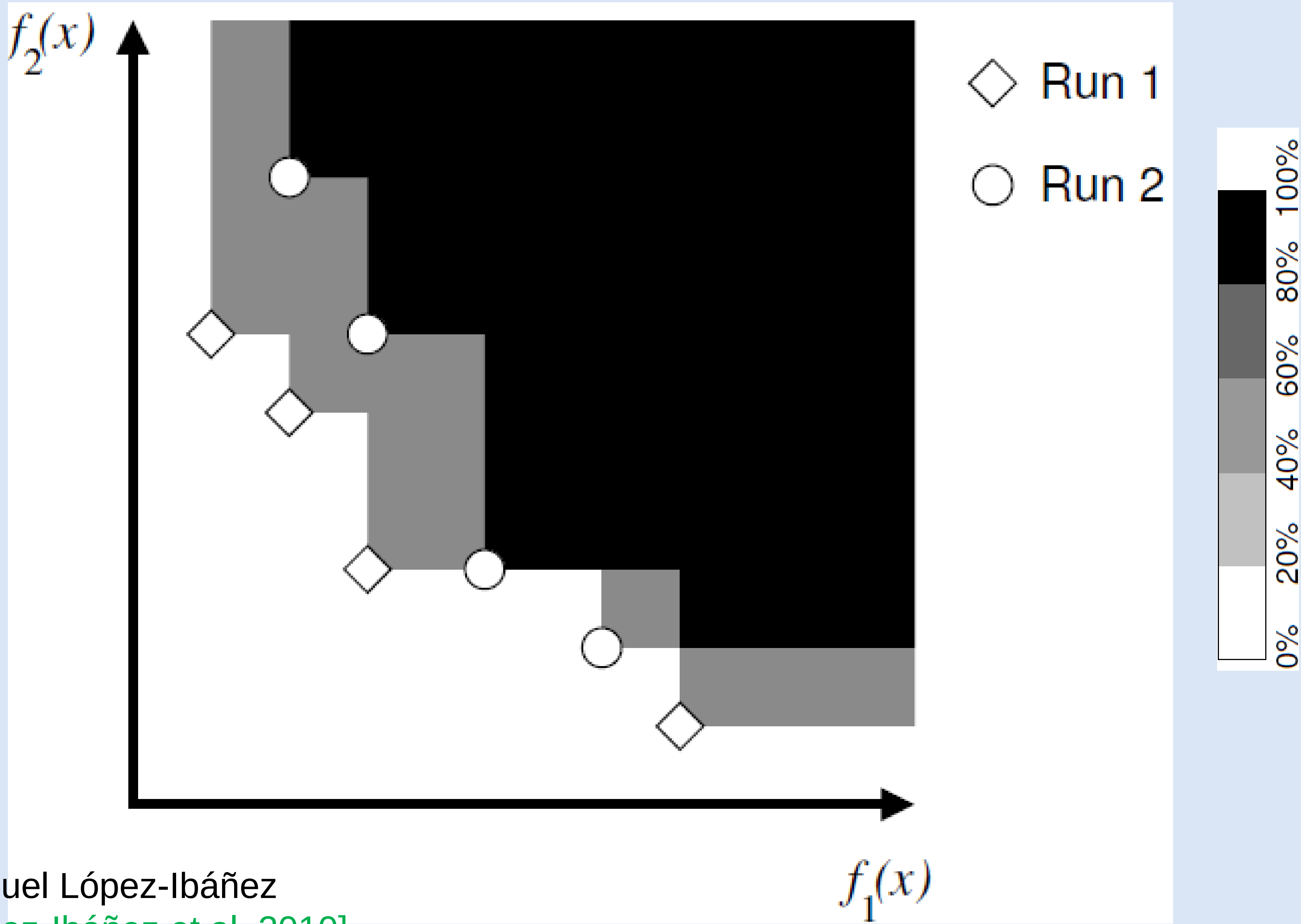
© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Idea



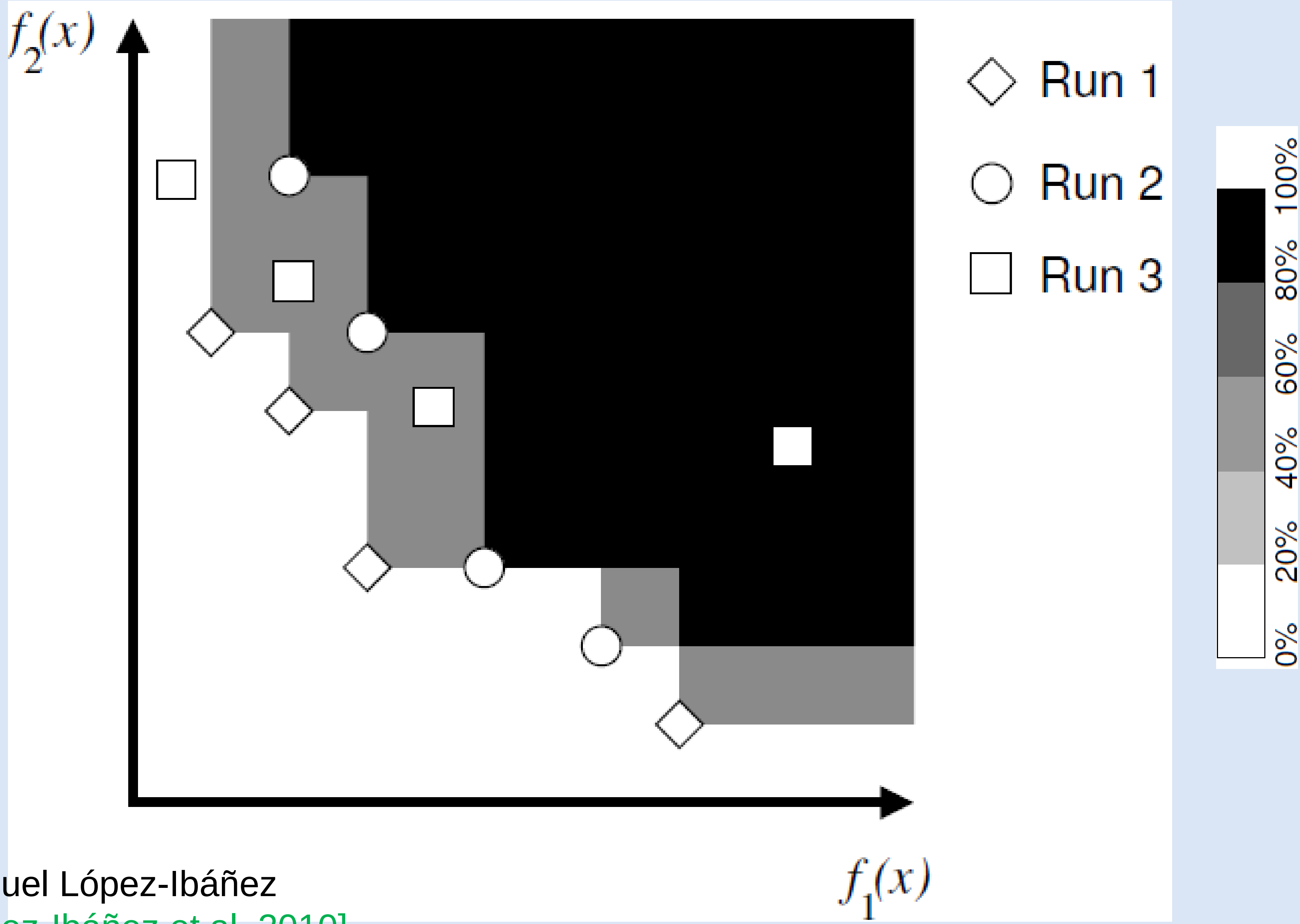
© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Idea



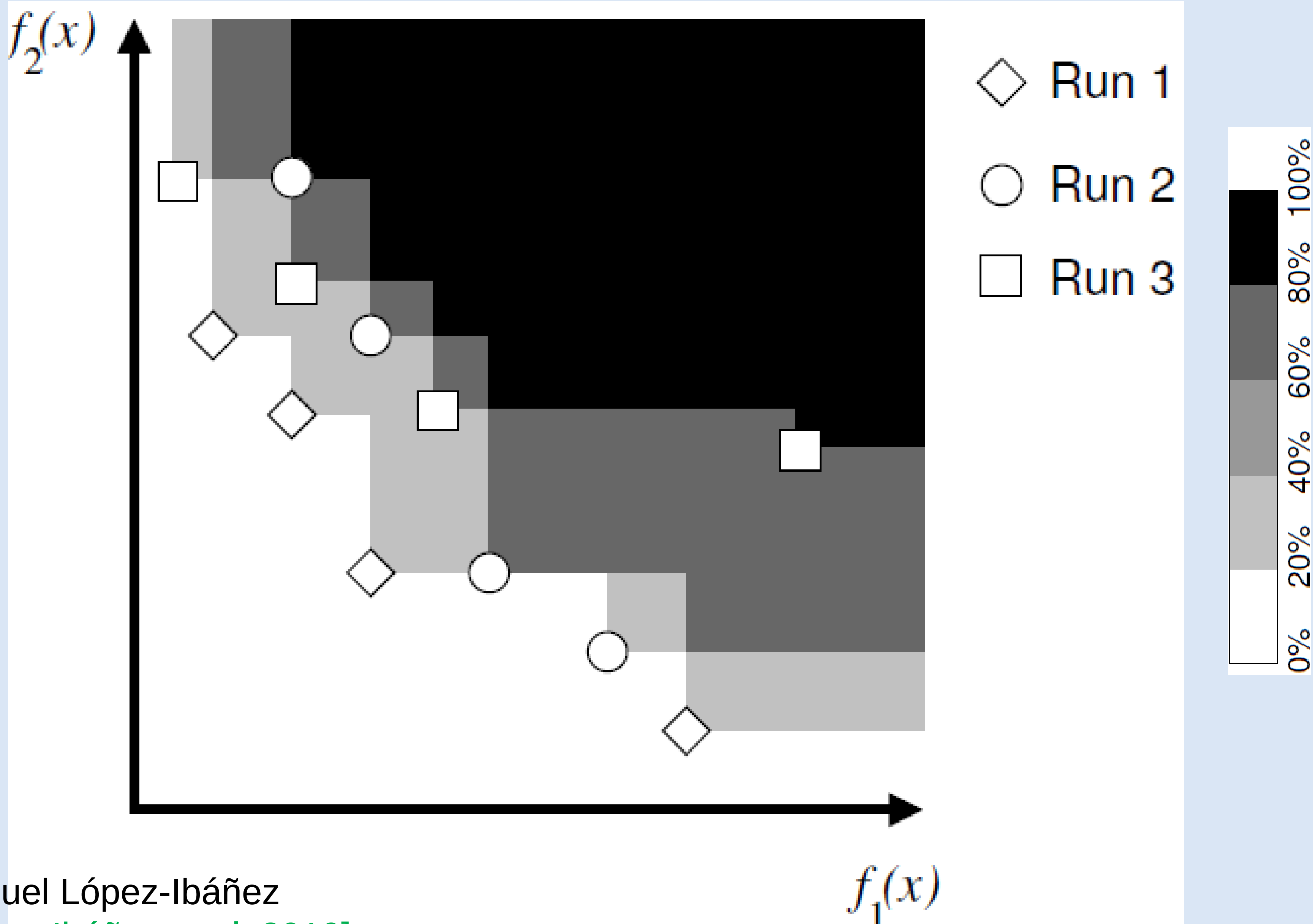
© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Idea



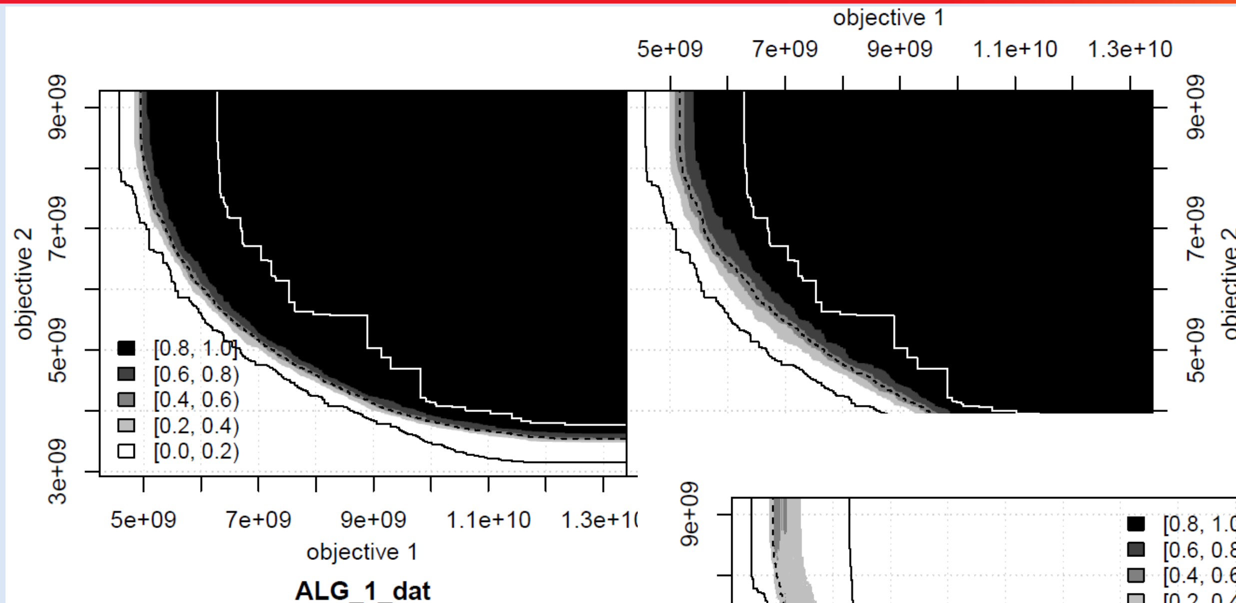
© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

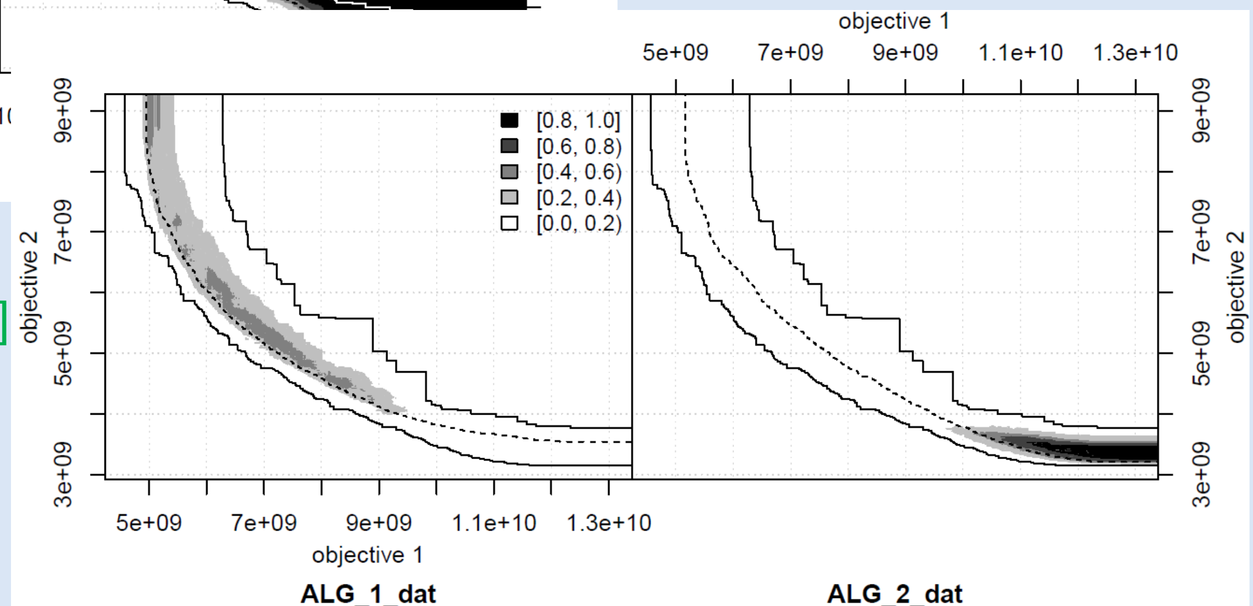
Empirical Attainment Functions: Idea



© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Attainment Plots in Practice



