Advanced Optimization

Lecture/Exercise 4: (Evolutionary) Multiobjective Optimmization

December 11, 2018

Master AIC

Université Paris-Saclay, Orsay, France

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Course Overview

	Date		Topic
1	Tue, 20.11.2018	Dimo	Randomized Algorithms for Discrete Problems
2	Tue, 27.11.2018	Dimo	Exercise: The Travelling Salesperson Problem
3	Tue, 4.12.2018	Dimo	Evolutionary Multiobjective Optimization I
4	Tue, 11.12.2018	Dimo	Evolutionary Multiobjective Optimization II
	vacation		
5	Tue, 8.1.2019	Dimo	Looking at Data
6	Tue, 15.1.2019	Anne	Continuous Optimization I
7	Tue, 29.1.2019	Anne	Continuous Optimization II
	Tue, 12.2.2019		oral presentations (individual time slots)

all lectures from 14h00 till 17h15

here in E107 in Nov/Dec and in E105 in January

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Organization Oral Exams

	Tuesday, Feb 12, 2019	
9:30am	Martin	
10am	Robin	
10:30am	Hao	
11am	Malik	
11:30am	Jiaxin	
12am	Samuel	
12:30pm	Nouredine	
1:30pm	Mirwaisse	
2:00pm		
2:30pm	Alexandre	
3pm	Luc	
3:30pm		
4pm		

to be assigned: Antoine, David, Cedric, Luca

Overview of the Two EMO Lectures

Introduction to multiobjective optimization

(a bit more detailed than in the introductory lecture)

- difference to single-objective optimization, the basics
- algorithms and their design principles
- performance assessment and benchmarking
- integration of preferences
- a few aspects of visualization

Exercise around COCO

- implement basic algorithm(s)
- benchmark on COCO
  - two goals: testing our software and producing data for the upcoming BBOB-2019 workshop

Approaches to Multiobjective Optimization

aggregation-based

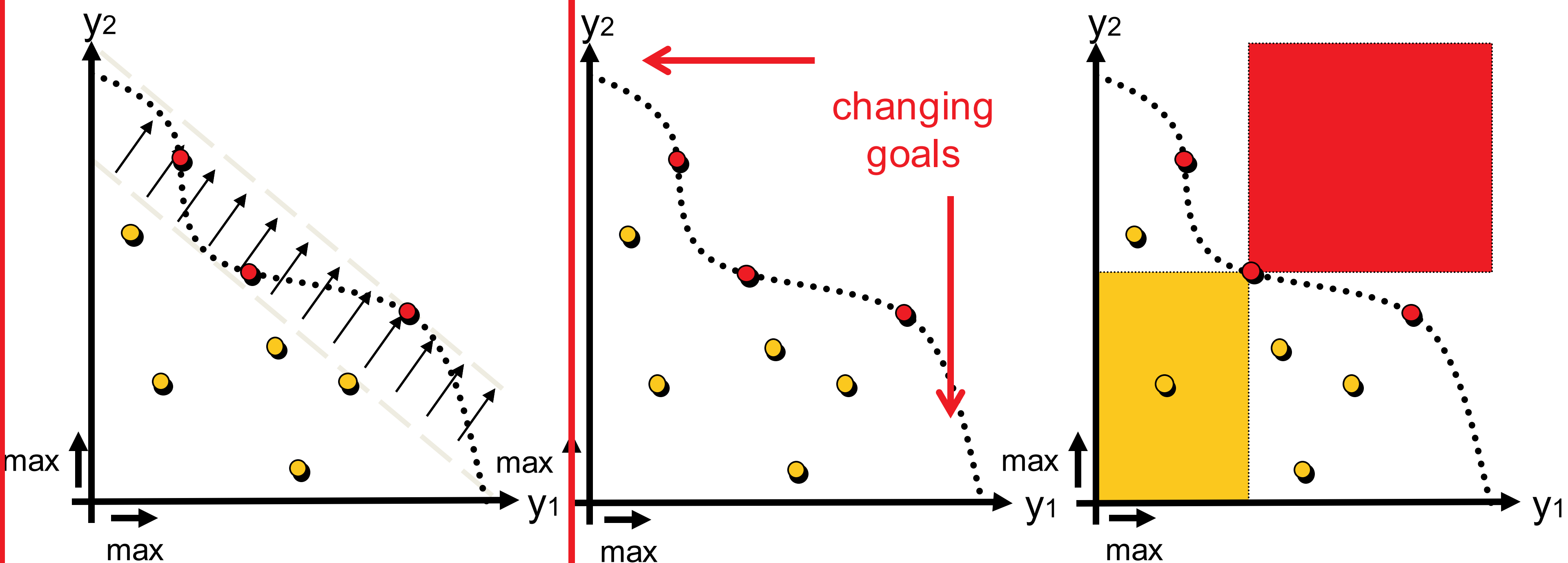
problem decomposition (multiple single-objective optimization problems)

criterion-based

VEGA

dominance-based

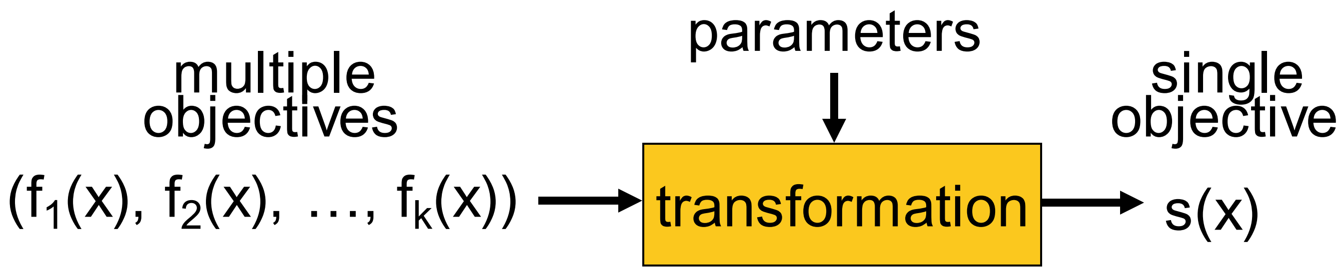
SPEA2, NSGA-II
“modern” EMOA



solution-oriented
scaling-dependent

set-oriented
less scaling-independent

Solution-Oriented Problem Transformations



A scalarizing function s is a function $s: Z \to \mathbb{R}$ that maps each objective vector $u = (u_1, \ldots, u_n) \in Z$ to a real value $s(u) \in \mathbb{R}$

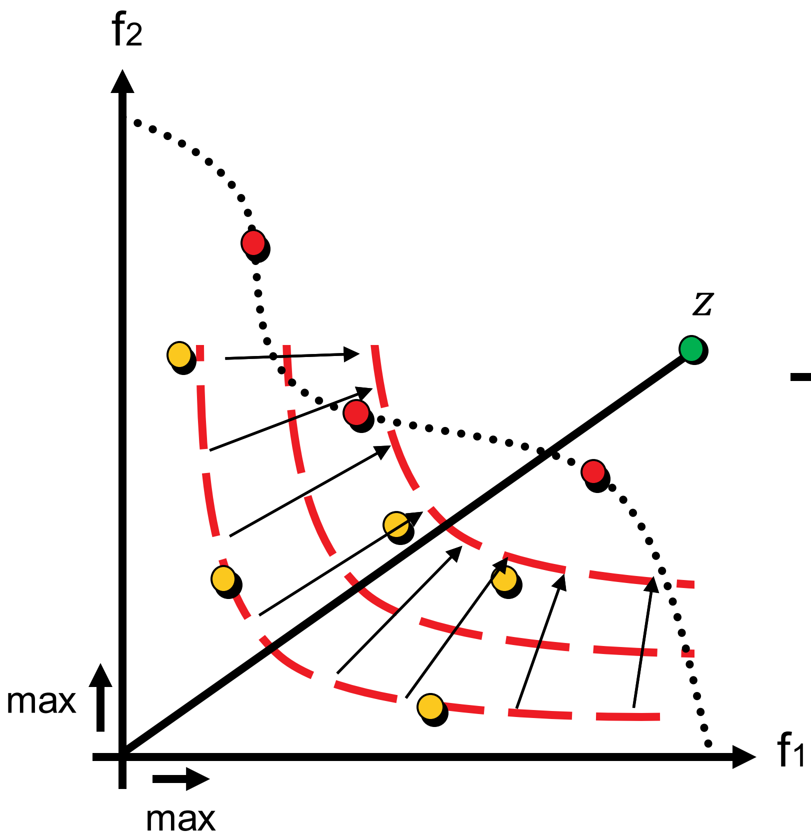
Solution-Oriented Problem Transformations

parameters

multiple objectives

$(f_1(x), f_2(x), \ldots, f_k(x))$ → **transformation** → $s(x)$

single objective



Example 2: weighted p-norm

$(w_1, w_2, \ldots, w_k)$

$$y = \sqrt[p]{\sum_{i=1}^{k} (|w_1(y_1 - z_i)|)^p}$$

$p = 1$: weighted sum

$p = \infty$: weighted Tchebycheff

Exercise: Benchmarking a Weighted Sum Approach on COCO

Exercise

a) download COCO (release 2.2.1) from **`https://github.com/numbbo/coco/`**

b) install and test it via **`python do.py run-python`**

c) run the previous example code of CMA-ES (e.g. in ipython shell) to get an idea how it works

d) start your implementation of a weighted sum optimizer from [**`http://www.cmap.polytechnique.fr/~dimo.brockhoff/advancedOptSaclay/2018/exercises/example_experiment_WS.py`**](http://www.cmap.polytechnique.fr/~dimo.brockhoff/advancedOptSaclay/2018/exercises/example_experiment_WS.py) within the function **`def weighted_sum(fun, budget)`**

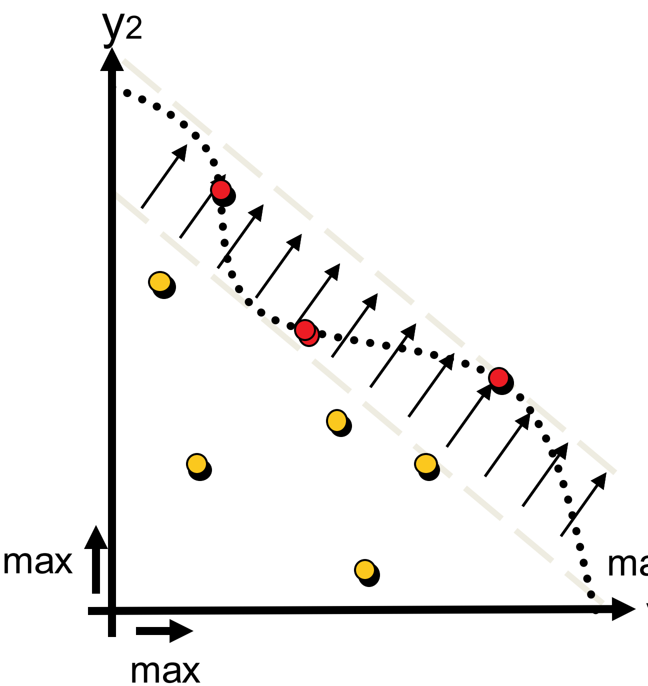
tip: start simple and extend!

e) run the experiments by typing
`python example_experiment_WS.py bbob-biobj-ext BUDGET`
with **`BUDGET`** any integer (start small and then increase) and send all data to me by email ☺