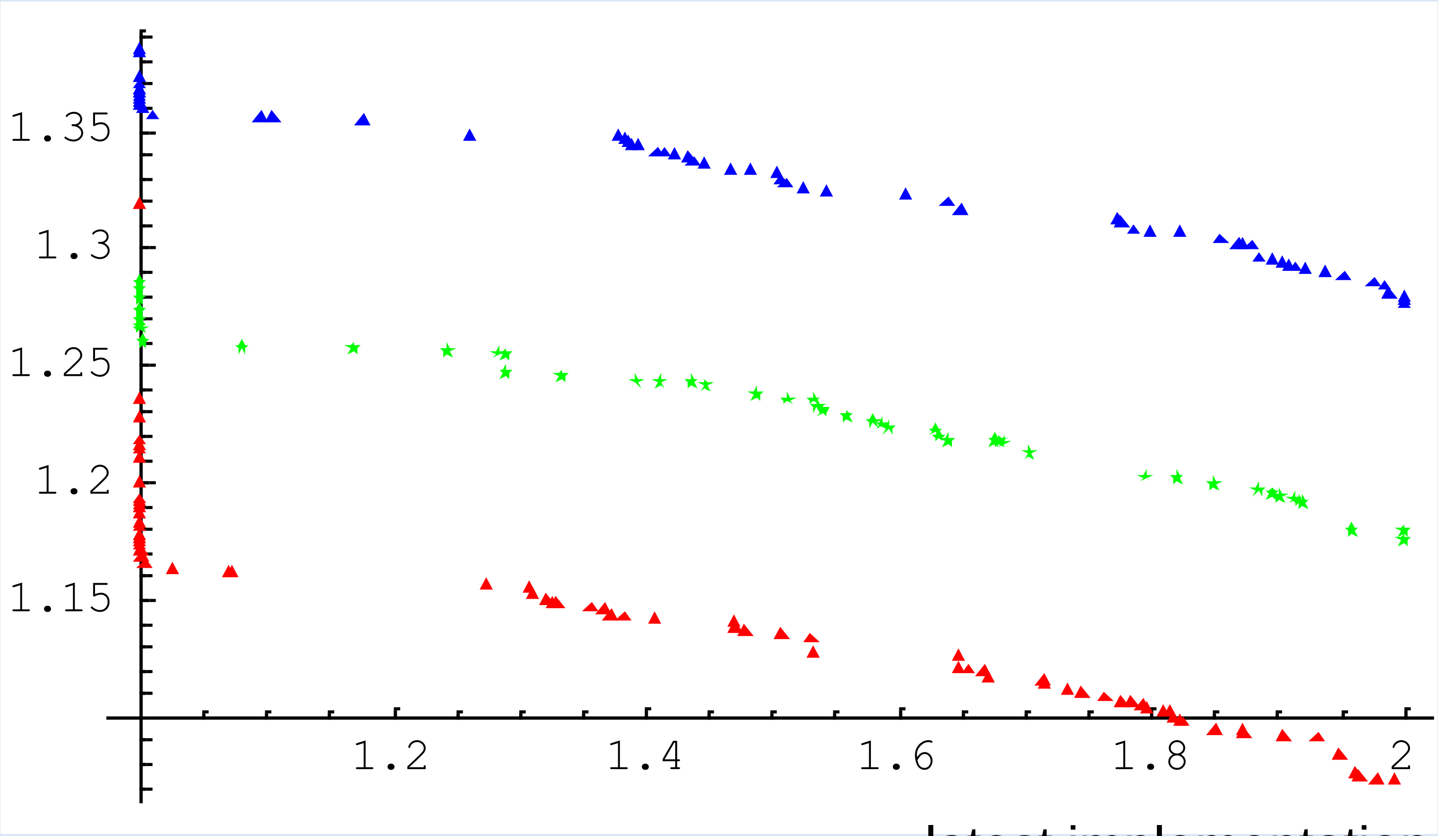


© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
R package: **`http://lopez-ibanez.eu/eaftools`**
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

Attainment Plots

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)



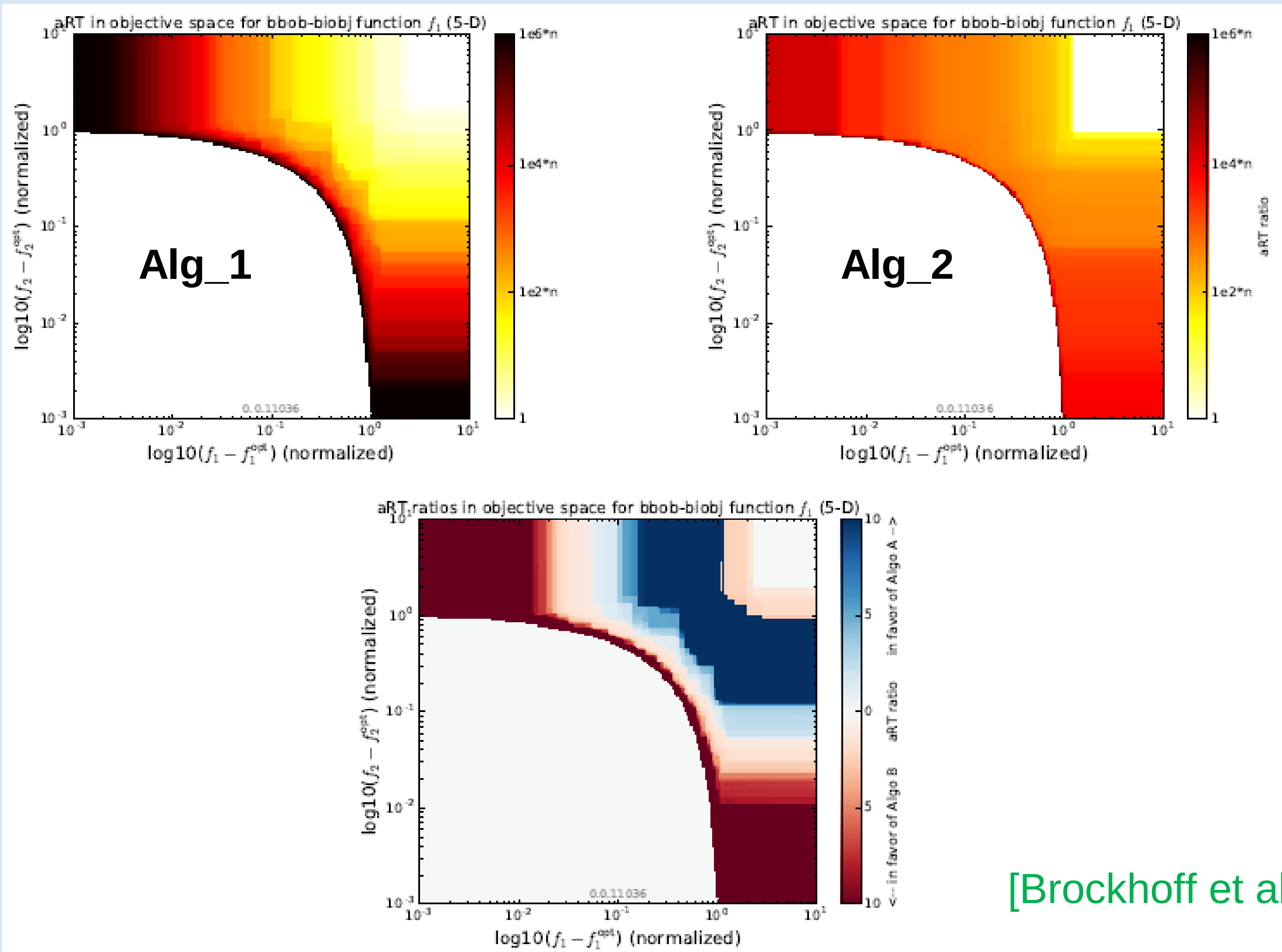
latest implementation online at

`http://eden.dei.uc.pt/~cmfonsec/software.html`

see [Fonseca et al. 2011]

Average Runtime Attainment Plots

...display not only the success probabilities, but the average runtime to attain points in objective space:



[Brockhoff et al. 2016]

Most Used Approach: Quality Indicators

A quality indicator

- maps a solution set to a real number
- can be used with standard performance assessment
  - report median, variance, ...
  - boxplots
  - statistical tests
- should optimally refine the dominance relation on sets

Recommendation:

- use hypervolume (refinement, i.e. it does not contradict the dominance relation)
- or epsilon indicator or R2 indicator (are weak refinements)

Also important:

- interpretation of the results (by knowing theoretical properties of the used indicator)

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function (“quality indicator”) on sets
- use the resulting total (pre-)order (on the quality values)

Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation should be reflected!

$$A \preceq B :\Leftrightarrow \forall_{y \in B} \exists_{x \in A} x \leq_{par} y$$

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$ fulfills requirement

❷ $\overset{\text{ref}}{\preccurlyeq}$ **weakly refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

$\Rightarrow$ does not fulfill requirement, but $\overset{\text{ref}}{\preccurlyeq}$ does not contradict $\preccurlyeq$

! sought are total refinements… [Zitzler et al. 2010]

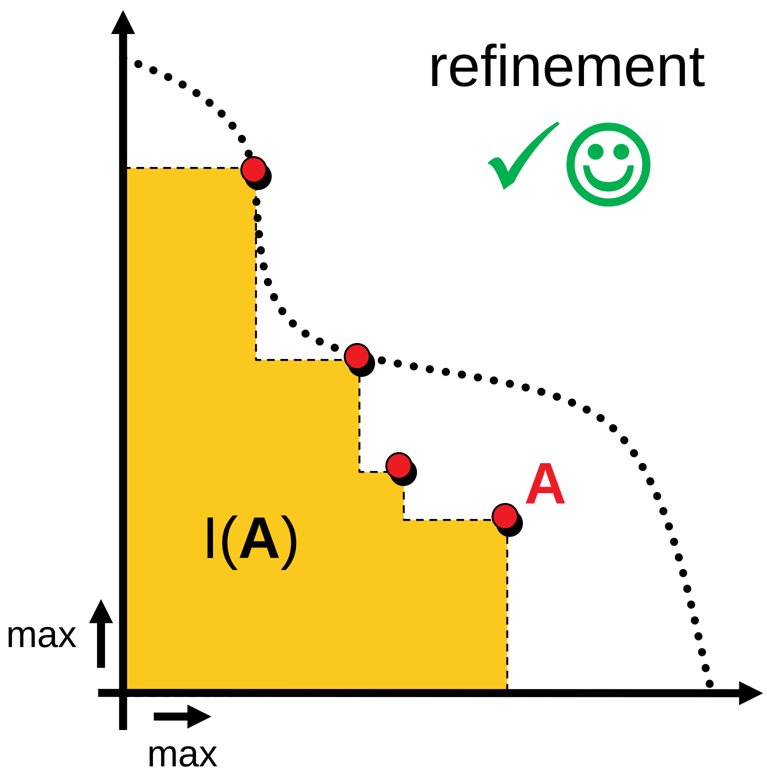
Example: Refinements Using Indicators

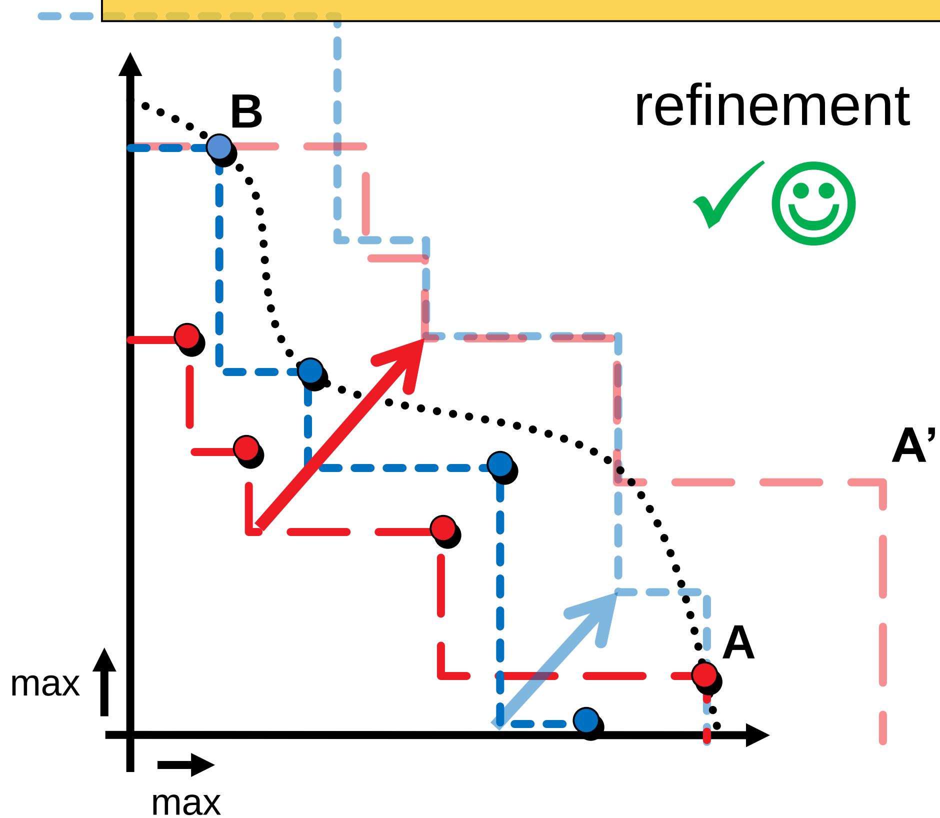
$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \geq I(B)$$

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,B) \leq I(B,A)$$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B





unary hypervolume indicator

binary epsilon indicator

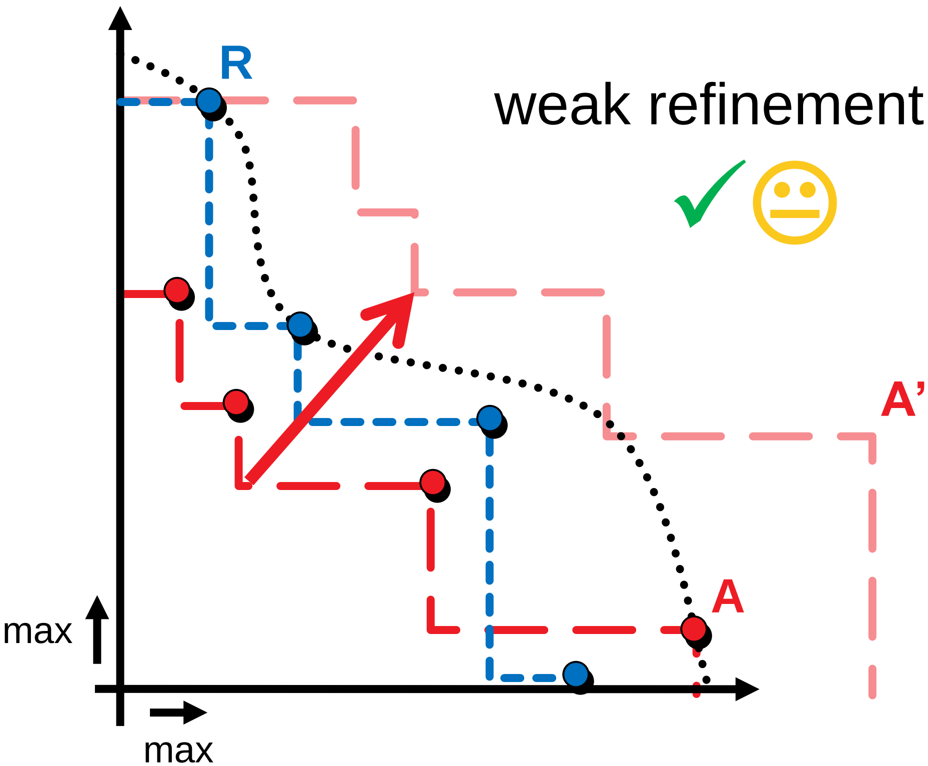
Example: Weak Refinement / No Refinement