Approaches to Multiobjective Optimization

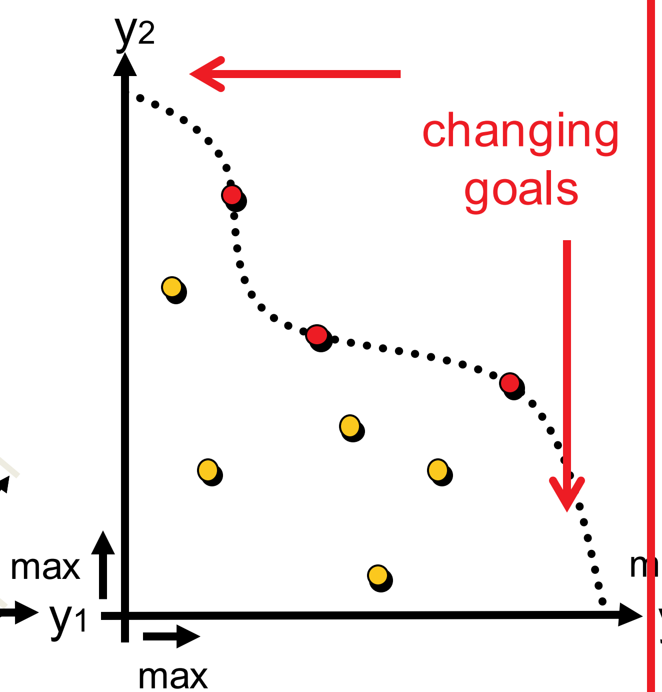
aggregation-based

problem decomposition (multiple single-objective optimization problems)



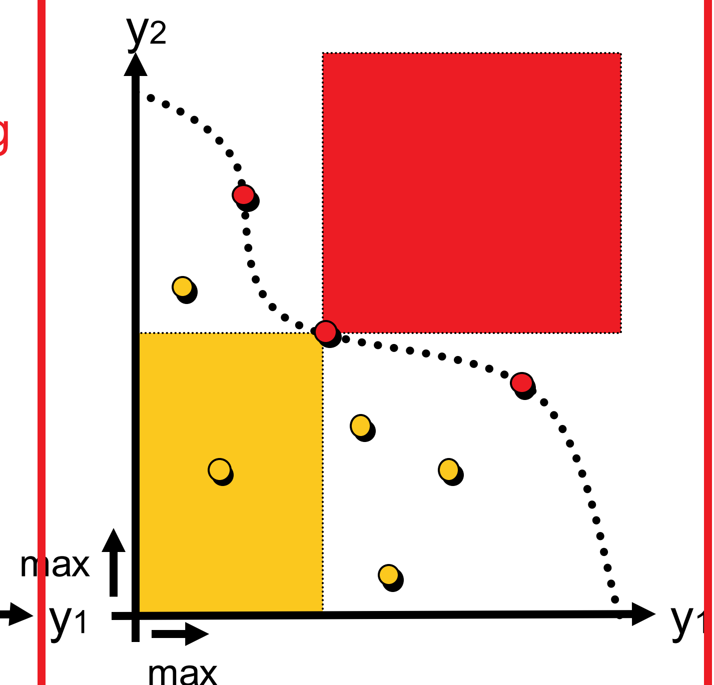
criterion-based

VEGA



dominance-based

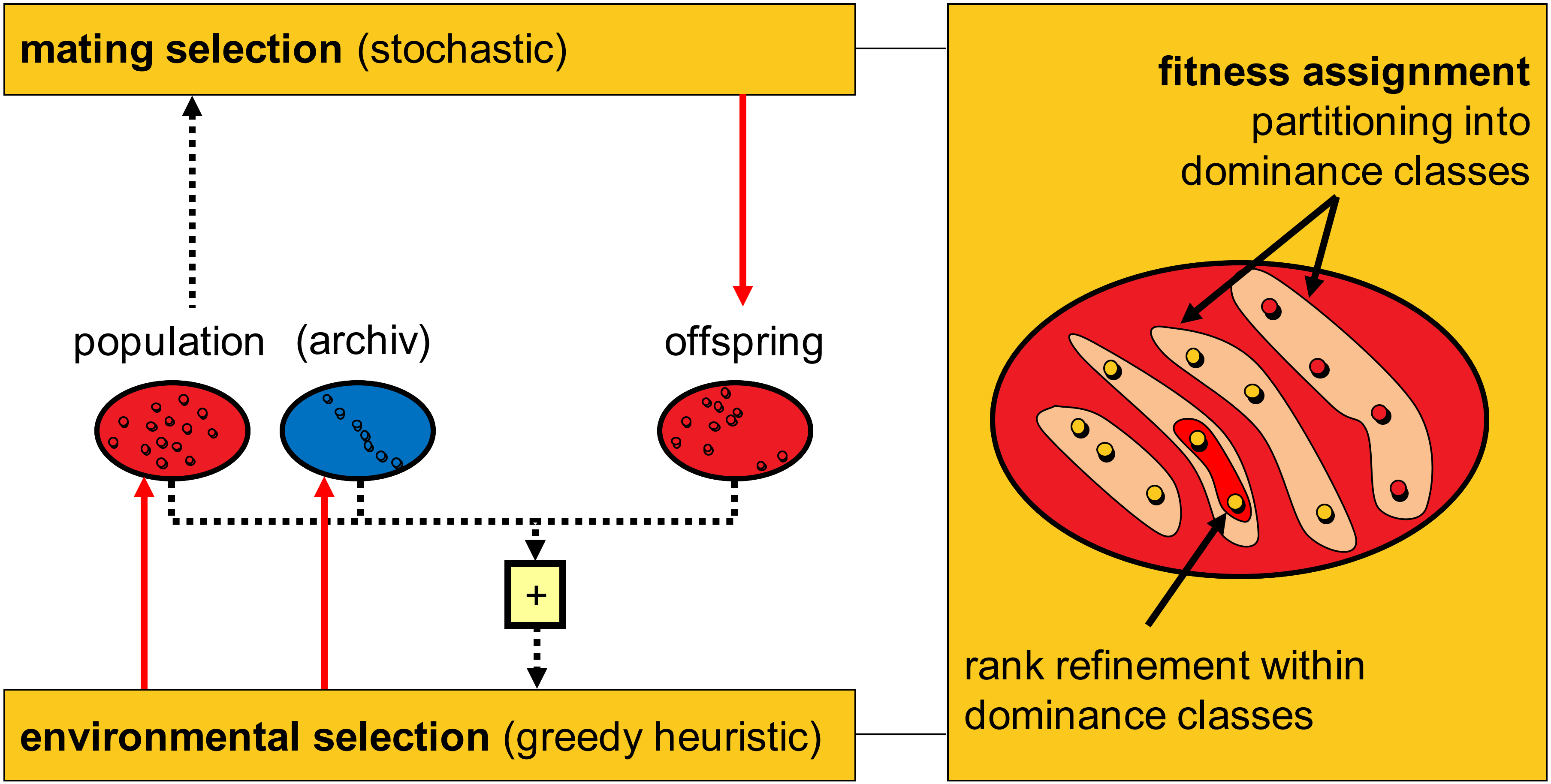
SPEA2, NSGA-II
"modern" EMOA



solution-oriented
scaling-dependent

set-oriented
less scaling-independent

General Scheme of Most Set-Oriented EMO

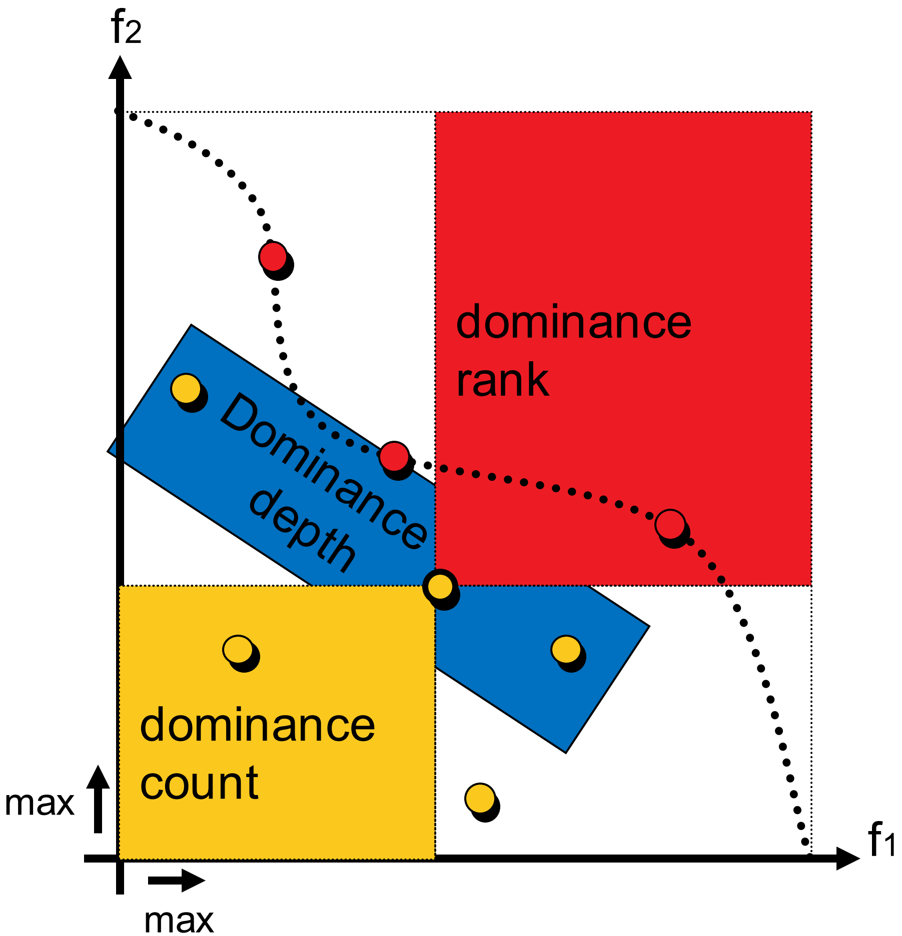


Ranking of the Population Using Dominance

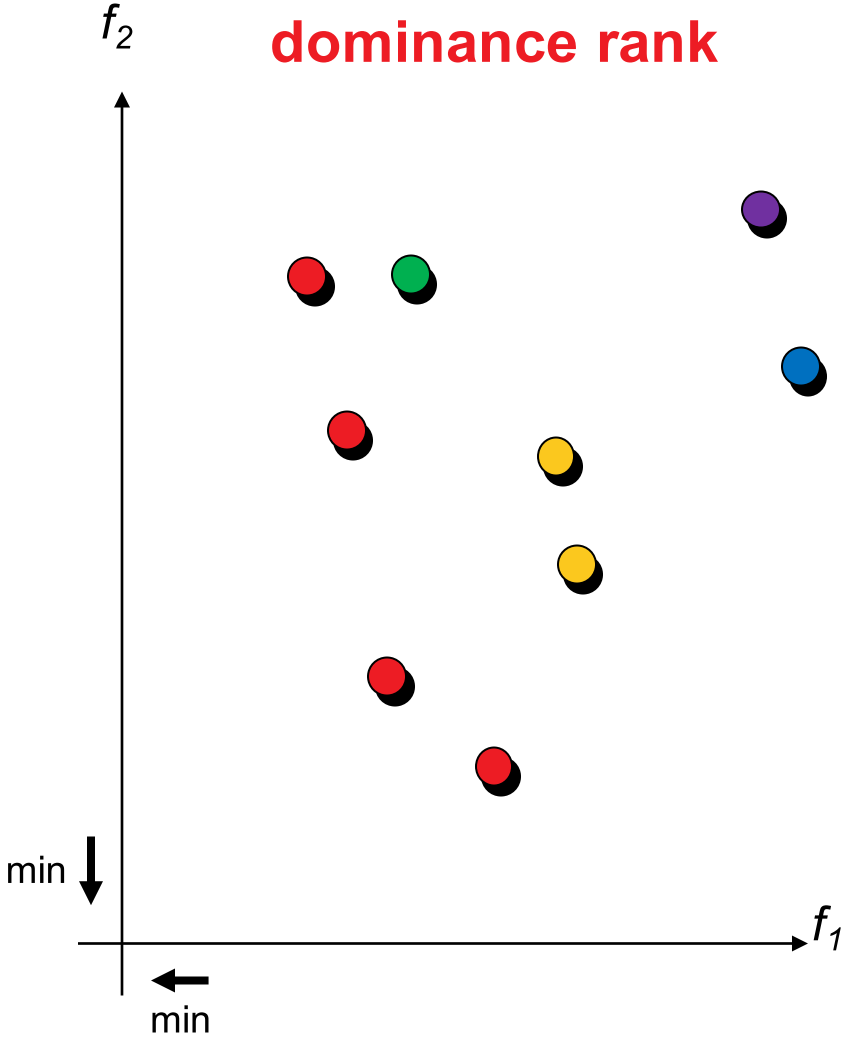
... goes back to a proposal by David Goldberg in 1989.

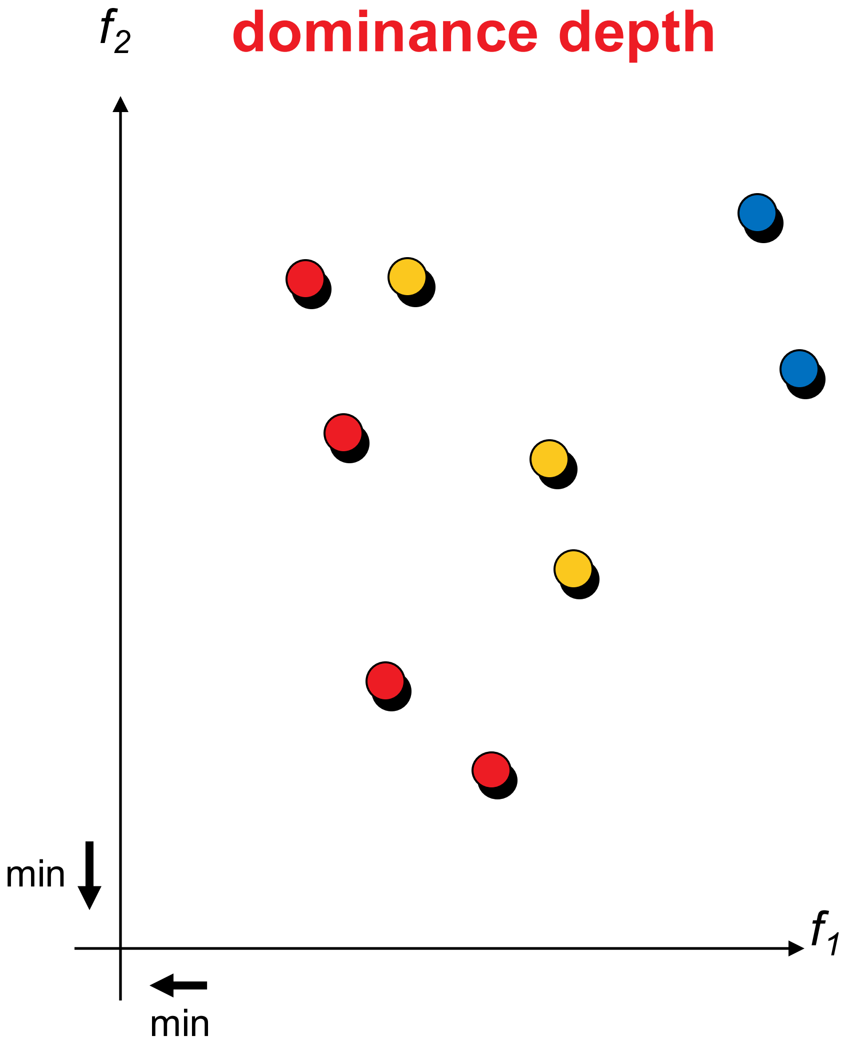
... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  MOGA, NPGA
- **dominance count:** how many individuals does an individual dominate?
  SPEA, SPEA2
- **dominance depth:** at which front is an individual located?
  NSGA, NSGA-II, most of the recently proposed algorithms



Exercise: Dominance-Based Partitioning





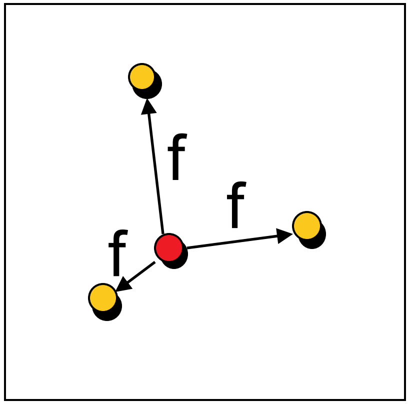
Refinement of Dominance Rankings

Goal: rank incomparable solutions within a dominance class

❶ Diversity information

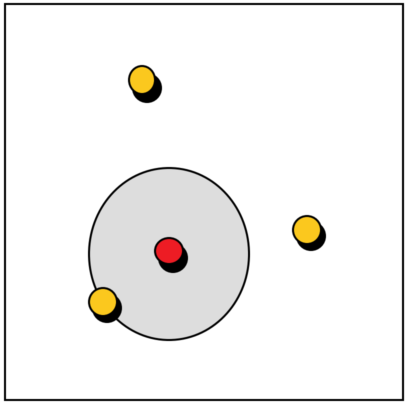
Kernel method

diversity =
function of the
distances



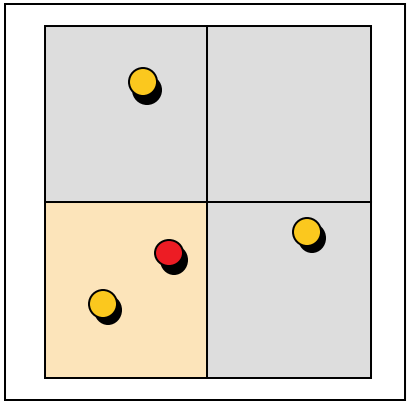
k-th nearest neighbor

diversity =
function of distance
to k-th nearest neighbor



Histogram method

diversity =
number of elements
within box(es)

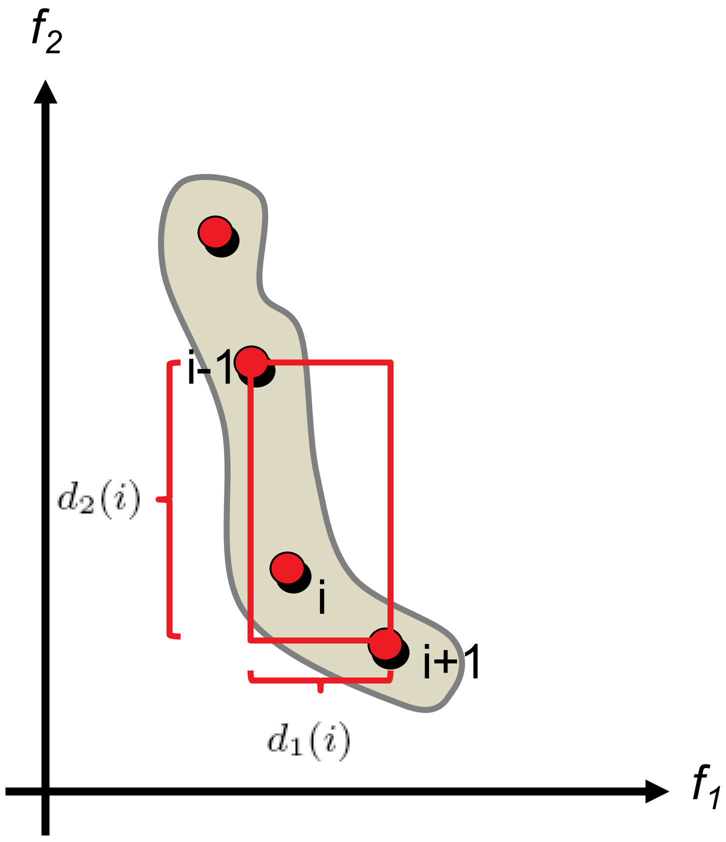


❷ (Contribution to a) quality indicator

Example: NSGA-II Diversity Preservation

Crowding Distance (CD)

- sort solutions with regard to each objective
- assign CD maximum value to extremal objective vectors
- compute CD based on the distance to the neighbors in each objective

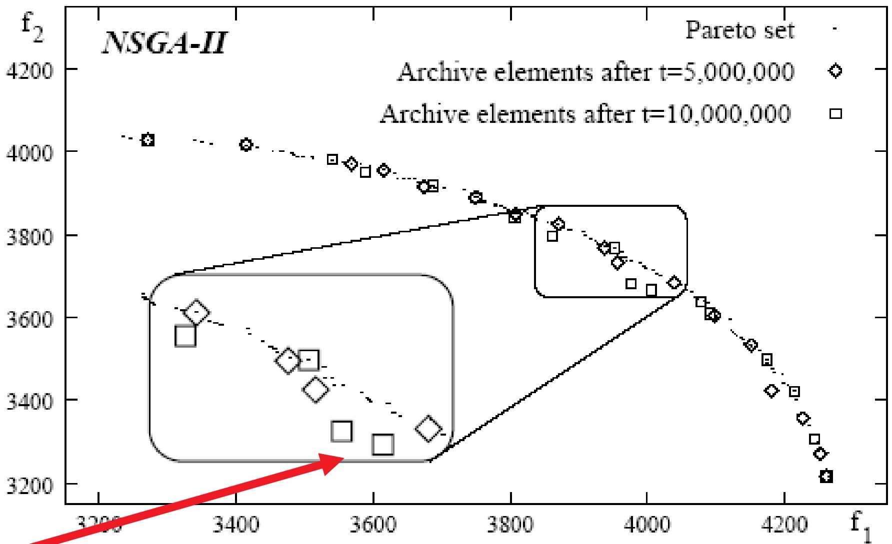


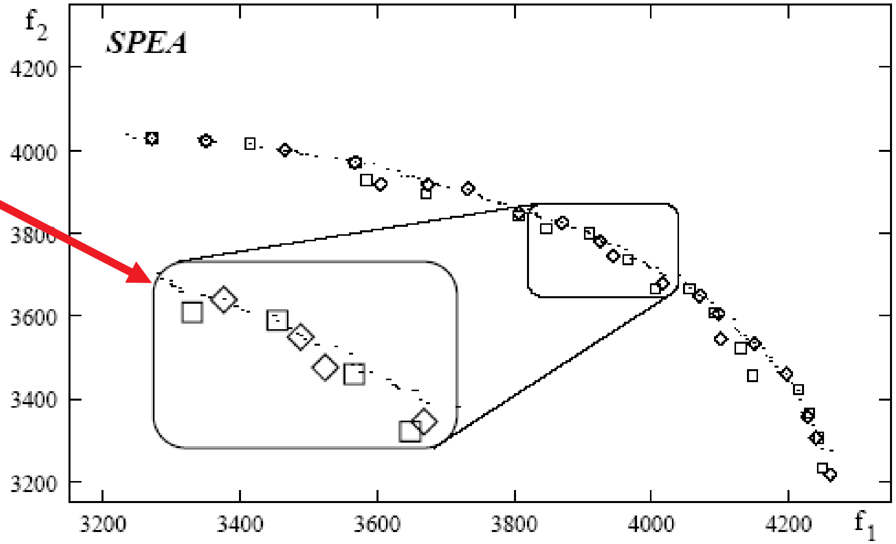
$$\mathrm{CD}(i) = \frac{d_1(i)}{f_{1,\max} - f_{1,\min}} + \cdots + \frac{d_m(i)}{f_{m,\max} - f_{m,\min}}$$