$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A,R) \leq I(B,R)$$

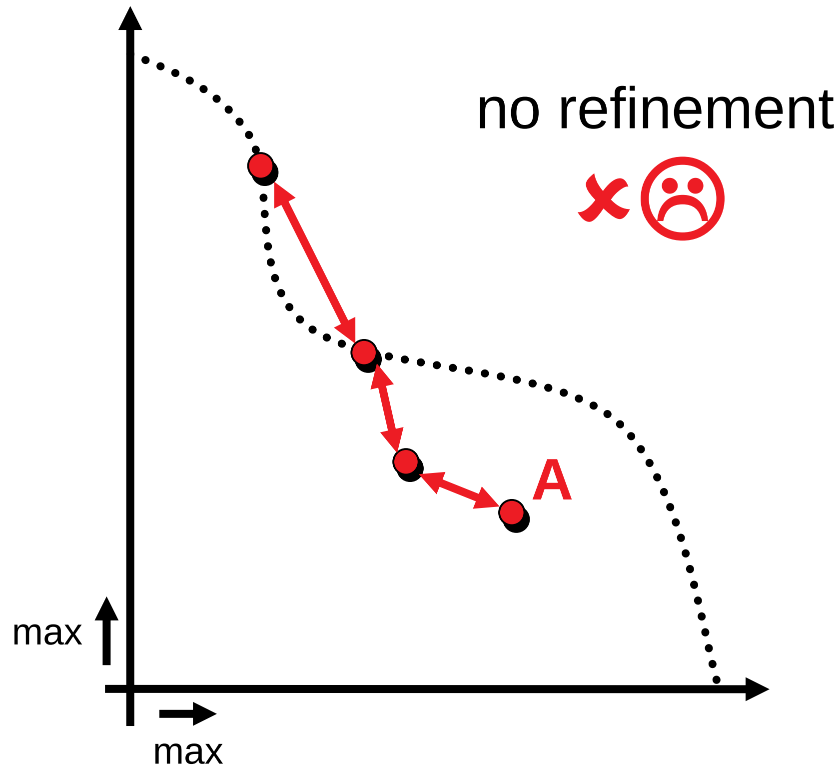
$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A) \leq I(B)$$

I(A,R) = how much needs A to be moved to weakly dominate R

I(A) = variance of pairwise distances



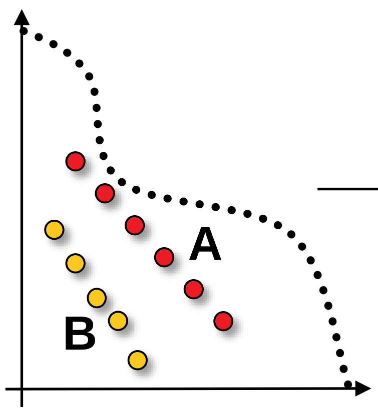
unary epsilon indicator



unary diversity indicator

Quality Indicator Approach

Goal: compare two Pareto set approximations A and B



→

	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

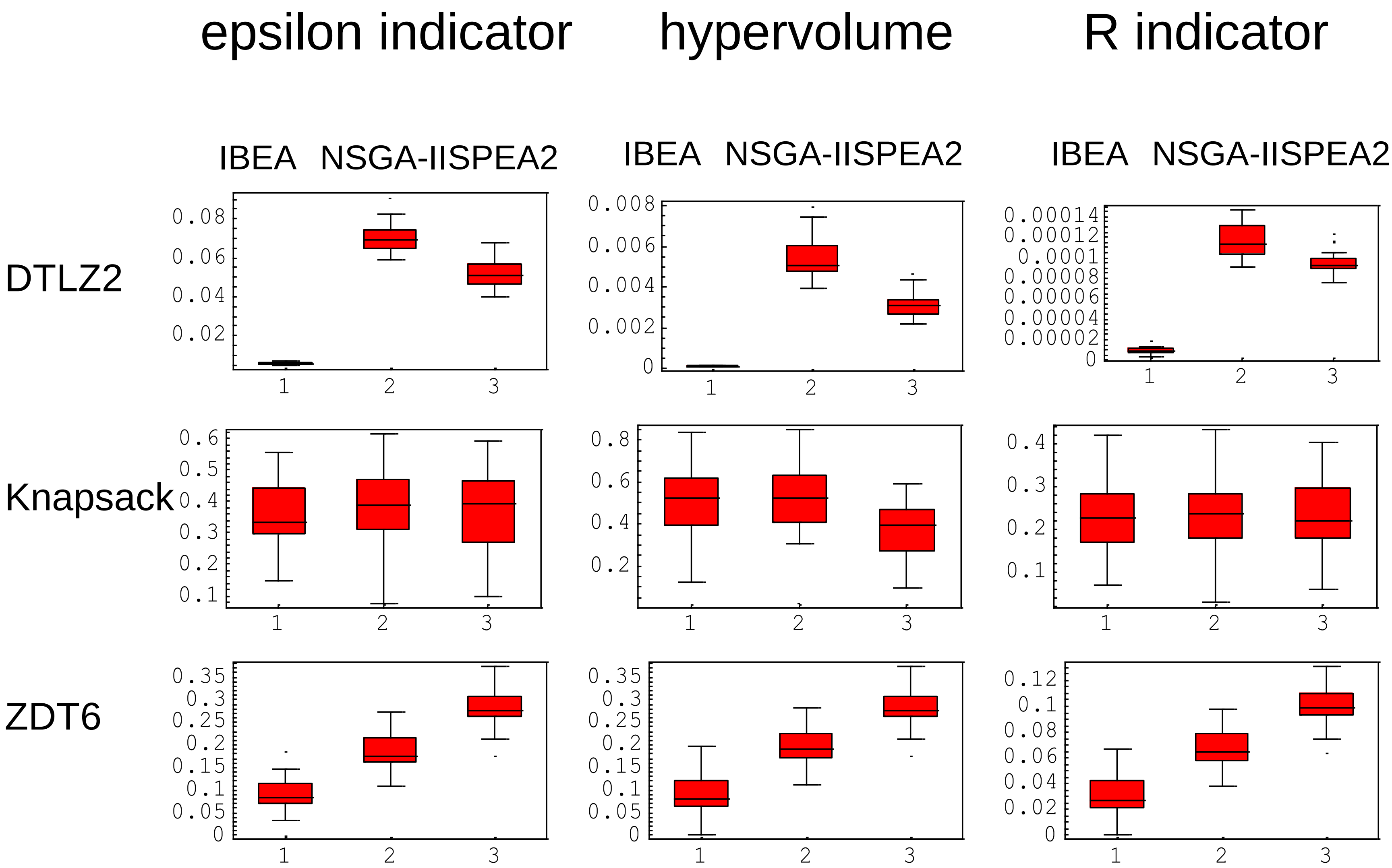
→ "**A** better"

Comparison method C = quality measure(s) + Boolean function

$$A, B \xrightarrow{\text{quality measure}} \mathbb{R}^n \xrightarrow{\text{Boolean function}} \text{statement}$$

reduction **interpretation**

Example: Box Plots



Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		~0 ☺
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

DTLZ2
R

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		1
SPEA2	1	~0 ☺	

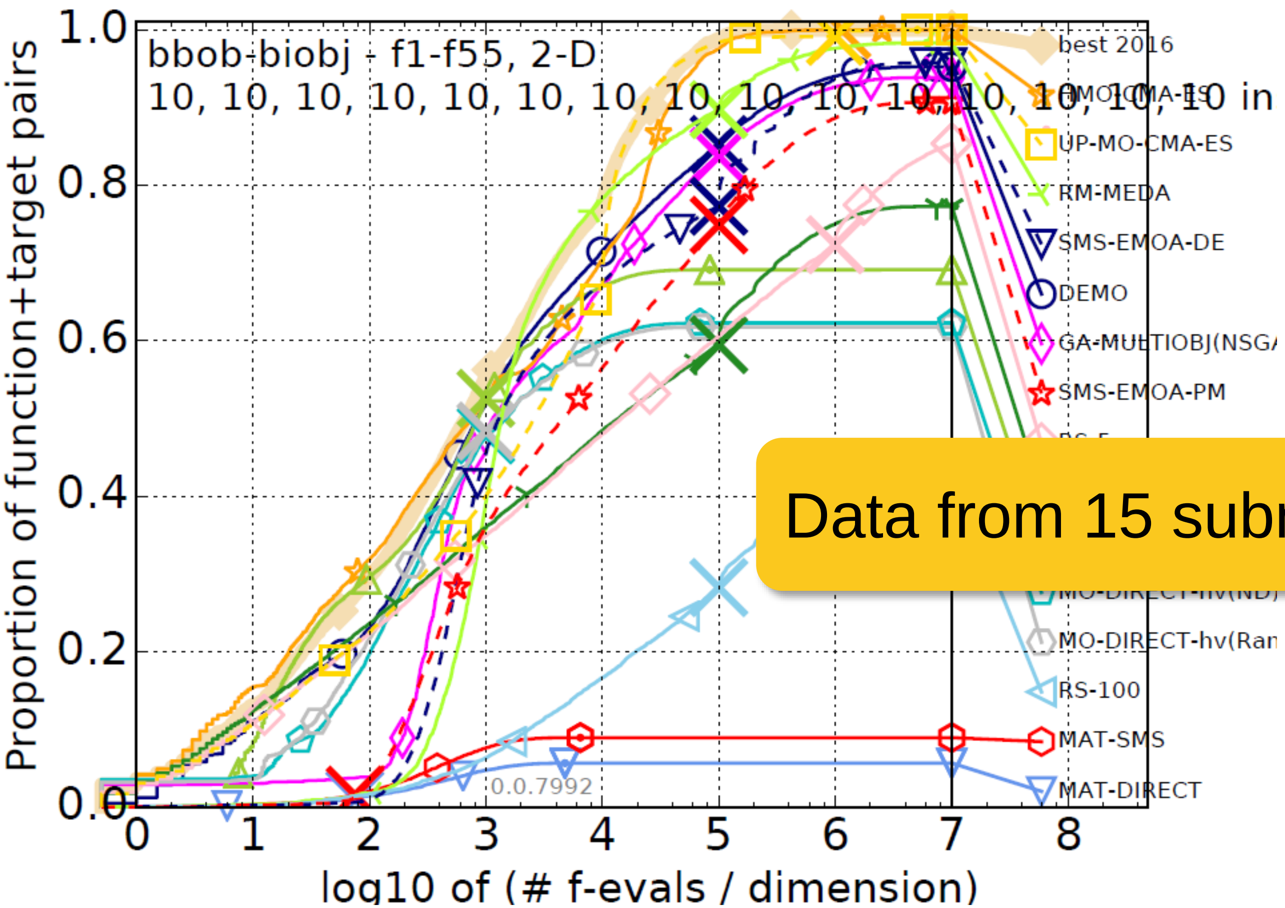
Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H_0 = No significance of any differences

© Anne Auger and Dimo Brockhoff, Inria

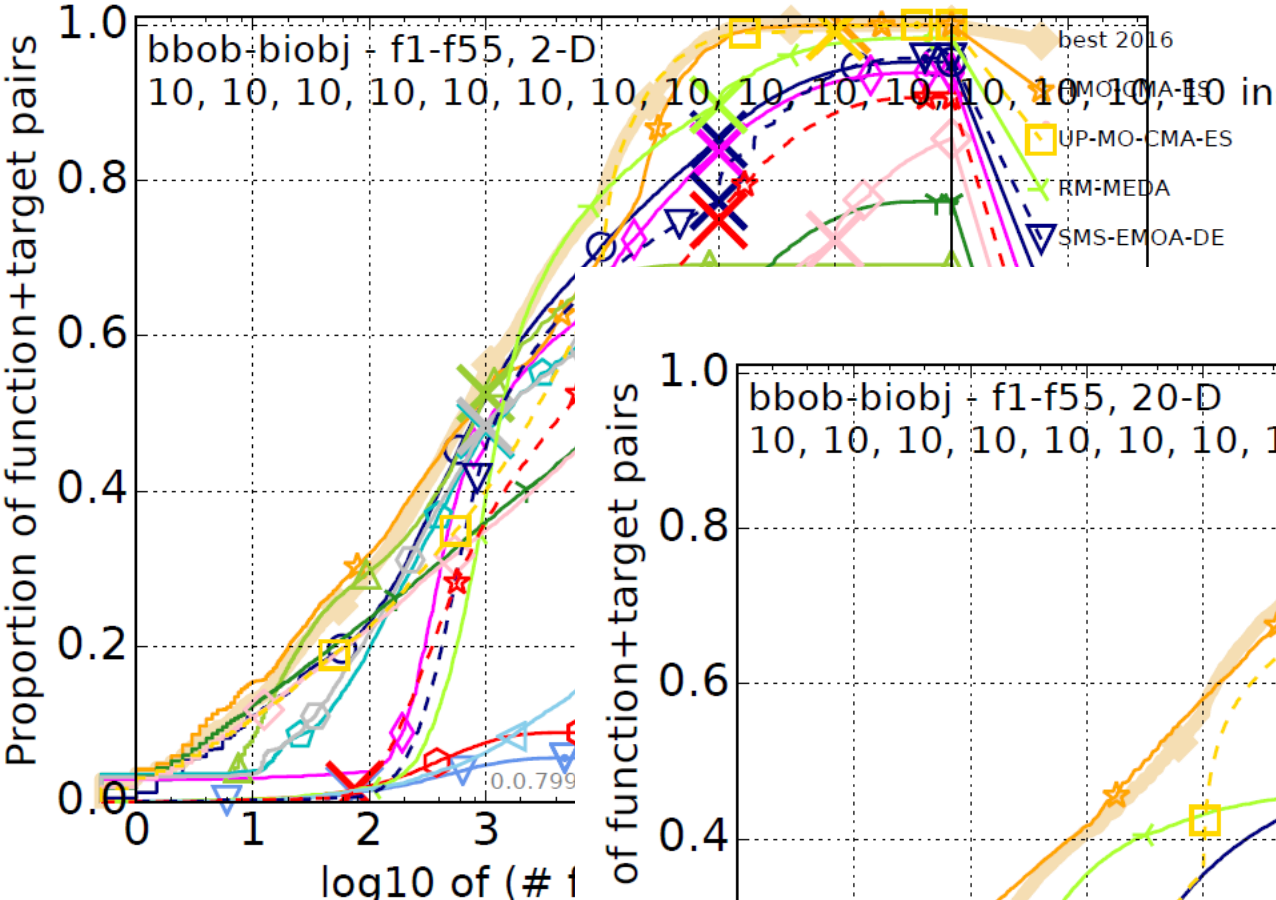
Automated Benchmarking

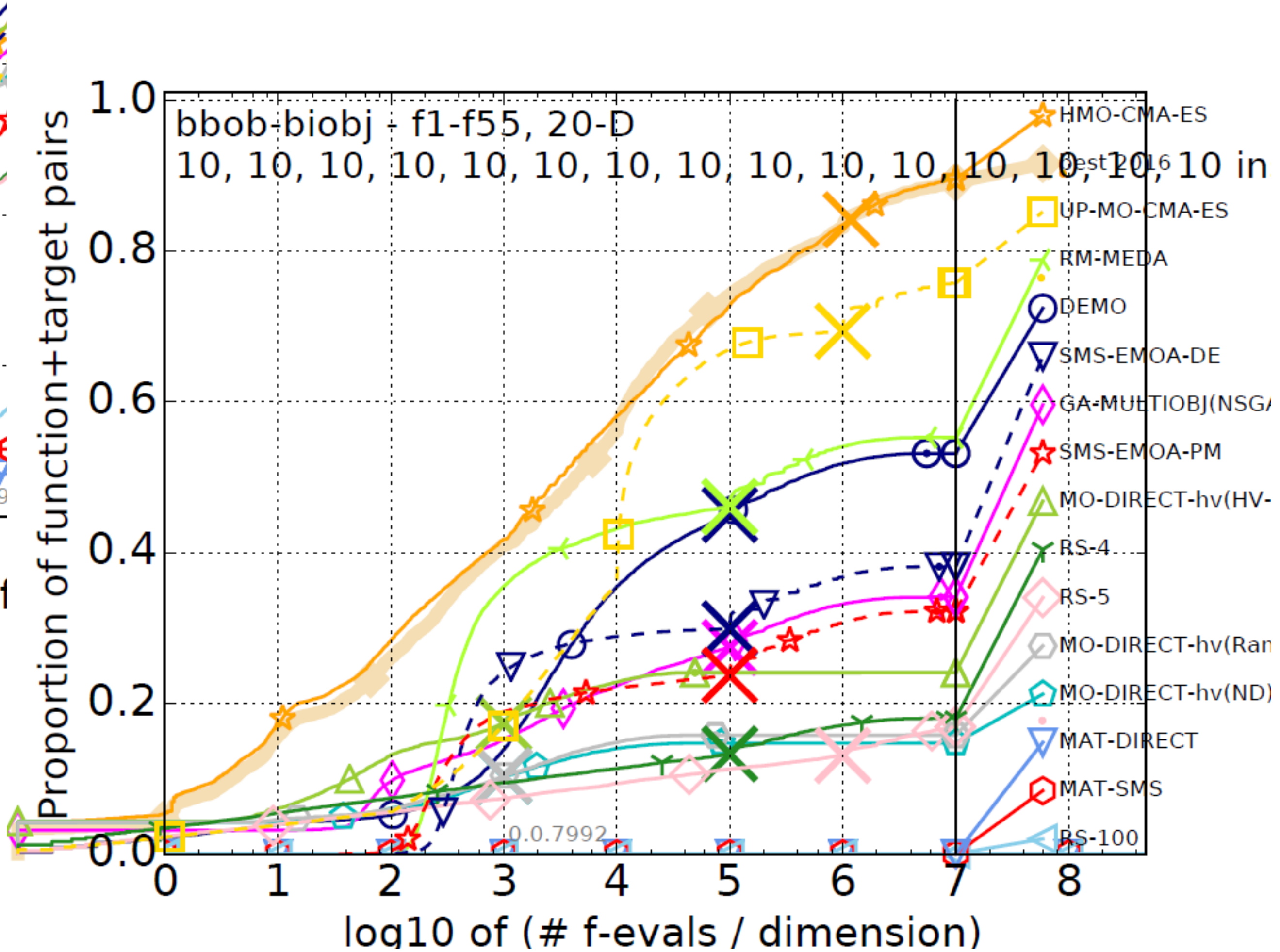
- State-of-the-art in single-objective optimization: Blackbox Optimization Benchmarking (BBOB) with COCO platform
  `https://github.com/numbbo/coco`
- In 2016: first release of a bi-objective test suite and corresponding BBOB-2016 workshop @ GECCO
- Focus on target-based runlengths
  - gives (nearly) anytime, interpretable results
  - defines problem=(test function instance, single-objective goal e.g. min. indicator difference to reference set, target precision)
  - reports average runtimes (aRT) to reach target precision
- COCO provides data profiles, scaling plots, scatter plots, tables, statistical tests, etc. automatically



Data from 15 submitted algorithms

Exemplary BBOB-2016 Results





Overview

The Big Picture

Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

Selected Advanced Concepts

- preference articulation
- visualization aspects

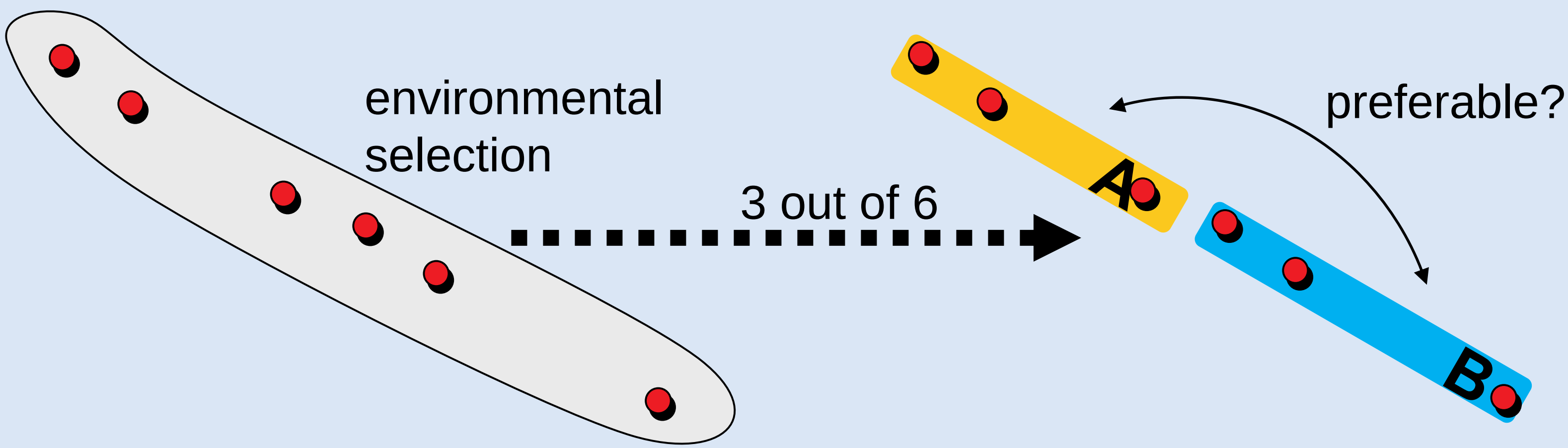
Articulating User Preferences During Search

What we thought: EMO is preference-less

Search before decision making: Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

[Zitzler 1999]

What we learnt: EMO just uses weaker preference information

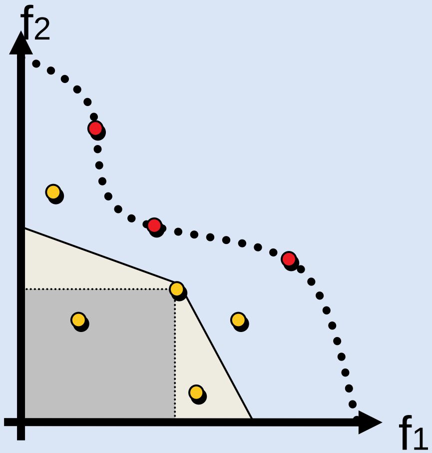


Nevertheless...

- the more (known) preferences incorporated the better
- in particular if search space is large

[Branke and Deb 2004] [Branke 2008] [Bechikh et al. 2015]

❶ Refine/modify dominance relation, e.g.:



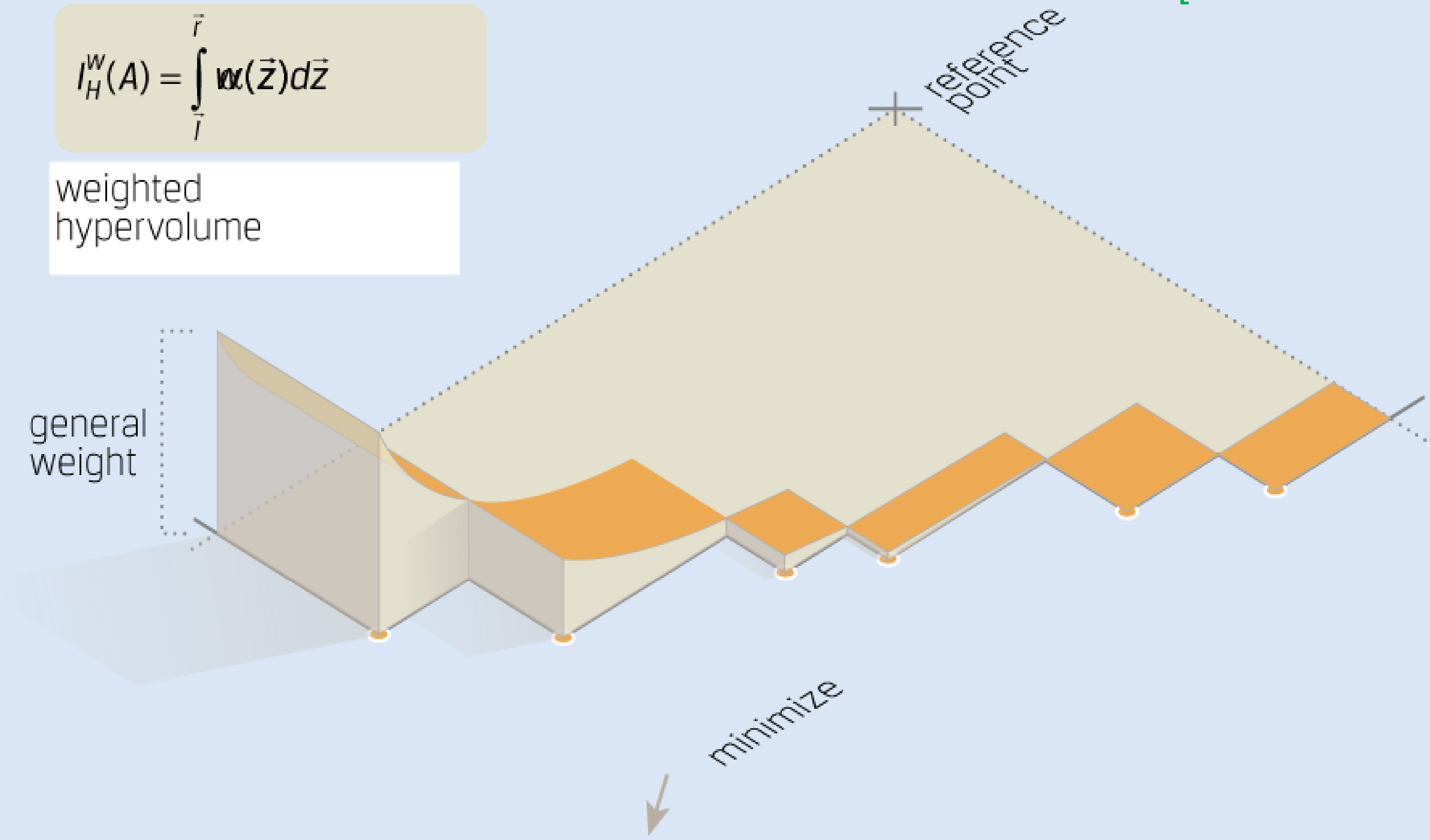
- using goals, priorities, constraints
  [Fonseca and Fleming 1998a,b]
- using different types of dominance cones
  [Branke and Deb 2004]

❷ Use quality indicators, e.g.:

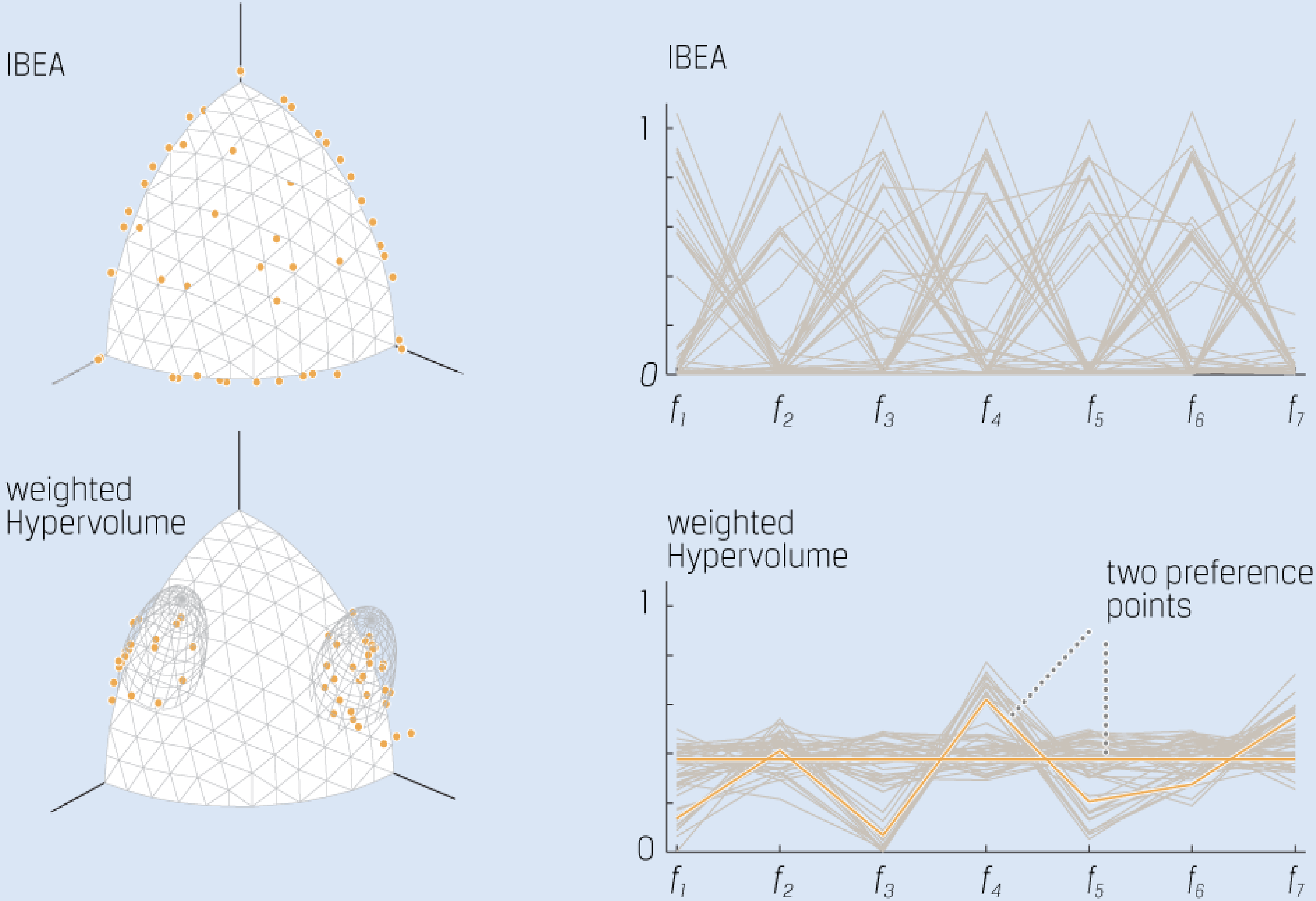
- based on reference points and directions [Deb and Sundar 2006, Deb and Kumar 2007]
- based on the hypervolume indicator
  [Brockhoff et al. 2013] [Wagner and Trautmann 2010]
- based on the R2 indicator [Trautmann et al. 2013]

Example: Weighted Hypervolume Indicator

[Brockhoff et al. 2013]



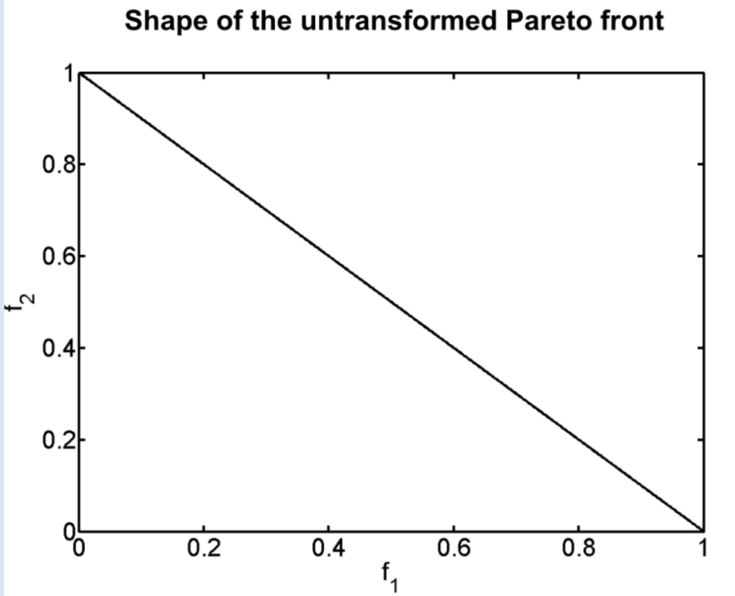
Weighted Hypervolume in Practice

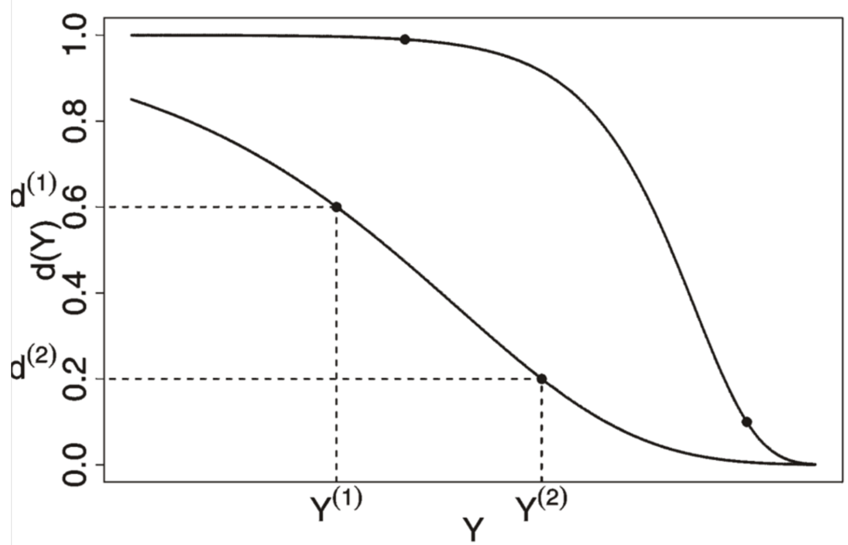


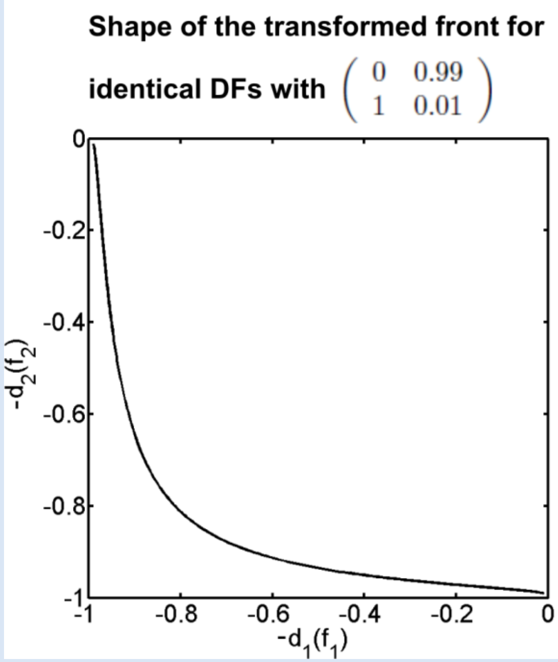
[Auger et al. 2009b]