SPEA2 and NSGA-II: Deteriorative Cycles

Selection in SPEA2 and NSGA-II can result in
deteriorative cycles

non-dominated solutions already found can be lost





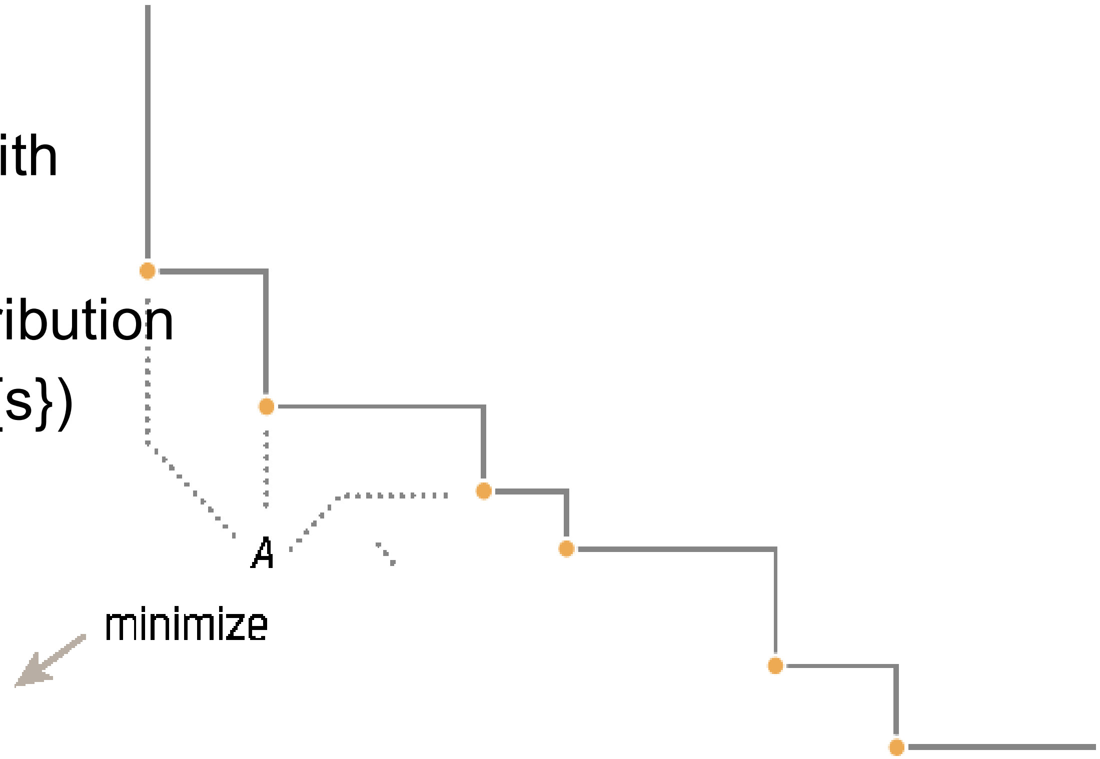
Hypervolume-Based Selection

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …)

use hypervolume indicator to guide the search: refines dominance

Main idea

Delete solutions with the smallest hypervolume contribution $d(s) = I_H(P) - I_H(P \ / \ \{s\})$ iteratively



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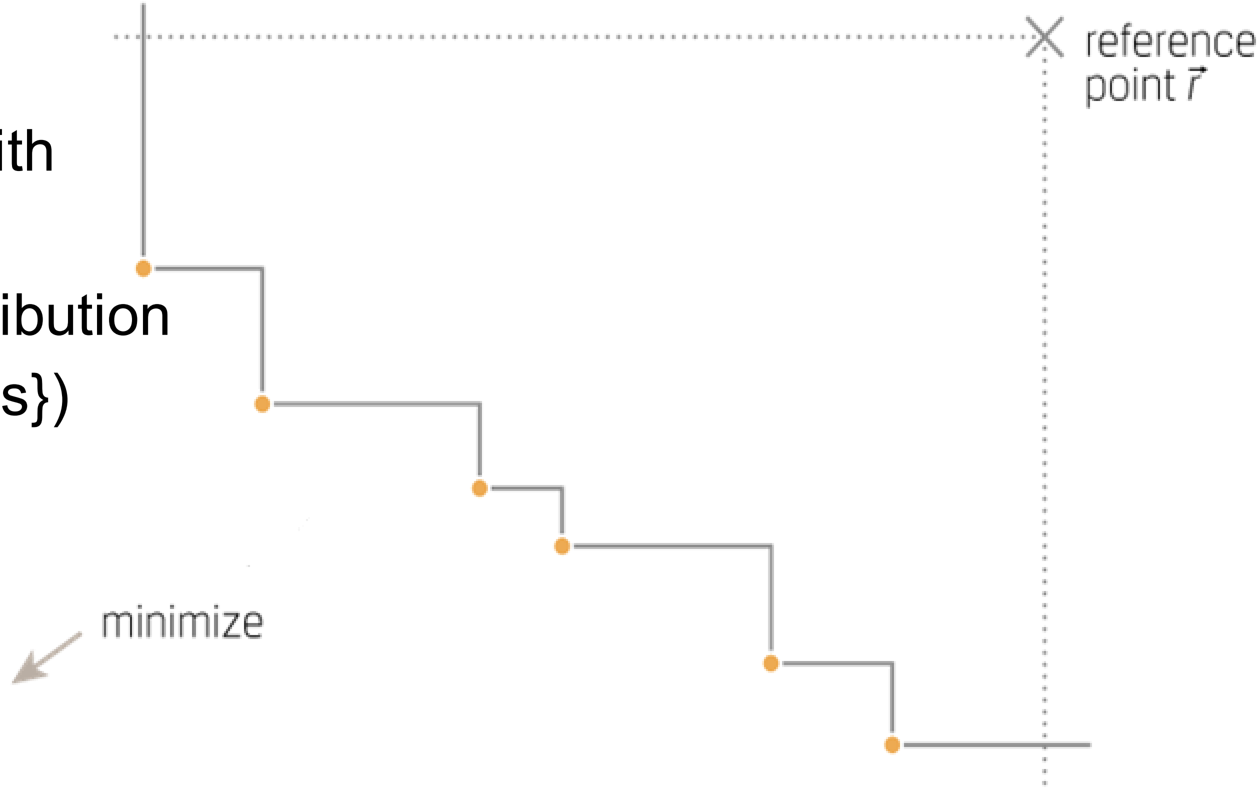

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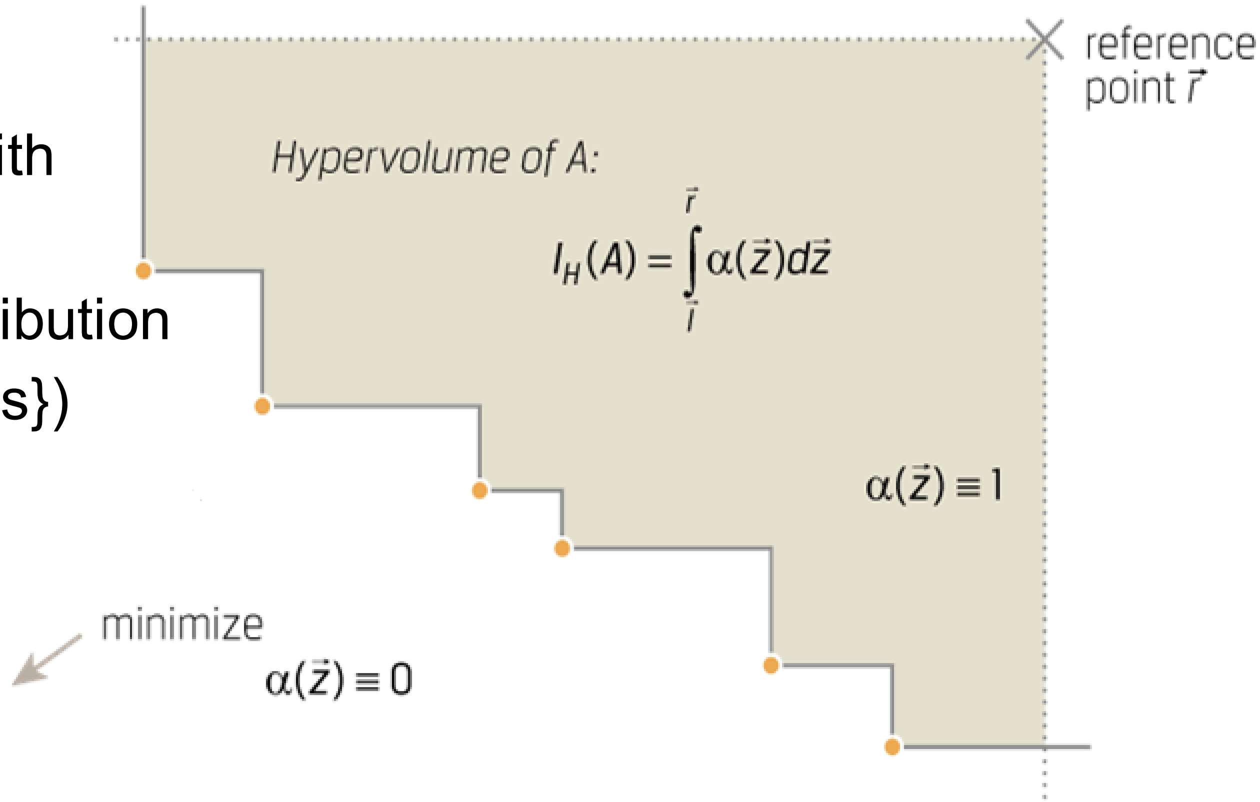

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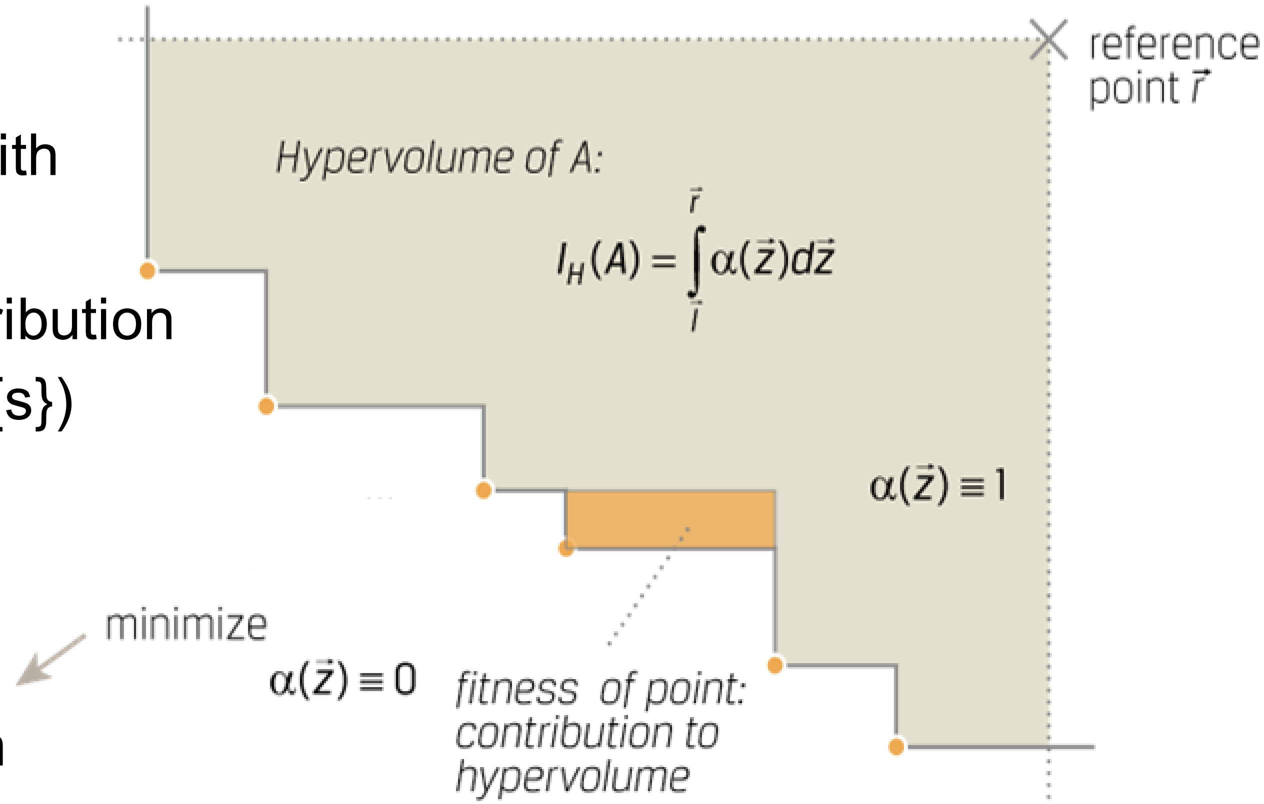


But: can also result in cycles if reference point is not constant [Judt et al. 2011] and is expensive to compute exactly [Bringmann and Friedrich 2009]

Indicator-Based Selection

- Concept can be generalized to any quality indicator

A (unary) quality indicator I is a function $I : \Psi = 2^X \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.

$$\text{Multiobjective Problem} \xrightarrow{\text{Indicator}} \text{Single-objective Problem}$$

- for example: R2-indicator [Brockhoff et al. 2012], [Trautmann et al. 2013], [Díaz-Manríquez et al. 2013]

- Generalizable also to contribution to larger sets

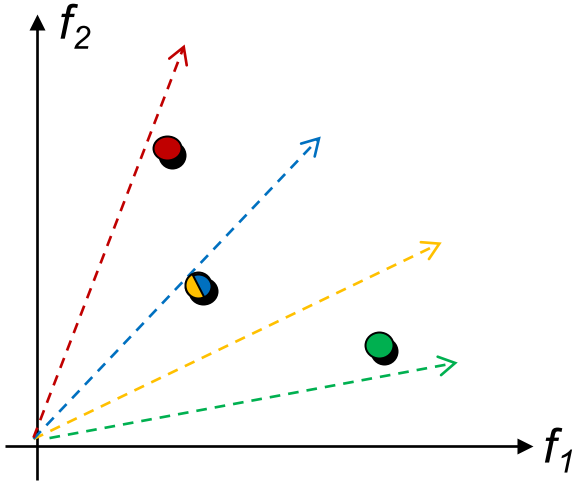
  HypE [Bader and Zitzler 2011]: Hypervolume sampling + contribution if more than 1 (random) solution deleted

Decomposition-Based Selection: MOEA/D

MOEA/D: Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

Ideas:

- optimize N scalarizing functions in parallel
- use best solutions of neighbor subproblems for mating
- keep the best solution for each scalarizing function
- update neighbors
- use external archive for non-dominated solutions
- several variants and enhancements



Remark: Variation in EMO

- at first sight not different from single-objective optimization
- most research on selection mechanisms (until now)
- but: convergence to a set ≠ convergence to a point

Open Question:

- how to achieve fast convergence to a *set*?

Related work:

- set-based gradient of the HV [Emmerich et al. 2007]
- multiobjective CMA-ES [Igel et al. 2007] [Voß et al. 2010]
- RM-MEDA [Zhang et al. 2008]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]
- offline and online configuration based on libraries of variation operators [Bezerra et al. 2015] [Hadka and Reed 2013]

The Big Picture

Basic Principles of Multiobjective Optimization

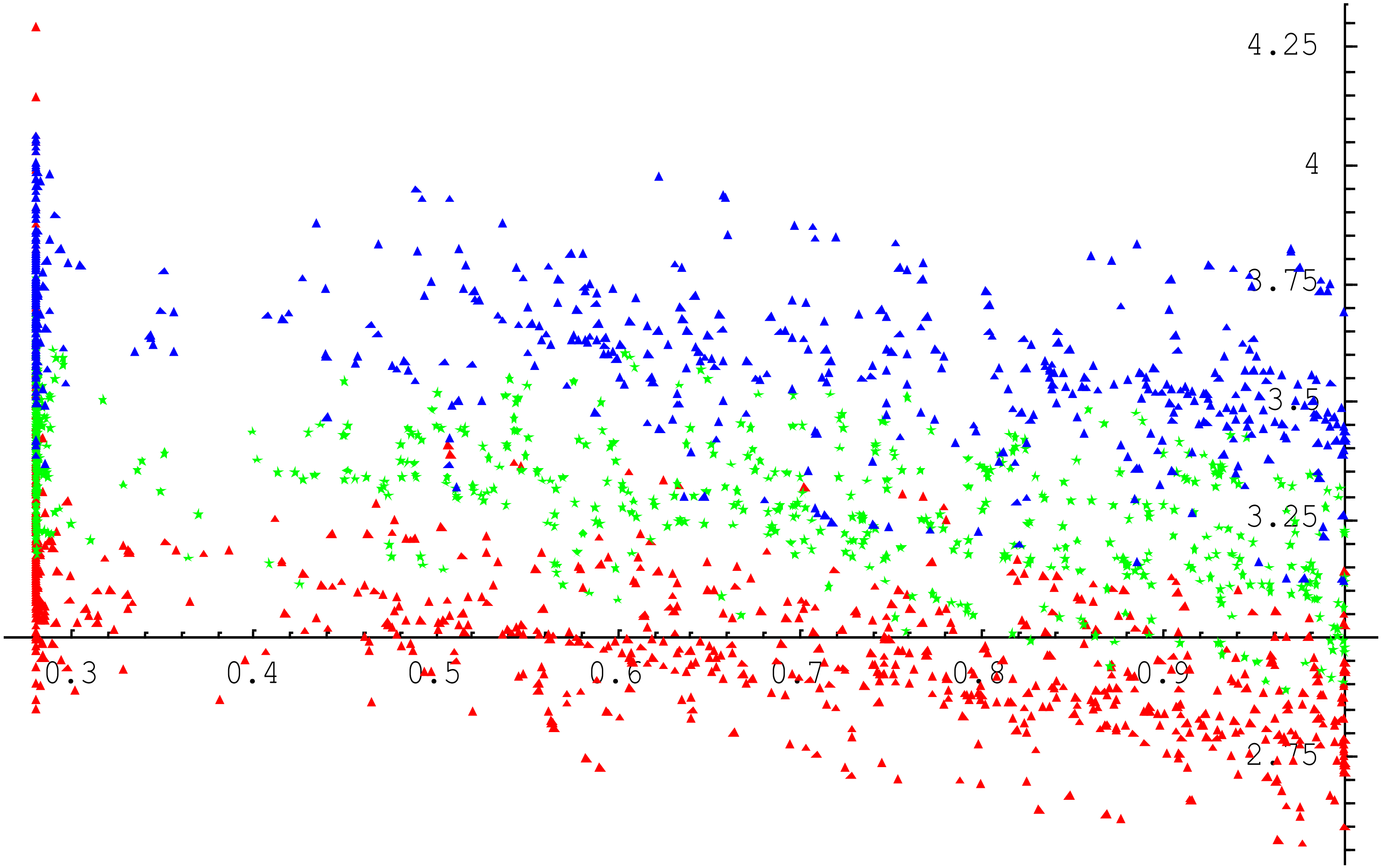
- algorithm design principles and concepts
- performance assessment

Selected Advanced Concepts

- preference articulation
- visualization aspects

Once Upon a Time...

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

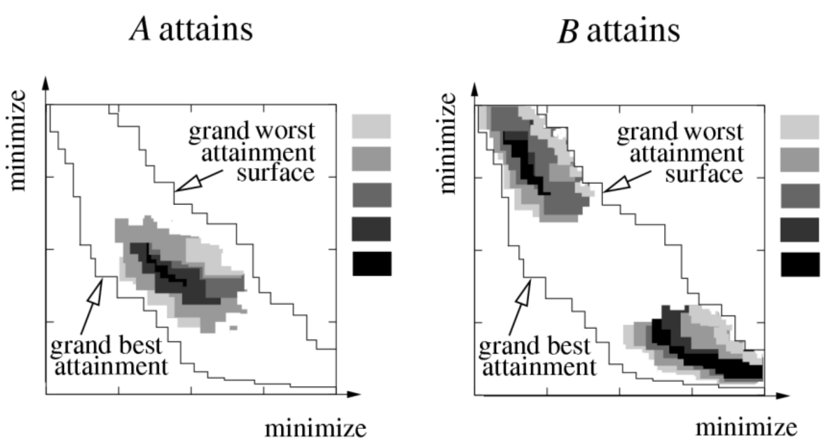
Two Main Approaches for Empirical Studies

Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur

Quality indicator approach

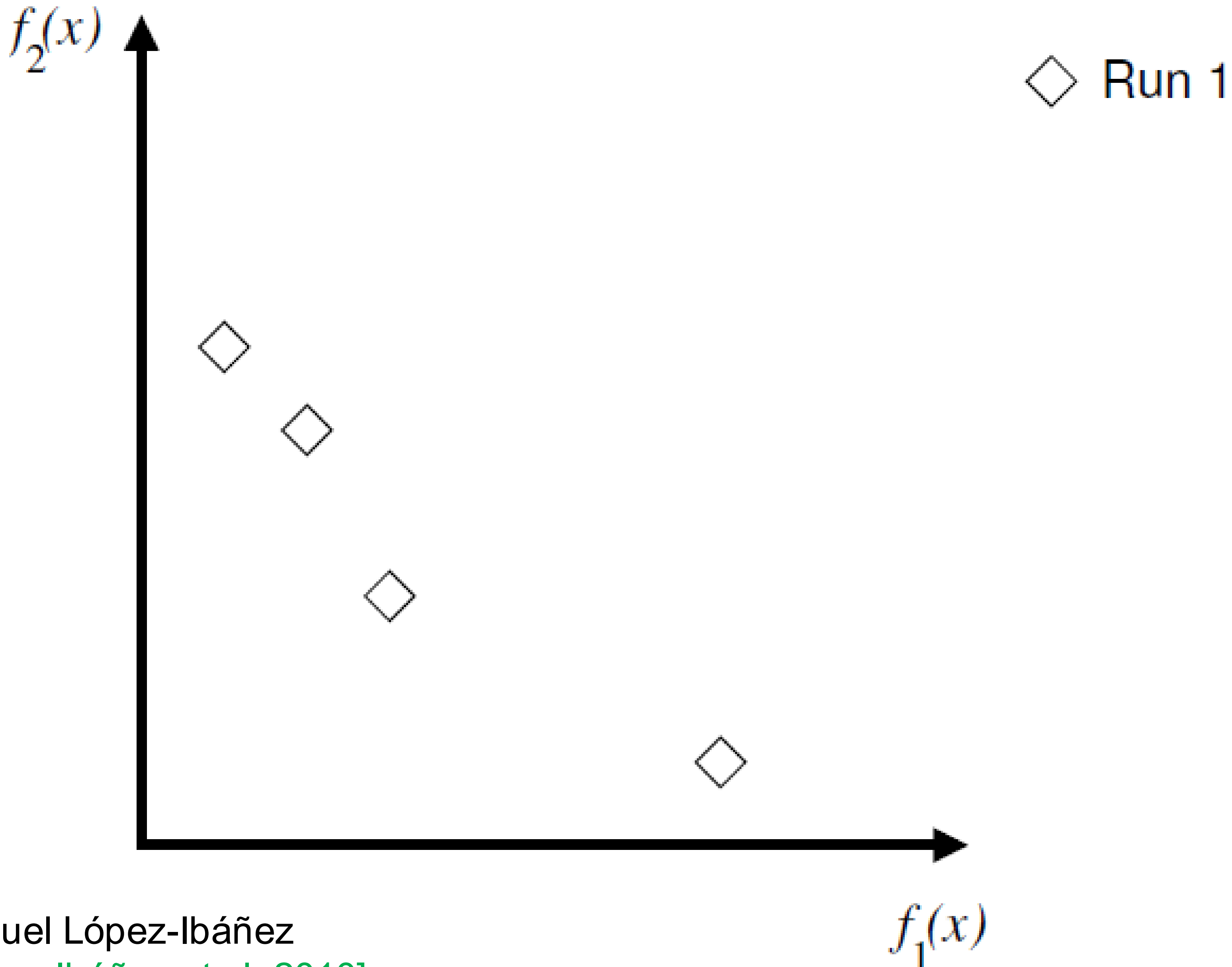
- reduces each approximation set to a single quality value
- applies statistical tests to the quality values



Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

Empirical Attainment Functions: Idea



[López-Ibáñez et al. 2010]

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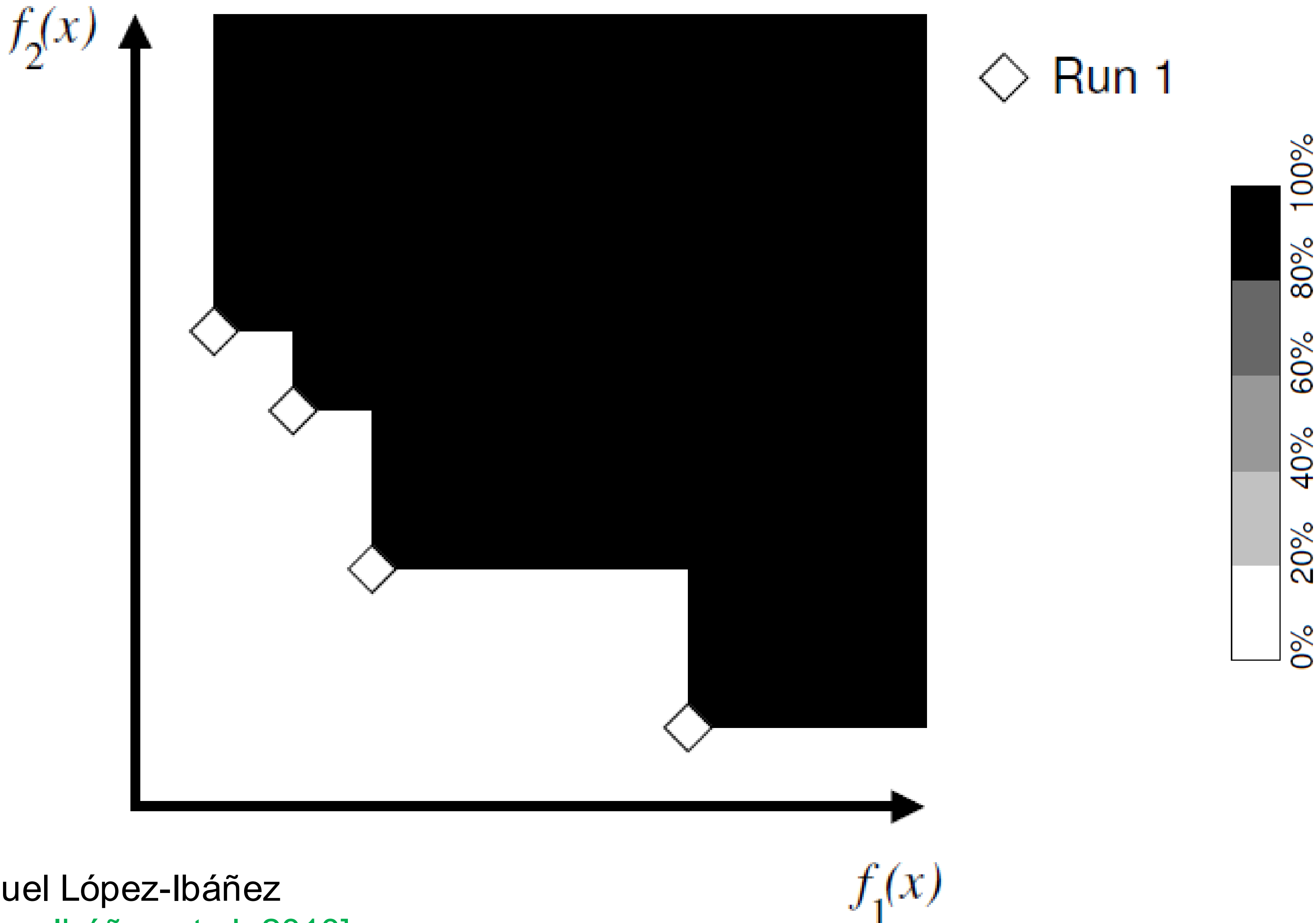

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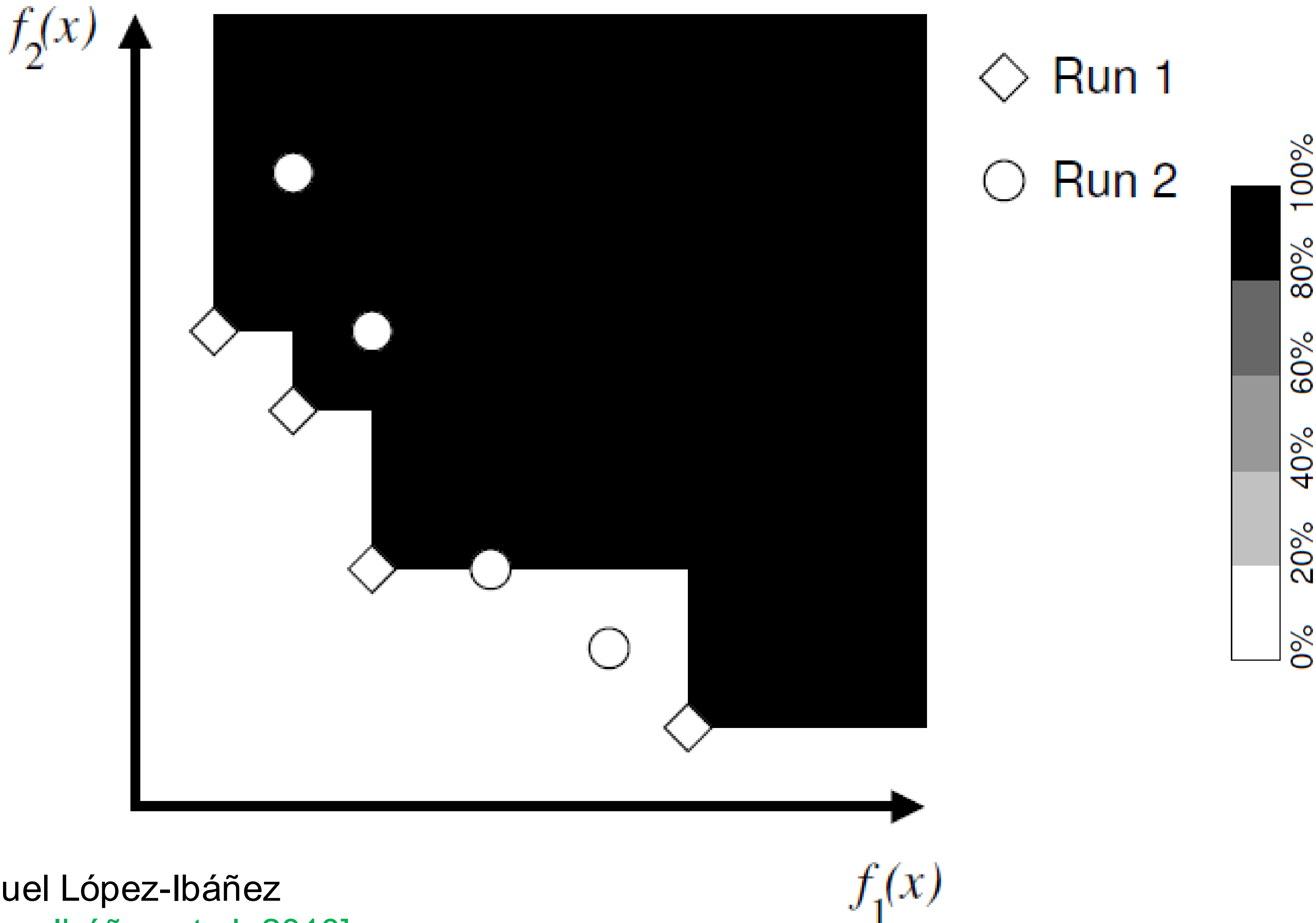

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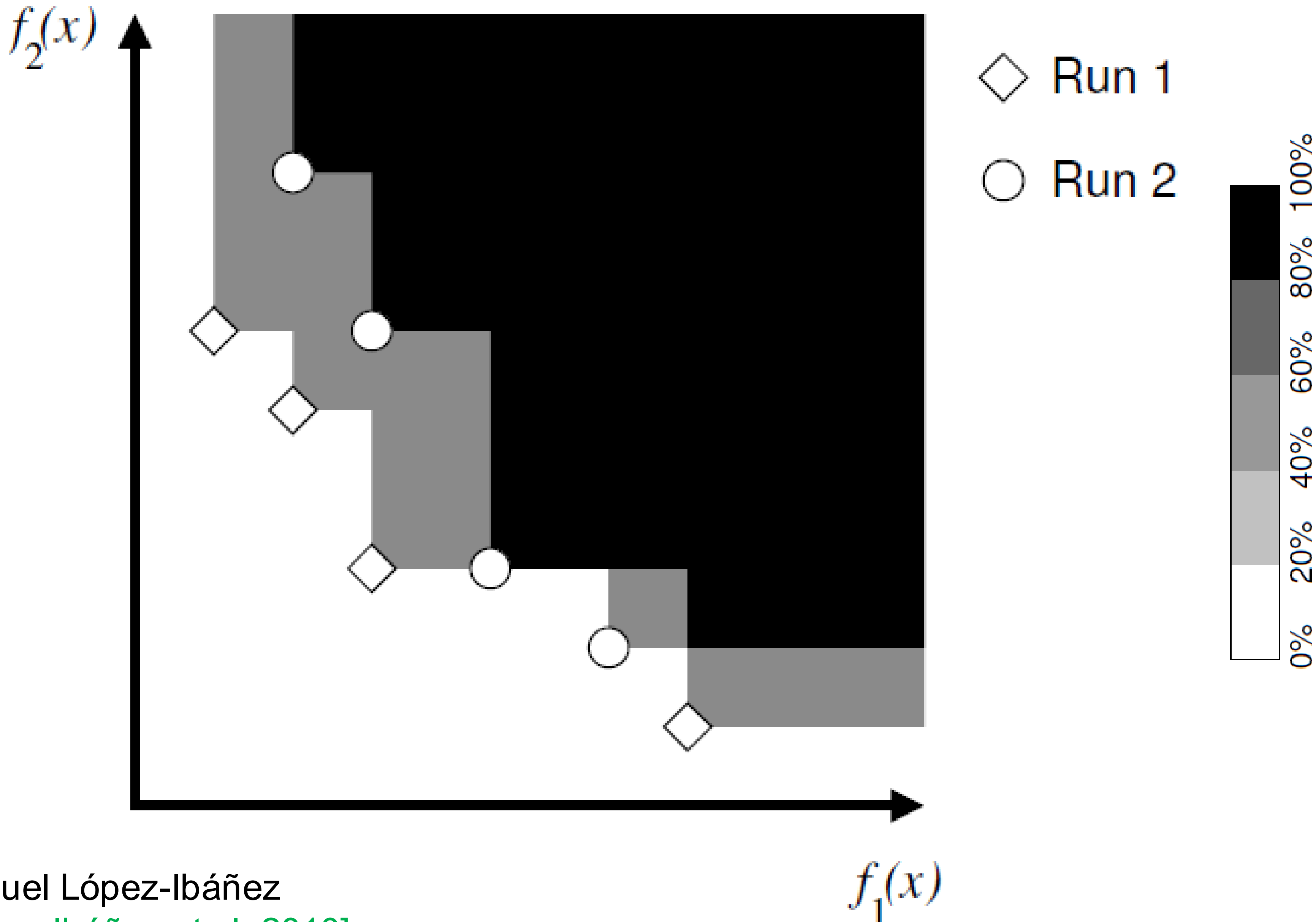