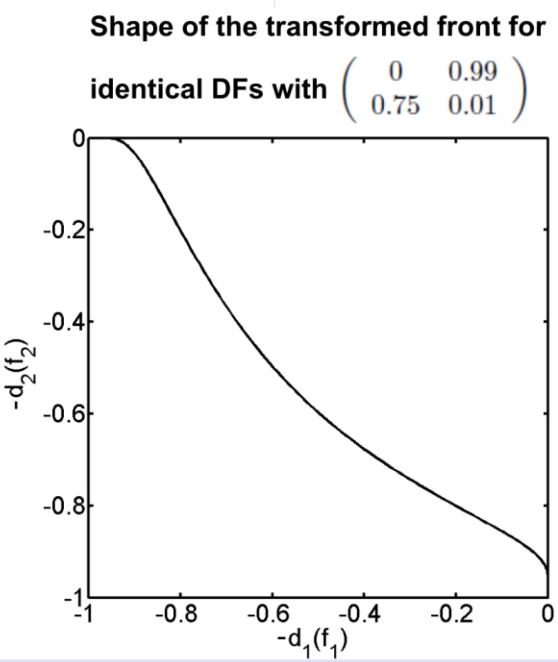
Example: Desirability Function (DF)-SMS-EMOA

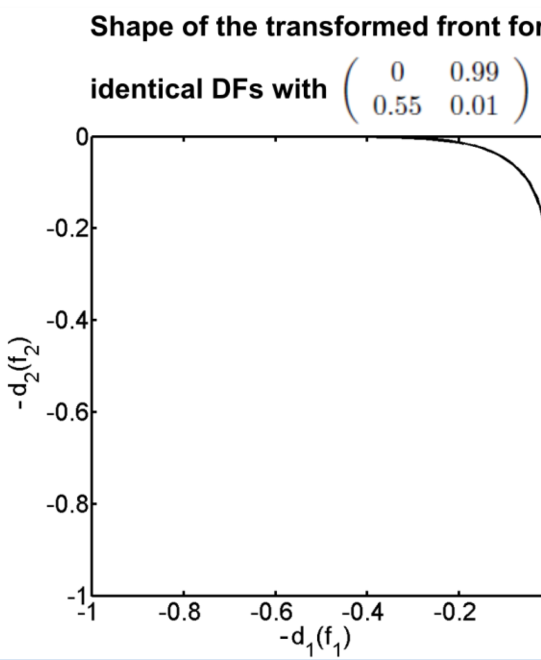
[Wagner and Trautmann 2010]



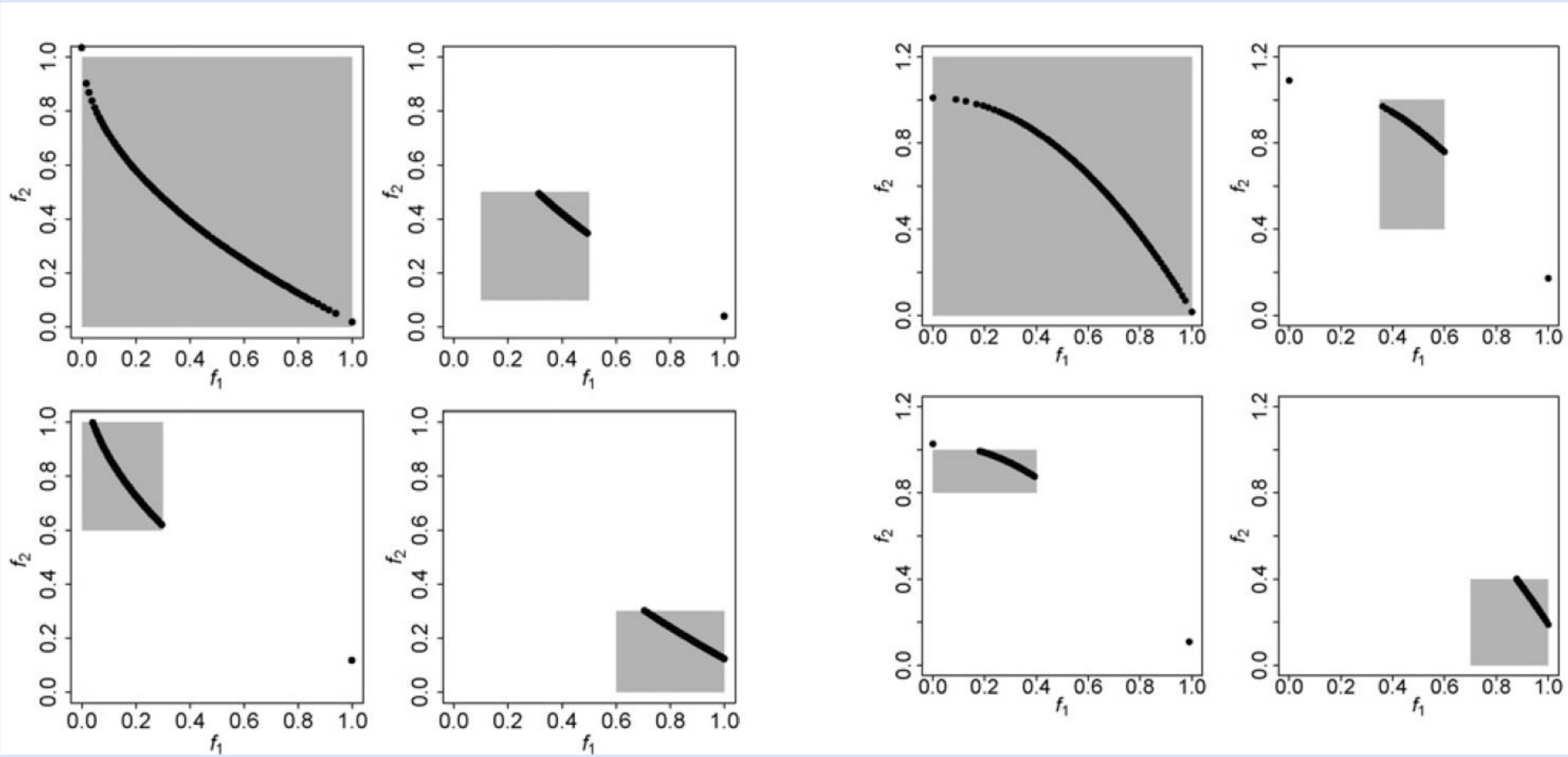








DF-SMS-EMOA in Practice

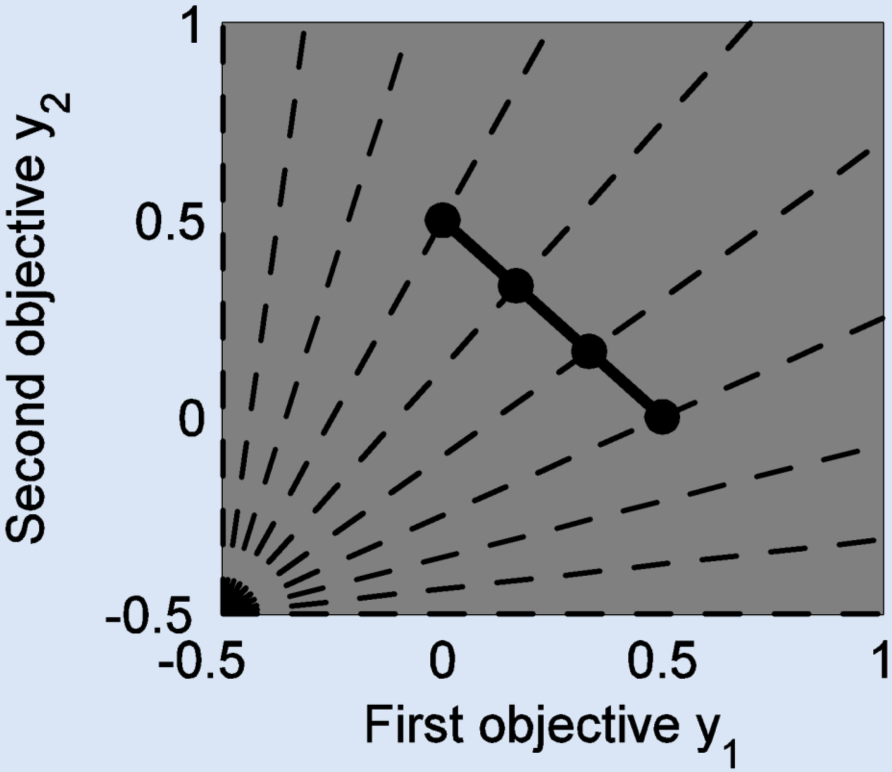


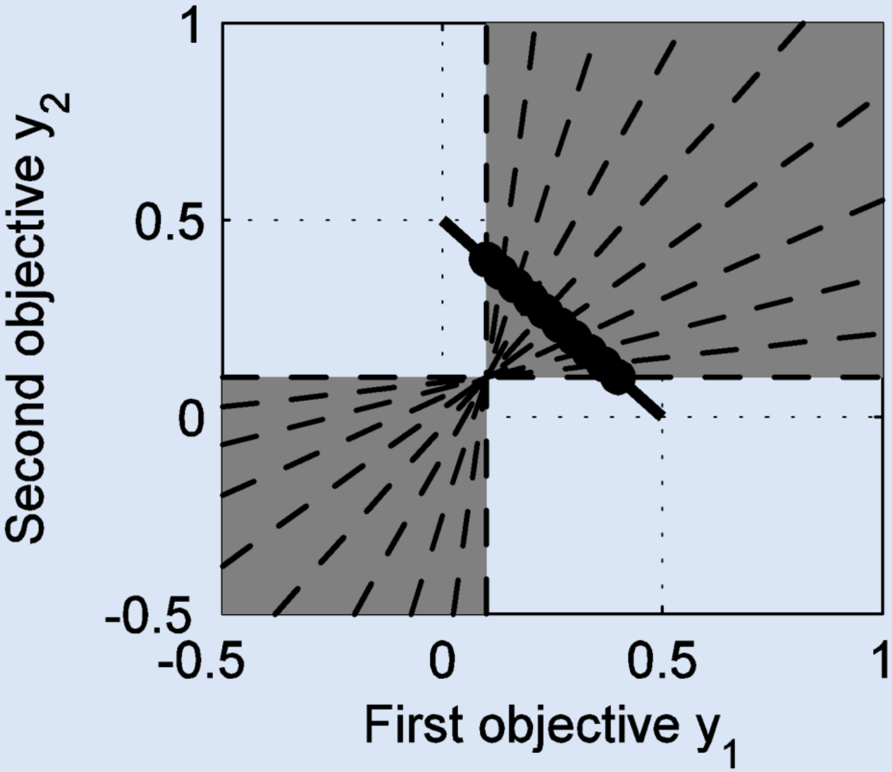
Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Position of ideal point



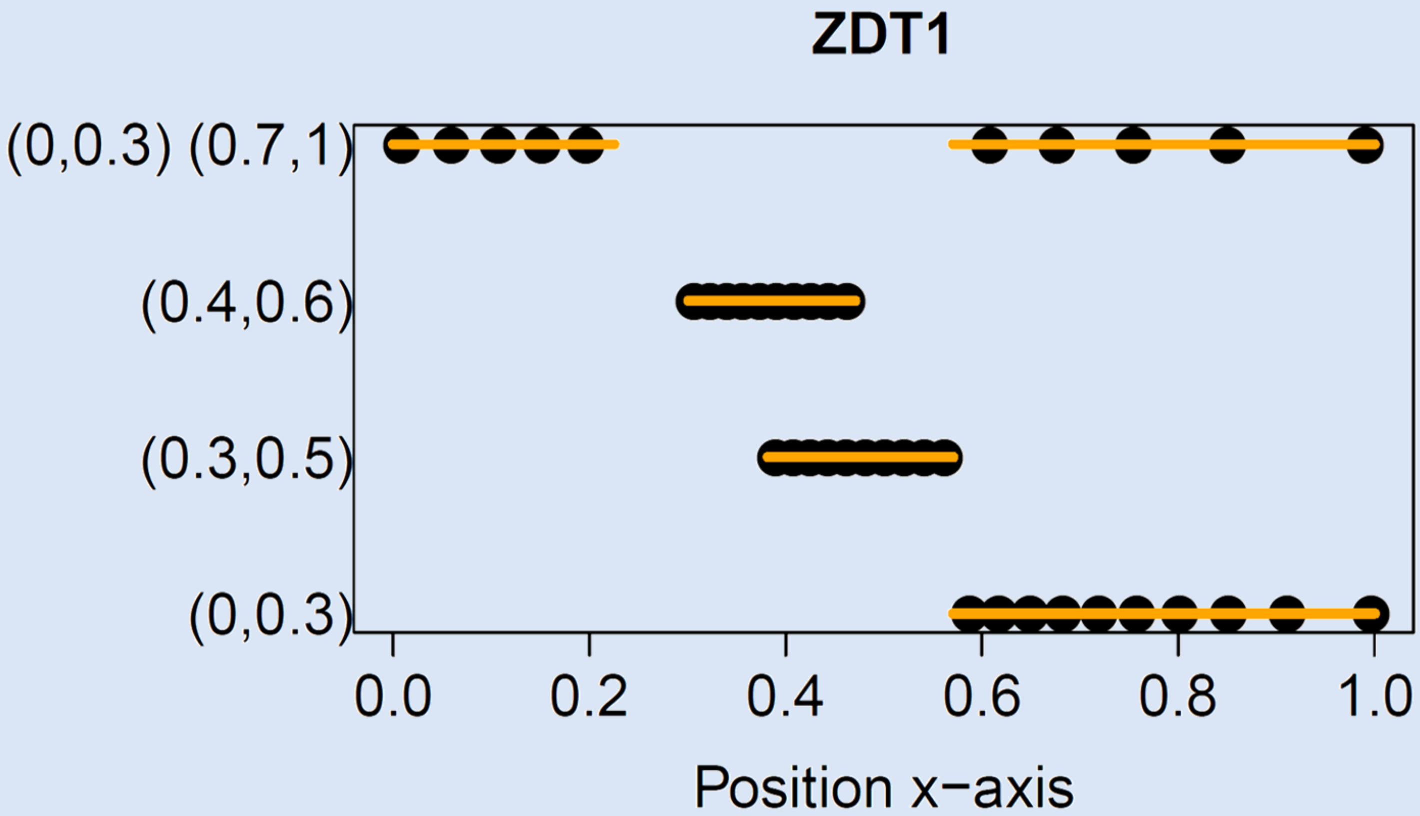


Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Restriction of the weight space



Interactive Approaches

Successive Preference Articulation = Interactive EMO

- recent interest of both EMO and MCDM community
- important in practice

Examples

- first interactive EMO: [Tanino et al. 1993]
- good overview: [Jaszkiewicz and Branke 2008]
- more recent work: [Brockhoff et al. 2014] [Branke et al. 2014]