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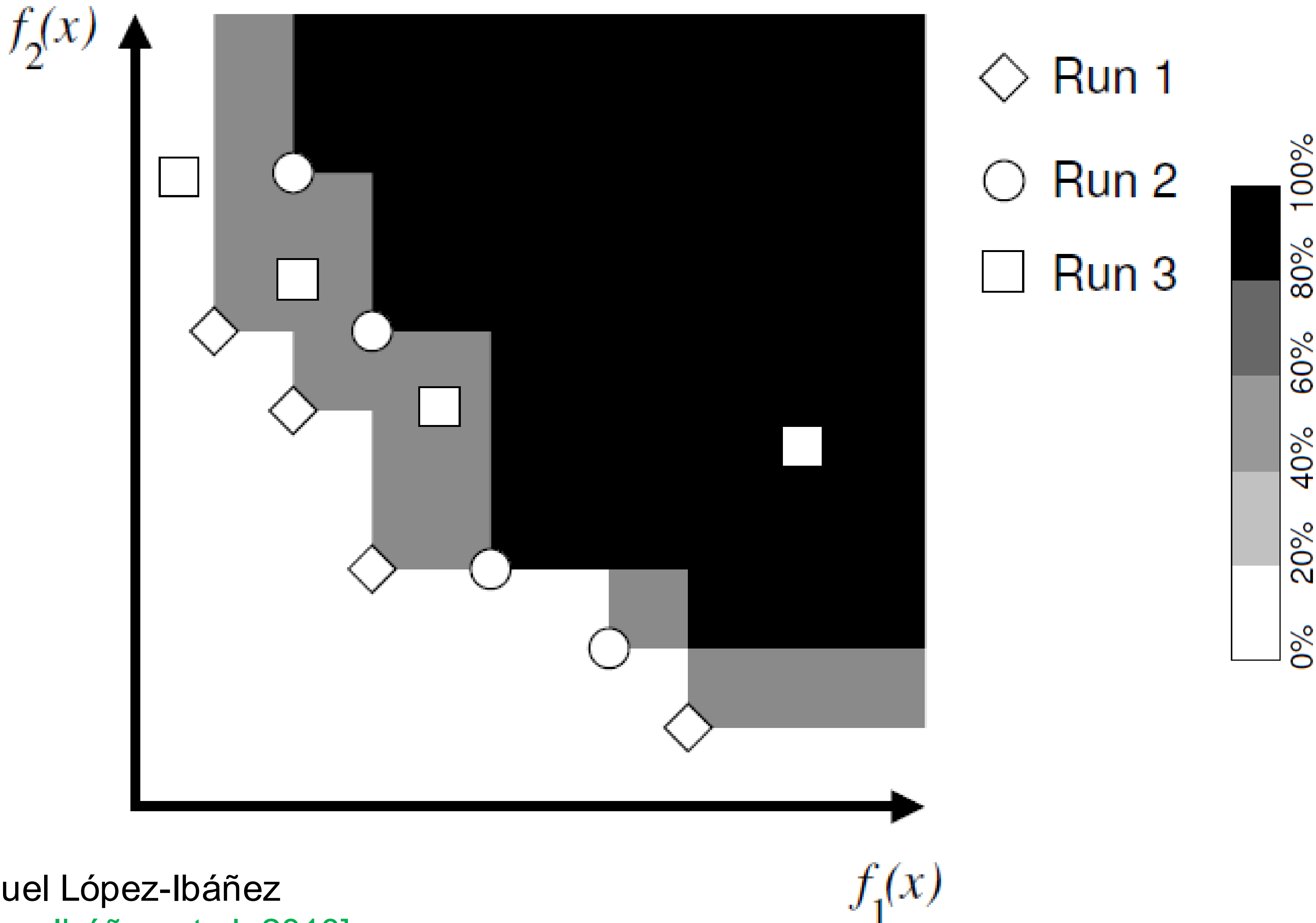

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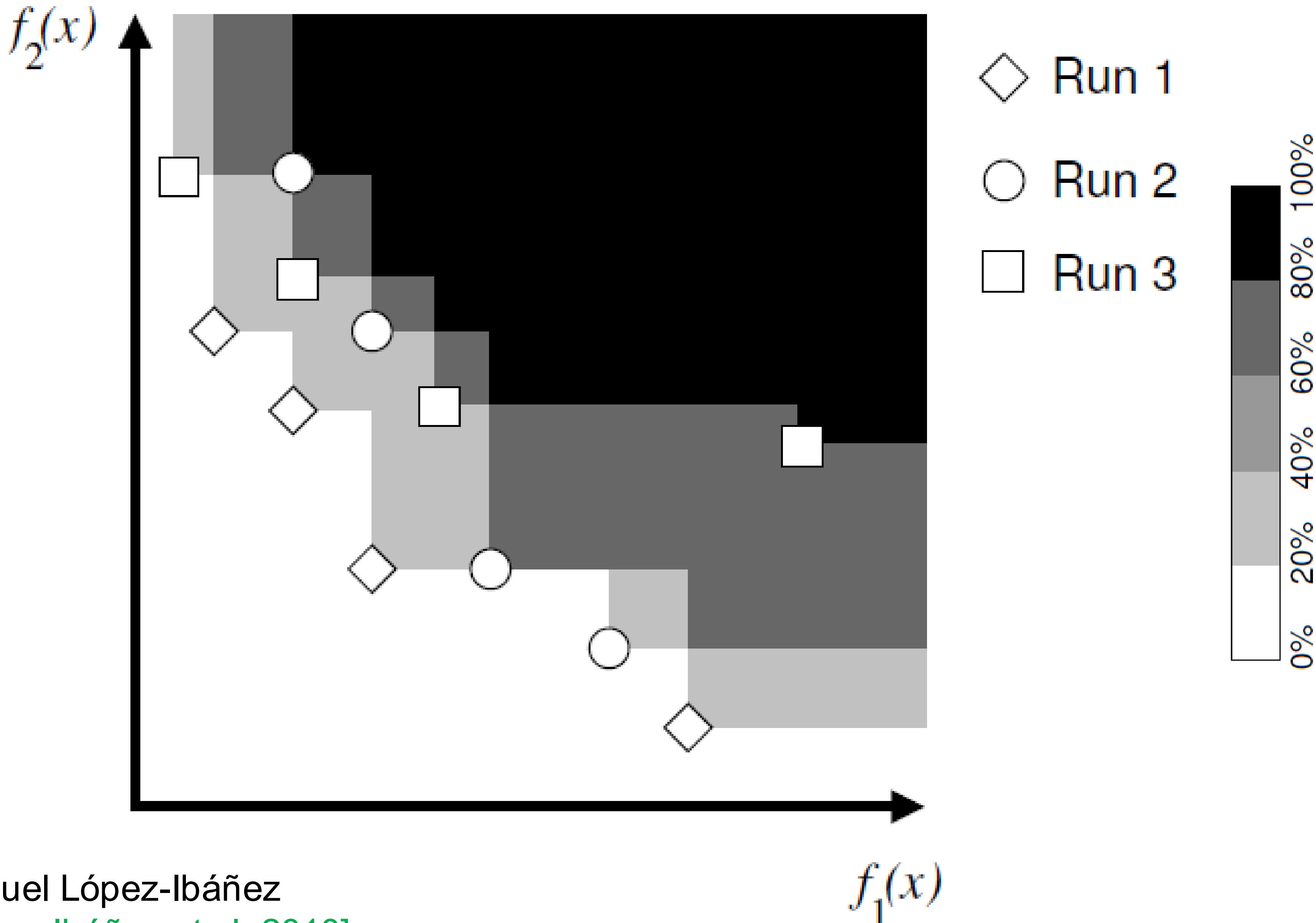
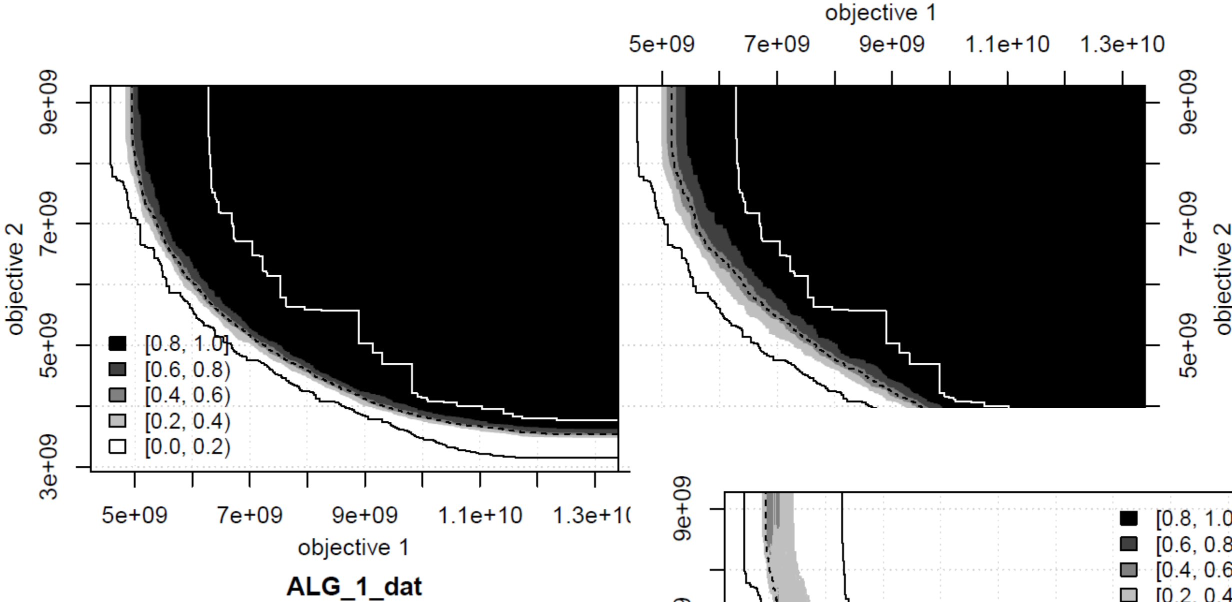


[López-Ibáñez et al. 2010]