Issues/Open Questions

- realistic scenarios/ value functions
- evaluation of interactive algorithms [López-Ibáñez and Knowles 2015]

Overview

The Big Picture

Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

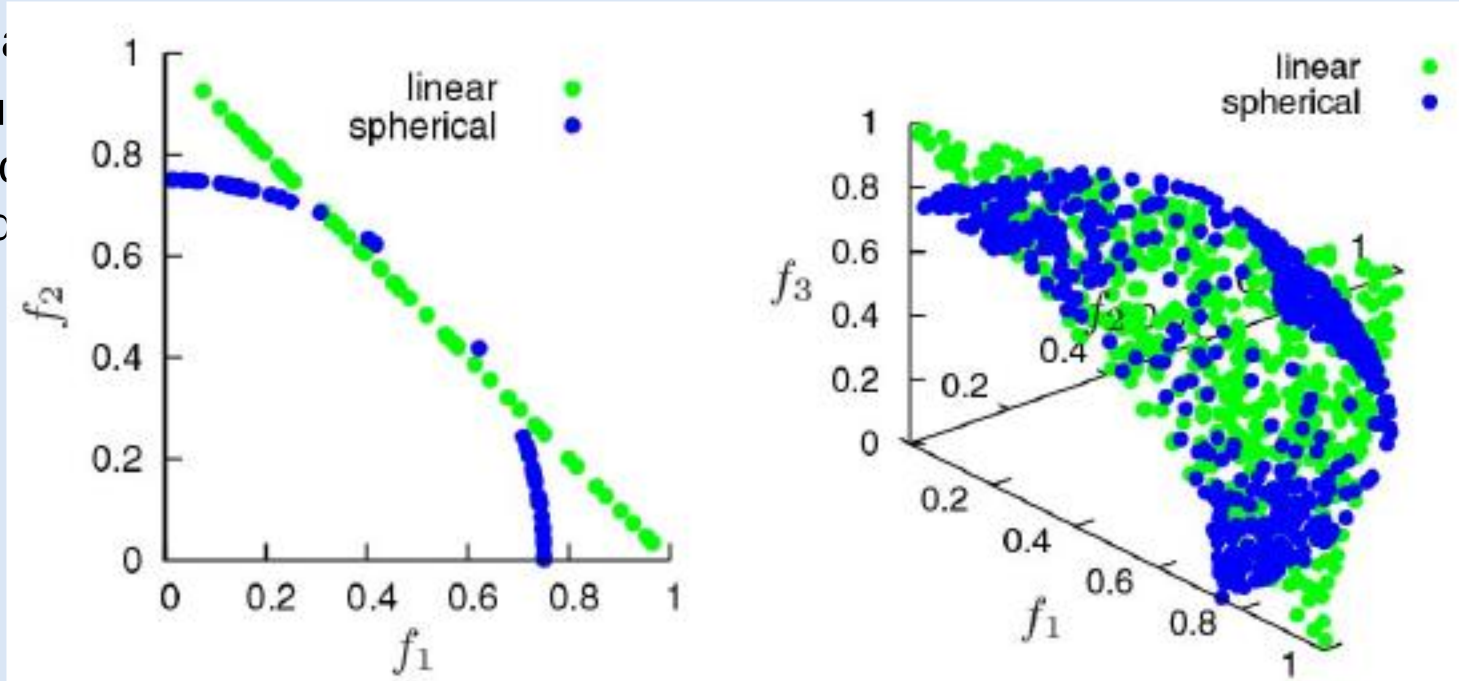
Selected Advanced Concepts

- preference articulation
- visualization aspects

Visualization is Difficult for Many Objectives

These a

Tea Tu … in
Evoluti … n
Method … 15.

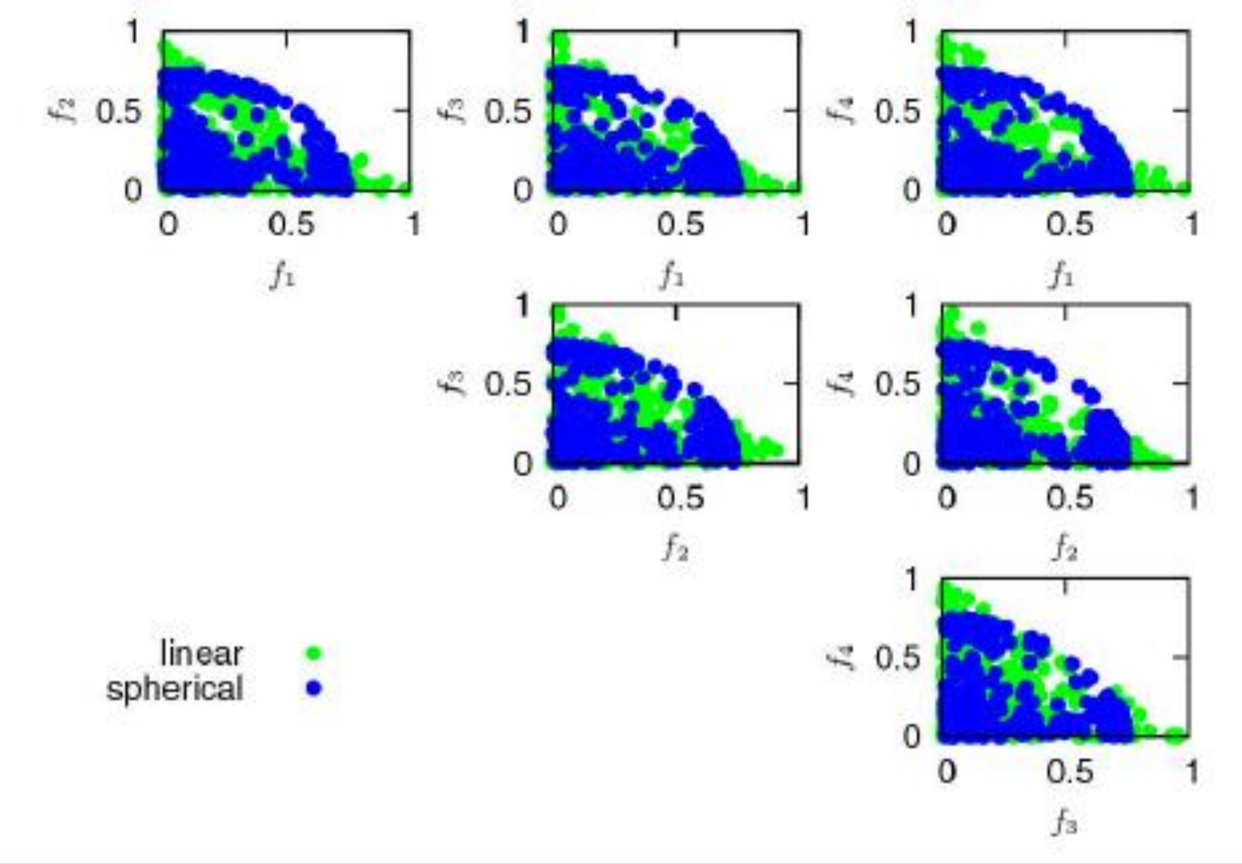


2 objective functions

3 objective functions

>3 objective functions?

Scatter Plots for all Objective Combinations



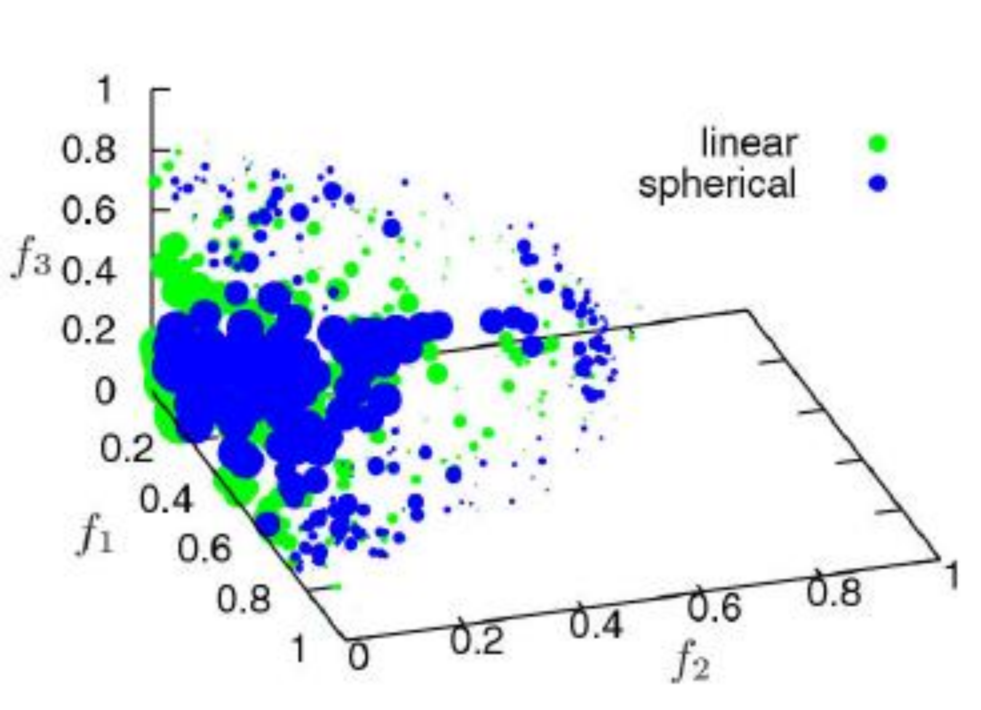
These and the following plots are taken from

Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

Bubble Chart

Bubble chart:

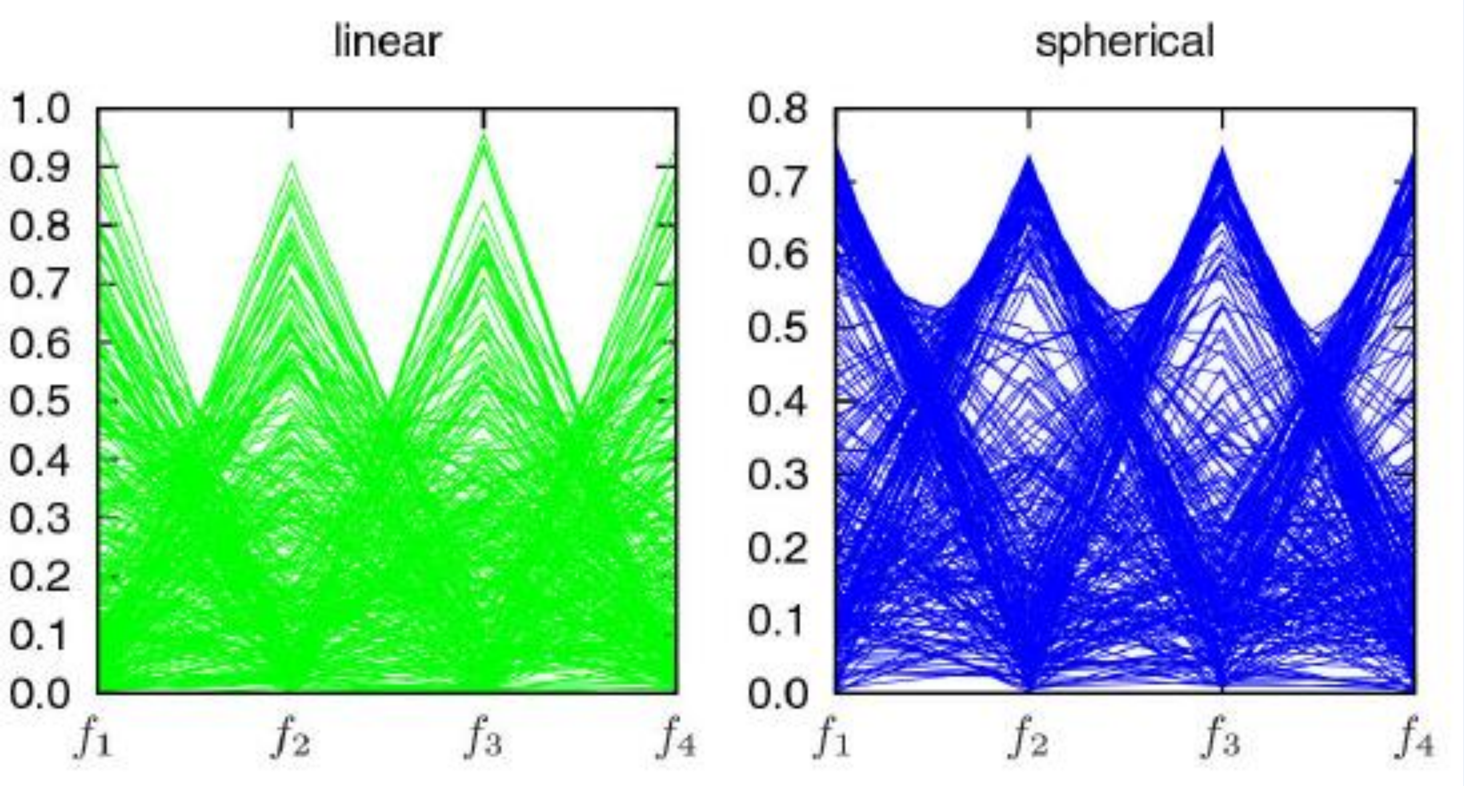
size of bubble = forth objective



This and the following plots are taken from

Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

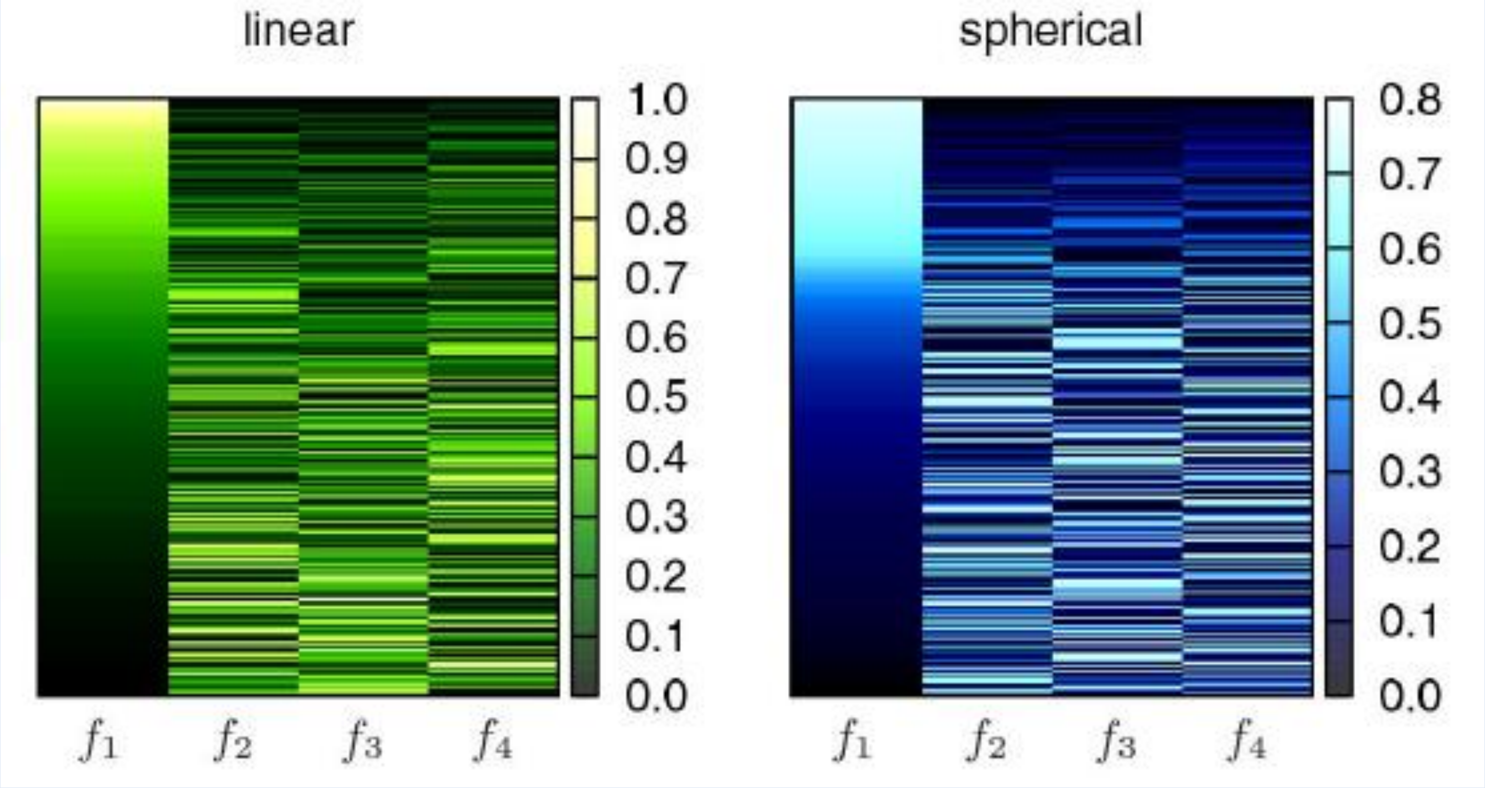
Parallel Coordinates



These and the following plots are taken from

Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

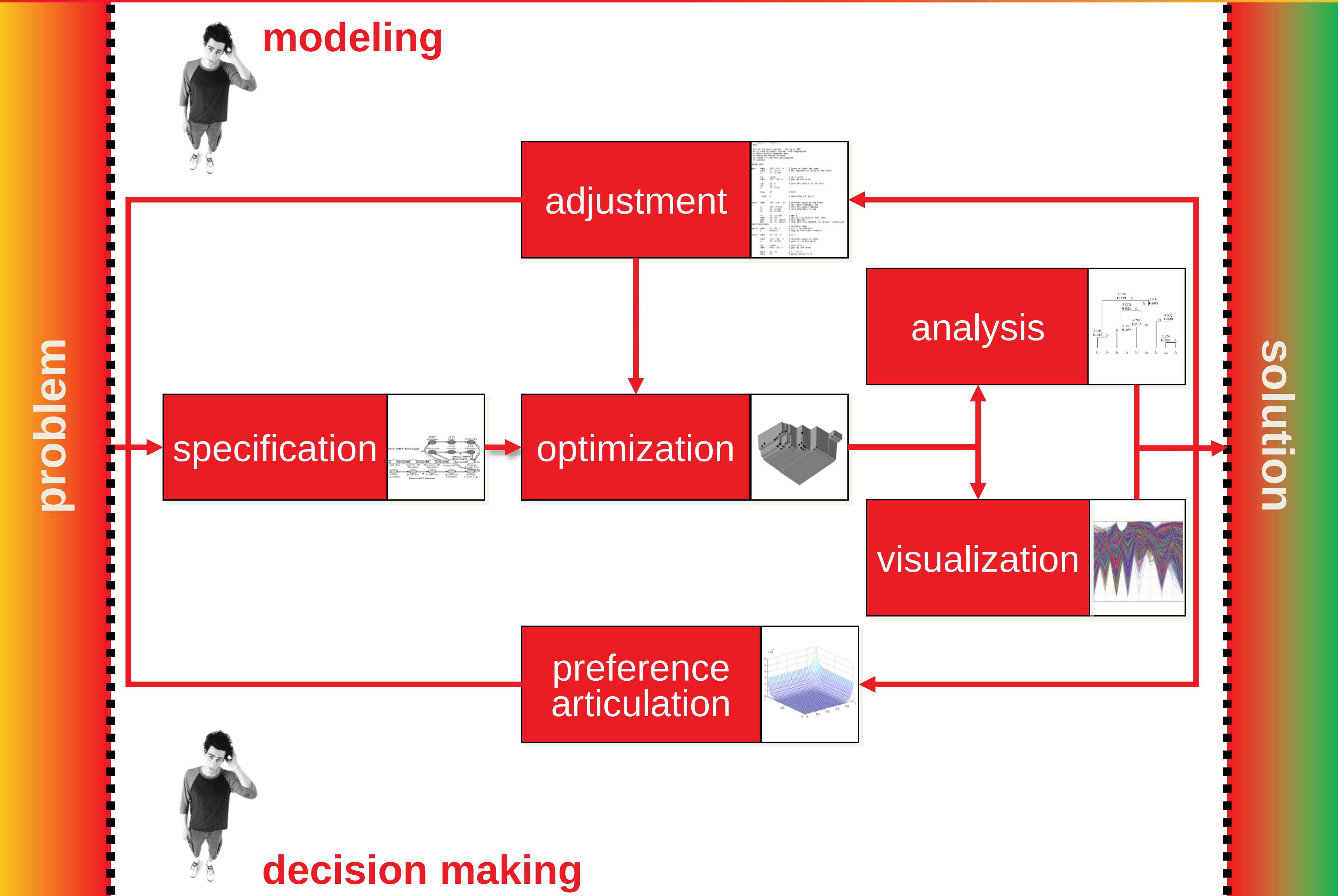
and many more...



These plots are taken from

Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

Conclusions: EMO as Interactive Decision Support



The EMO Community

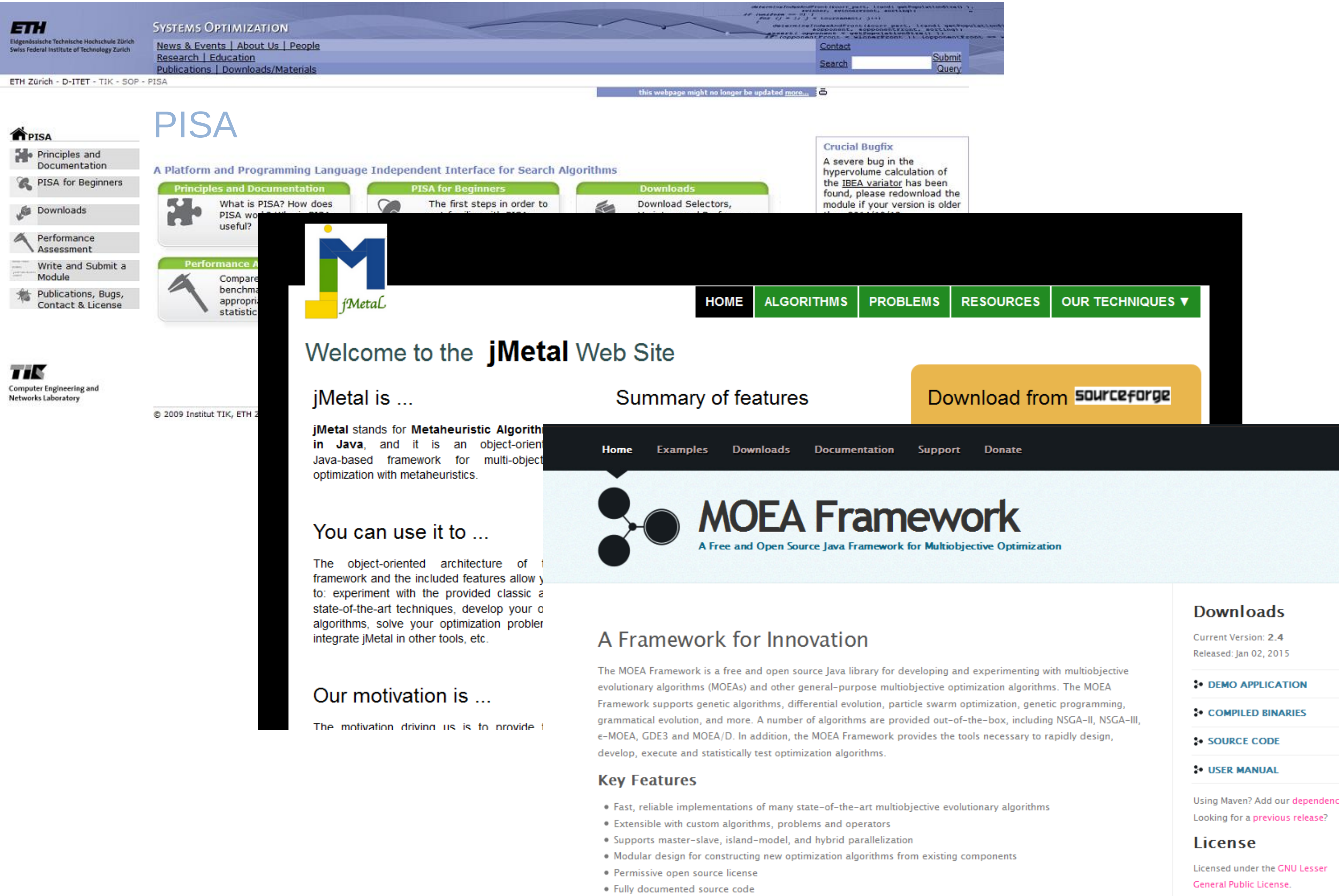
Links:

- EMO mailing list: *https://lists.dei.uc.pt/mailman/listinfo/emo-list*
- MCDM mailing list: *http://lists.jyu.fi/mailman/listinfo/mcdm-discussion*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.dep.uminho.pt/EMO2015/*

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms***
  Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [(still) many open questions!]
- and more…

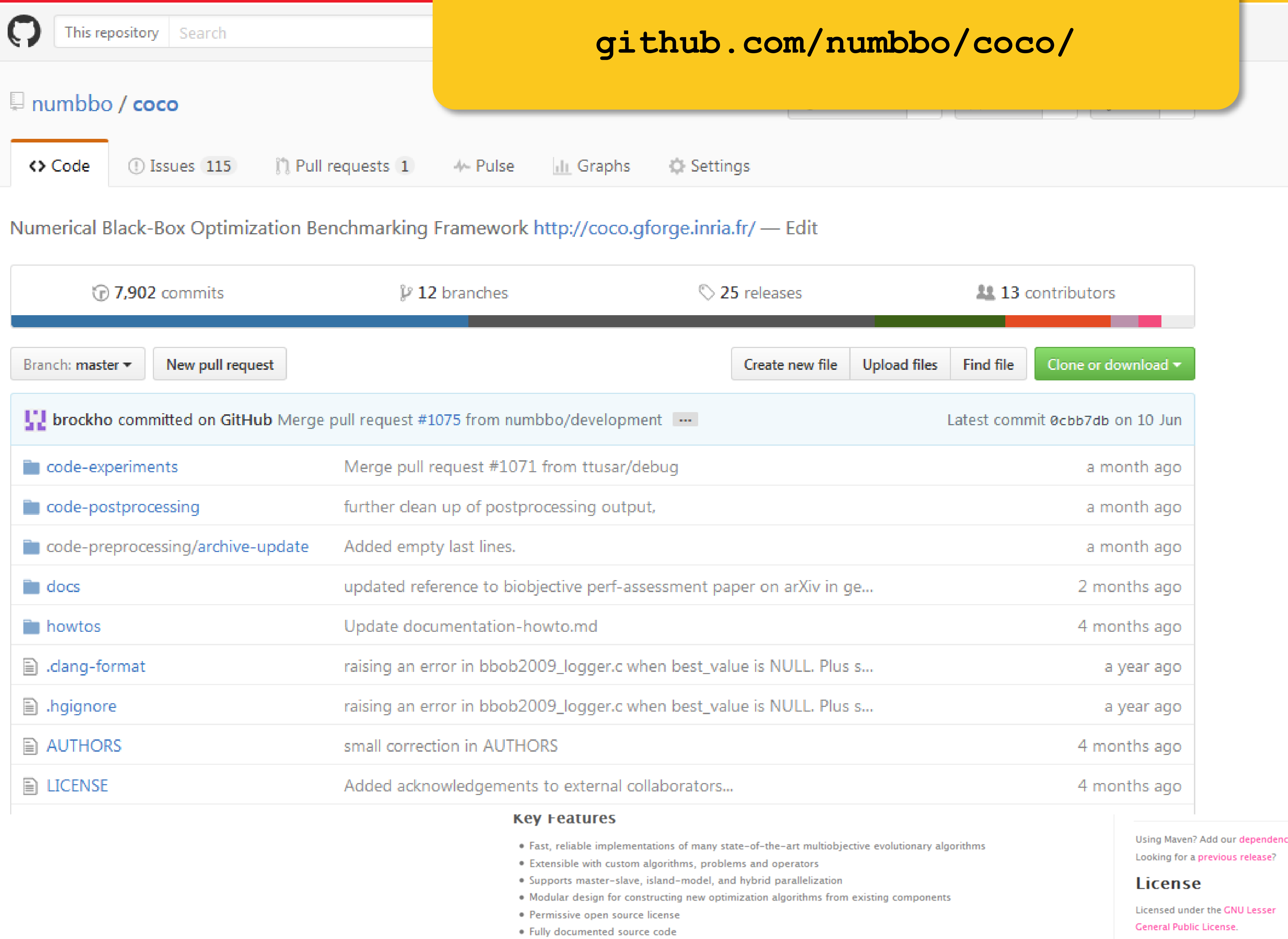
Software



ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich
Systems Optimization
News & Events | About Us | People
Research | Education
Publications | Downloads/Materials
Contact
Search
Submit Query
ETH Zürich - D-ITET - TIK - SOP - PISA
this webpage might no longer be updated more...
PISA
PISA
Principles and Documentation
PISA for Beginners
Downloads
Performance Assessment
Write and Submit a Module
Publications, Bugs, Contact & License
A Platform and Programming Language Independent Interface for Search Algorithms
Principles and Documentation
What is PISA? How does
PISA for Beginners
The first steps in order to
Downloads
Download Selectors,
Crucial Bugfix
A severe bug in the hypervolume calculation of the IBEA variator has been found, please redownload the module if your version is older
TIK
Computer Engineering and Networks Laboratory
© 2009 Institut TIK, ETH
jMetal
HOME
ALGORITHMS
PROBLEMS
RESOURCES
OUR TECHNIQUES ▼
Welcome to the jMetal Web Site
jMetal is ...
Summary of features
Download from sourceforge
jMetal stands for Metaheuristic Algorith in Java, and it is an object-orient Java-based framework for multi-object optimization with metaheuristics.
You can use it to ...
The object-oriented architecture of framework and the included features allow y to: experiment with the provided classic a state-of-the-art techniques, develop your o algorithms, solve your optimization probler integrate jMetal in other tools, etc.
Our motivation is ...
Home
Examples
Downloads
Documentation
Support
Donate
MOEA Framework
A Free and Open Source Java Framework for Multiobjective Optimization
A Framework for Innovation
The MOEA Framework is a free and open source Java library for developing and experimenting with multiobjective evolutionary algorithms (MOEAs) and other general-purpose multiobjective optimization algorithms. The MOEA Framework supports genetic algorithms, differential evolution, particle swarm optimization, genetic programming, grammatical evolution, and more. A number of algorithms are provided out-of-the-box, including NSGA-II, NSGA-III, ε-MOEA, GDE3 and MOEA/D. In addition, the MOEA Framework provides the tools necessary to rapidly design, develop, execute and statistically test optimization algorithms.
Key Features
Fast, reliable implementations of many state-of-the-art multiobjective evolutionary algorithms
Extensible with custom algorithms, problems and operators
Supports master-slave, island-model, and hybrid parallelization
Modular design for constructing new optimization algorithms from existing components
Permissive open source license
Fully documented source code
Downloads
Current Version: 2.4
Released: Jan 02, 2015
DEMO APPLICATION
COMPILED BINARIES
SOURCE CODE
USER MANUAL
Looking for a previous release?
License
Licensed under the GNU Lesser General Public License.

Software

github.com/numbbo/coco/



References

[Auger et al. 2009a] A. Auger, J. Bader, D. Brockhoff, and E. Zitzler. Theory of the Hypervolume Indicator: Optimal μ-Distributions and the Choice of the Reference Point. In Foundations of Genetic Algorithms (FOGA 2009), pages 87–102, New York, NY, USA, 2009. ACM.

[Auger et al. 2009b] A. Auger, J. Bader, D. Brockhoff, and E. Zitzler. Articulating User Preferences in Many-Objective Problems by Sampling the Weighted Hypervolume. In G. Raidl et al., editors, Genetic and Evolutionary Computation Conference (GECCO 2009), pages 555–562, New York, NY, USA, 2009. ACM

[Bader 2010] J. Bader. Hypervolume-Based Search For Multiobjective Optimization: Theory and Methods. PhD thesis, ETH Zurich, 2010

[Bader and Zitzler 2011] J. Bader and E. Zitzler. HypE: An Algorithm for Fast Hypervolume-Based Many-Objective Optimization. Evolutionary Computation 19(1):45-76, 2011.

[Bader et al. 2009] J. Bader, D. Brockhoff, S. Welten, and E. Zitzler. On Using Populations of Sets in Multiobjective Optimization. In M. Ehrgott et al., editors, Conference on Evolutionary Multi-Criterion Optimization (EMO 2009), volume 5467 of LNCS, pages 140–154. Springer, 2009

[Bandaru and Deb 2015] S. Bandaru and K. Deb. Temporal Innovization: Evolution of Design Principles Using Multi-objective Optimization. In A. Gaspar-Cunha et al., editors, Proc. EMO 2015, volume 9018 of LNCS, pages 79-93, Springer, 2015

[Bechikh et al. 2015] S. Bechikh, M. Kessentini, L. Ben Said and K. Ghedira. Preference Incorporation in Evolutionary Multiobjective Optimization: A Survey of the State-of-the-Art. Advances in Computers, 98:141–207, 2015

[Bezerra et al. 2015] L. Bezerra, M. Lopez-Ibanez, T. Stützle. To DE or Not to DE? Multi-objective Differential Evolution Revisited from a Component-Wise Prespective. In A. Gaspar-Cunha et al., editors, Proc. EMO 2015, volume 9018 of LNCS, pages 48-63, Springer, 2015

[Branke 2008] J. Branke. Consideration of Partial User Preferences in Evolutionary Multiobjective Optimization. In Multiobjective Optimization, volume 5252 of LNCS, pages 157-178. Springer, 2008

References

[Branke and Deb 2004] J. Branke and K. Deb. Integrating User Preferences into Evolutionary Multi-Objective Optimization. In Y. Jin, editor, Knowledge Incorporation in Evolutionary Computation, pages 461–477. Springer, 2004

[Branke et al. 2014] J. Branke, S. Greco, R. Slowinski and P. Zielniewicz. Learning Value Functions in Interactive Evolutionary Multiobjective Optimization. IEEE Transactions on Evolutionary Computation, 19: 88-102, 2014

[Bringmann 2012] K. Bringmann. An improved algorithm for Klee's measure problem on fat boxes. Computational Geometry: Theory and Applications, 45:225–233, 2012.

[Bringmann 2013] K. Bringmann. Bringing Order to Special Cases of Klee's Measure Problem. arXiv preprint arXiv:1301.7154 (2013).

[Bringmann and Friedrich 2009] K. Bringmann and T. Friedrich. Approximating the Least Hypervolume Contributor: NP-hard in General, But Fast in Practice. In M. Ehrgott et al., editors, Conference on Evolutionary Multi-Criterion Optimization (EMO 2009),pages 6–20. Springer, 2009

[Bringmann, et al. 2014] K. Bringmann, T. Friedrich, and and Patrick Klitzke. Two-dimensional subset selection for hypervolume and epsilon-indicator. Genetic and Evolutionary Computation Conference (GECCO 2014), pages 589–596. ACM, 2014

[Brockhoff et al. 2009] D. Brockhoff, T. Friedrich, N. Hebbinghaus, C. Klein, F. Neumann, and E. Zitzler. On the Effects of Adding Objectives to Plateau Functions. IEEE Transactions on Evolutionary Computation, 13(3):591–603, 2009

[Brockhoff et al. 2012] D. Brockhoff, T. Wagner, and H. Trautmann. On the Properties of the R2 Indicator. In Genetic and Evolutionary Computation Conference (GECCO 2012), pages 465–472. ACM, 2012

[Brockhoff et al. 2013] D. Brockhoff, J. Bader, L. Thiele and E. Zitzler. Directed Multiobjective Optimization Based on the Weighted Hypervolume Indicator. Journal of Multicriteria Decision Analysis, 20(5-6):291–317, 2013

References

[Brockhoff et al. 2014] D. Brockhoff, Y. Hamadi, and S. Kaci. Using Comparative Preference Statements in Hypervolume-Based Interactive Multiobjective Optimization. Learning and Intelligent Optimization (LION 2014), pages 121–136. Springer, 2014

[Calonder et al. 2006] M. Calonder, S. Bleuler, and E. Zitzler. Module Identification from Heterogeneous Biological Data Using Multiobjective Evolutionary Algorithms. In T. P. Runarsson et al., editors, Conference on Parallel Problem Solving from Nature (PPSN IX), volume 4193 of LNCS, pages 573–582. Springer, 2006

[Camerini et al. 1984] P. M. Camerini, G. Galbiati, and F. Maffioli. The complexity of multi-constrained spanning tree problems. In Theory of algorithms, Colloquium PECS 1984, pages 53-101, 1984.

[Deb and Kumar 2007] K. Deb and A. Kumar. Light Beam Search Based Multi-objective Optimization Using Evolutionary Algorithms. In Congress on Evolutionary Computation (CEC 2007), pages 2125–2132. IEEE Press, 2007

[Deb and Srinivasan 2006] K. Deb and A. Srinivasan. Innovization: Innovating Design Principles through Optimization. In Proc. GECCO 2006, pages 1629–1636. ACM, 2006

[Deb and Sundar 2006] K. Deb and J. Sundar. Reference Point Based Multi-Objective Optimization Using Evolutionary Algorithms. In Maarten Keijzer et al., editors, Conference on Genetic and Evolutionary Computation (GECCO 2006), pages 635–642. ACM Press, 2006

[Deb et al. 2014] K. Deb, S. Bandaru, D. Greiner, A. Gaspar-Cunha and C. Celal Tutum. An integrated approach to automated innovization for discovering useful design principles: Case studies from engineering. Applied Soft Computing, 15:42-56, 2014

[Díaz-Manríquez et al. 2013] A. Díaz-Manríquez, G. Toscano-Pulido, C. A. C. Coello and R. Landa-Becerra. A ranking method based on the R2 indicator for many-objective optimization. In IEEE Congress on Evolutionary Computation (CEC), pages 1523-1530. IEEE.

[Emmerich et al. 2007] M. Emmerich, A. Deutz and N. Beume. Gradient-Based/Evolutionary Relay Hybrid for Computing Pareto Front Approximations Maximizing the *S*-Metric. In Bartz-Beielstein et al., editors, Proc. Hybrid Metaheuristics, pages 140-156. Springer, 2007

References

[Fonseca and Fleming 1998a] C. M. Fonseca and Peter J. Fleming. Multiobjective Optimization and Multiple Constraint Handling with Evolutionary Algorithms—Part I: A Unified Formulation. IEEE Transactions on Systems, Man, and Cybernetics, 28(1):26–37, 1998

[Fonseca and Fleming 1998b] C. M. Fonseca and Peter J. Fleming. Multiobjective Optimization and Multiple Constraint Handling with Evolutionary Algorithms—Part II: Application Example. IEEE Transactions on Systems, Man, and Cybernetics, 28(1):38–47, 1998

[Fonseca et al. 2011] C. M. Fonseca, A. P. Guerreiro, M. López-Ibáñez, and L. Paquete. On the computation of the empirical attainment function. In Takahashi et al., editors, Proc. EMO, volume 6576 of LNCS, pages 106-120. Springer, 2011

[Friedrich et al. 2011] T. Friedrich, K. Bringmann, T. Voß, C. Igel. The Logarithmic Hypervolume Indicator. In Beyer and Langdon, editors, Proc. FOGA. ACM, 2011.

[Guerreiro et al. 2015] A. P. Guerreiro, C. M. Fonseca, and L. Paquete. Greedy Hypervolume Subset Selection in the Three-Objective Case. In Genetic and Evolutionary Computation Conference (GECCO 2015), pages 671-678. ACM, 2015

[Greiner et al. 2007] D. Greiner, J. M. Emperador, G. Winter, and B. Galván. Improving Computational Mechanics Optimium Design Using Helper Objectives: An Application in Frame Bar Structures. In Conference on Evolutionary Multi-Criterion Optimization (EMO 2007), volume 4403 of LNCS, pages 575–589. Springer, 2007

[Hadka and Reed 2013] D. Hadka and P. Reed. Borg: An Auto-Adaptive Many-Objective Evolutionary Computing Framework. Evolutionary Computation, 21(2):231–259, 2013

[Handl et al. 2008a] J. Handl, S. C. Lovell, and J. Knowles. Investigations into the Effect of Multiobjectivization in Protein Structure Prediction. In G. Rudolph et al., editors, Conference on Parallel Problem Solving From Nature (PPSN X), volume 5199 of LNCS, pages 702–711. Springer, 2008

[Handl et al. 2008b] J. Handl, S. C. Lovell, and J. Knowles. Multiobjectivization by Decomposition of Scalar Cost Functions. In G. Rudolph et al., editors, Conference on Parallel Problem Solving From Nature (PPSN X), volume 5199 of LNCS, pages 31–40. Springer, 2008

References

[Igel et al. 2007] C. Igel, N. Hansen, and S. Roth. Covariance Matrix Adaptation for Multi-objective Optimization. Evolutionary Computation, 15(1):1–28, 2007

[Jaszkiewicz and Branke 2008] A. Jaszkiewicz and J. Branke. Interactive Multiobjective Evolutionary Algorithms. In: Multiobjective Optimization: Interactive and Evolutionary Approaches, pages 179–193, Springer, 2008

[Jensen 2004] M. T. Jensen. Helper-Objectives: Using Multi-Objective Evolutionary Algorithms for Single-Objective Optimisation. Journal of Mathematical Modelling and Algorithms, 3(4):323–347, 2004

[Judt et al. 2011] L. Judt, O. Mersmann, and B. Naujoks. Non-monotonicity of obtained hypervolume in 1-greedy S-Metric Selection. In: Conference on Multiple Criteria Decision Making (MCDM 2011), 2011

[Knowles et al. 2001] J. D. Knowles, R. A. Watson, and D. W. Corne. Reducing Local Optima in Single-Objective Problems by Multi-objectivization. In E. Zitzler et al., editors, Conference on Evolutionary Multi-Criterion Optimization (EMO 2001), volume 1993 of LNCS, pages 269–283. Springer, 2001

[Kuhn et al. 2014] T. Kuhn, C. M. Fonseca, L. Paquete, S. Ruzika, and J. R. Figueira. Hypervolume subset selection in two dimensions: Formulations and algorithms. Technical report. Technische Universität Kaiserslautern, Fachbereich Mathematik, 2014

[Lopez-Ibanez and Knowles 2015] M. Lopez-Ibanez and J. D. Knowles. Machine Decision Makers as a Laboratory for Interactive EMO. In A. Gaspar-Cunha et al., editors, Proc. EMO, volume 9019 of LNCS, pages 295-309. Springer, 2015

[Miettienen 1999] K. Miettinen. Nonlinear Multiobjective Optimization. Kluwer, Boston, MA, USA, 1999

[Neumann and Wegener 2006] F. Neumann and I. Wegener. Minimum Spanning Trees Made Easier Via Multi-Objective Optimization. Natural Computing, 5(3):305–319, 2006

[Obayashi and Sasaki 2003] S. Obayashi and D. Sasaki. Visualization and Data Mining of Pareto Solutions Using Self-Organizing Map. In Conference on Evolutionary Multi-Criterion Optimization (EMO 2003), volume 2632 of LNCS, pages 796–809. Springer, 2003

[Sacks et al. 1989] J. Sacks, W. Welch, T. Mitchell, H. Wynn. : Design and Analysis of Computer Experiments. Statistical Science, 4(4):409–423, 1989

References

[Schaffer 1985] J. D. Schaffer. Multiple Objective Optimization with Vector Evaluated Genetic Algorithms. In John J. Grefenstette, editor, Conference on Genetic Algorithms and Their Applications, pages 93–100, 1985.

[Segura et al. 2013] C. Segura, C. A. Coello Coello, M. Gara and L. Coromoto. Using multi-objective evolutionary algorithms for single-objective optimization. In: 4OR, 11(3):201-228. Springer, 2013.

[Siegfried et al. 2009] T. Siegfried, S. Bleuler, M. Laumanns, E. Zitzler, and W. Kinzelbach. Multi-Objective Groundwater Management Using Evolutionary Algorithms. IEEE Transactions on Evolutionary Computation, 13(2):229–242, 2009

[Tanino et al. 1993] T. Tanino, M. Tanaka, and C. Hojo. An Interactive Multicriteria Decision Making Method by Using a Genetic Algorithm. In: Conference on Systems Science and Systems Engineering, pages 381–386, 1993

[Thiele et al. 2002] L. Thiele, S. Chakraborty, M. Gries, and S. Künzli. Design Space Exploration of Network Processor Architectures. In Network Processor Design 2002: Design Principles and Practices. Morgan Kaufmann, 2002

[Trautmann et al. 2013] H. Trautmann, T. Wagner, and D. Brockhoff. R2-EMOA: Focused Multiobjective Search Using R2-Indicator-Based Selection. Learning and Intelligent Optimization Conference (LION 2013), pages 70–74, Springer, 2013. Short paper.

[Ulrich et al. 2007] T. Ulrich, D. Brockhoff, and E. Zitzler. Pattern Identification in Pareto-Set Approximations. In M. Keijzer et al., editors, Genetic and Evolutionary Computation Conference (GECCO 2008), pages 737–744. ACM, 2008.

[Verel et al. 2011] S. Verel, C. Dhaenens, A. Liefooghe. Set-based Multiobjective Fitness Landscapes: A Preliminary Study. In Genetic and Evolutionary Computation Conference (GECCO 2011), pages 769–776. ACM, 2011.

References

[Voß et al. 2010] T. Voß, N. Hansen, and C. Igel. Improved Step Size Adaptation for the MO-CMA-ES. In J. Branke et al., editors, Genetic and Evolutionary Computation Conference (GECCO 2010), pages 487–494. ACM, 2010

[Wagner et al. 2008] T. Wagner, D. Passmann, K. Weinert, D. Biermann and A. Bledzki. Efficient Modeling and Optimization of the Property Gradation of Self-Reinforced Polypropylene Sheets within a Thermo-Mechanical Compaction Process. In R. Teti, editor, Proc. ICME, pages 447–452. Edizione Ziino, 2008

[Wagner et al. 2010] T. Wagner, M. Emmerich, A. Deutz and W. Ponweiser. Improvement Criteria for Model-Based Multi-Objective Optimization. In R. Schaefer et al., editors, Proc. PPSN, volume 6238 of LNCS, pages 718–727. Springer, 2010

[Watanabe and Sakakibara 2007] S. Watanabe and K. Sakakibara. A multiobjectivization approach for vehicle routing problems. In Conference on Evolutionary Multi-Criterion Optimization (EMO 2007), volume 4403 of LNCS, pages 660–672. Springer, 2007

[Weinert et al. 2009] K. Weinert, A. Zabel, P. Kersting, T. Michelitsch and T. Wagner. On the Use of Problem-Specic Candidate Generators for the Hybrid Optimization of Multi-Objective Production Engineering Problems. Evolutionary Computation, 17(4):527–544, 2009

[Yildiz and Suri 2012] H. Yildiz and S. Suri. On Klee's measure problem for grounded boxes. Proceedings of the 2012 symposium on Computational Geometry. ACM, 2012

[Zhang and Li 2007] Q. Zhang and H. Li. MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition. IEEE Transactions on Evolutionary Computation, 11(6):712--731, 2007

[Zhang et al. 2008] Q. Zhang, A. Zhou and Y. Jin. RM-MEDA: A Regularity Model-Based Multiobjective Estimation of Distribution Algorithm. IEEE Transactions on Evolutionary Computation, 12(1):41–63, 2008

[Zhang et al. 2012] L. Zhang, T. Wagner and D. Biermann. Optimization of Cutting Parameters for Drilling Nickel-Based Alloys using Statistical Experimental Design Techniques. In S. Hinduja and L. Li, editors, Proc. MATADOR, pages 123-126. Springer, 2012

References

[Zitzler 1999] E. Zitzler. Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications. PhD thesis, ETH Zurich, Switzerland, 1999

[Zitzler and Künzli 2004] E. Zitzler and S. Künzli. Indicator-Based Selection in Multiobjective Search. In X. Yao et al., editors, Conference on Parallel Problem Solving from Nature (PPSN VIII), volume 3242 of LNCS, pages 832–842. Springer, 2004

[Zitzler et al. 2000] E. Zitzler, K. Deb, and L. Thiele. Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. Evolutionary Computation, 8(2):173–195, 2000

[Zitzler et al. 2003] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. Grunert da Fonseca. Performance Assessment of Multiobjective Optimizers: An Analysis and Review. IEEE Transactions on Evolutionary Computation, 7(2):117–132, 2003

[Zitzler et al. 2010] E. Zitzler, L. Thiele, and J. Bader. On Set-Based Multiobjective Optimization. IEEE Transactions on Evolutionary Computation, 14(1):58–79, 2010