Attainment Plots in Practice



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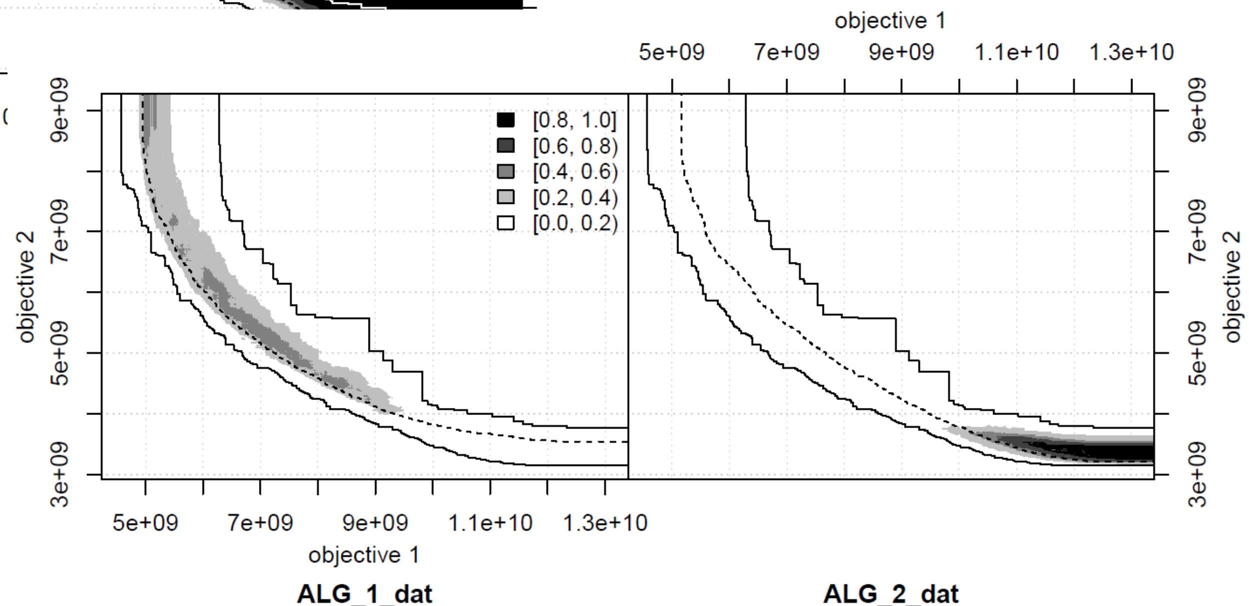
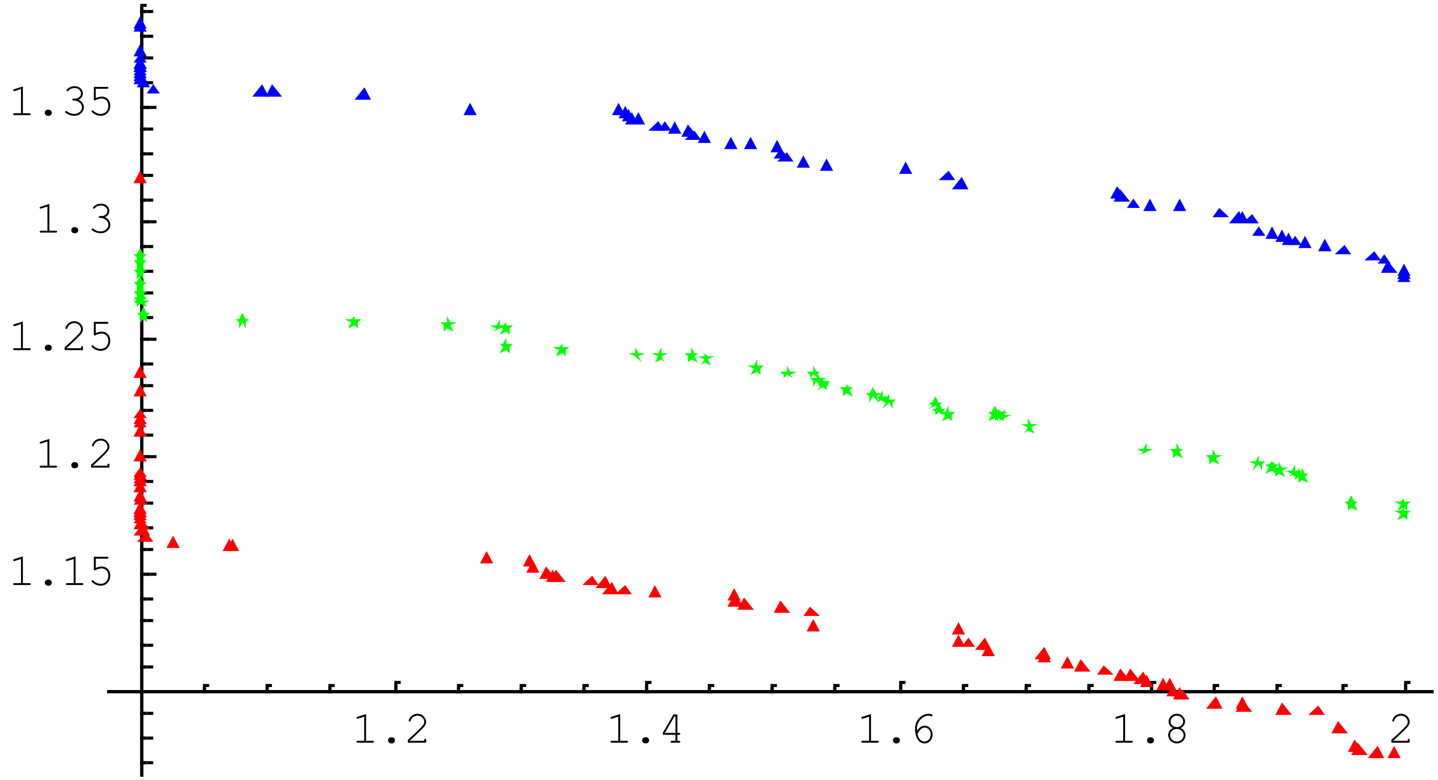


latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
R package: **`http://lopez-ibanez.eu/eaftools`**
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)



latest implementation online at
http://eden.dei.uc.pt/~cmfonsec/software.html
see [Fonseca et al. 2011]

Most Used Approach: Quality Indicators

A quality indicator

- maps a solution set to a real number
- can be used with standard performance assessment
  - report median, variance, ...
  - boxplots
  - statistical tests
- should optimally refine the dominance relation on sets

Recommendation:

- use hypervolume (refinement, i.e. it does not contradict the dominance relation)
- or epsilon indicator or R2 indicator (are weak refinements)

Also important:

- interpretation of the results (by knowing theoretical properties of the used indicator)

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function ("quality indicator") on sets
- use the resulting total (pre-)order (on the quality values)

Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation should be reflected!

$$A \preceq B :\Leftrightarrow \forall_{y \in B} \exists_{x \in A} x \leq_{par} y$$

Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$ fulfills requirement

❷ $\overset{\text{ref}}{\preccurlyeq}$ **weakly refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

$\Rightarrow$ does not fulfill requirement, but $\overset{\text{ref}}{\preccurlyeq}$ does not contradict $\preccurlyeq$

! sought are total refinements… [Zitzler et al. 2010]

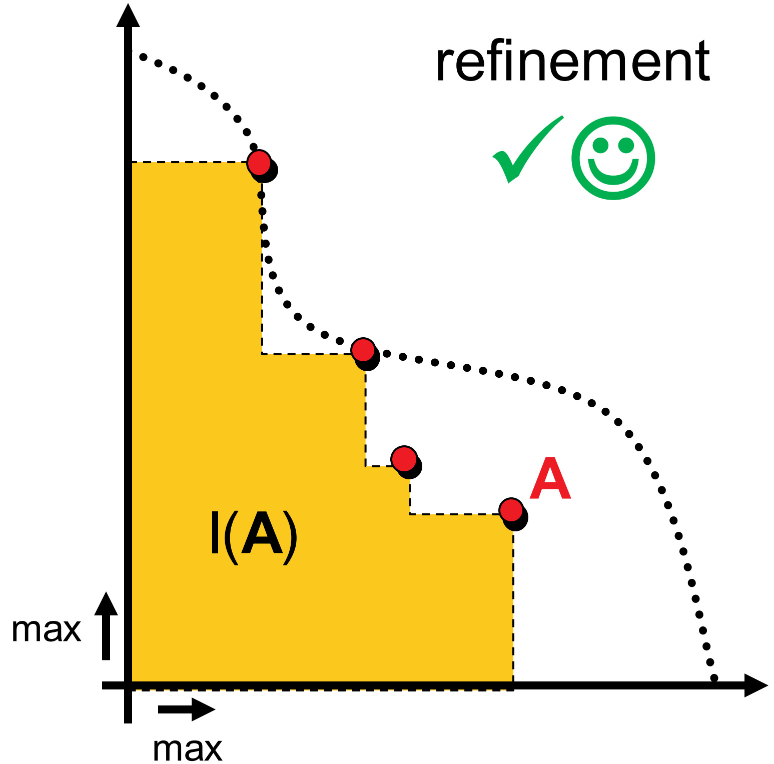
Example: Refinements Using Indicators

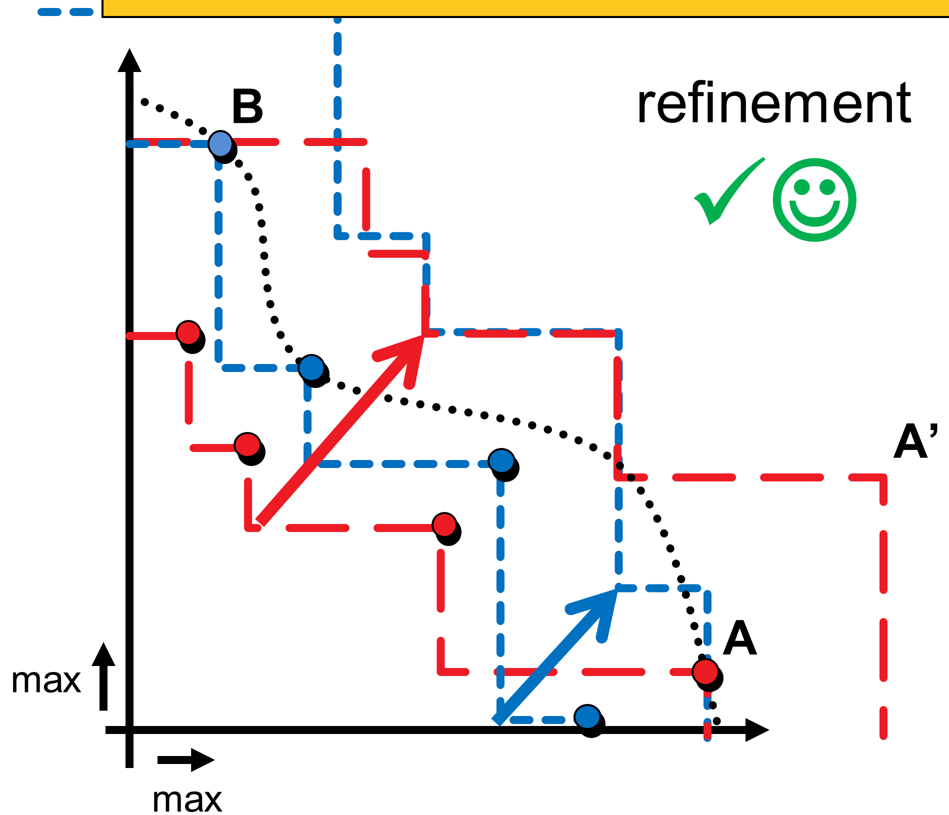
$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \geq I(B)$$

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,B) \leq I(B,A)$$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B





unary hypervolume indicator

binary epsilon indicator

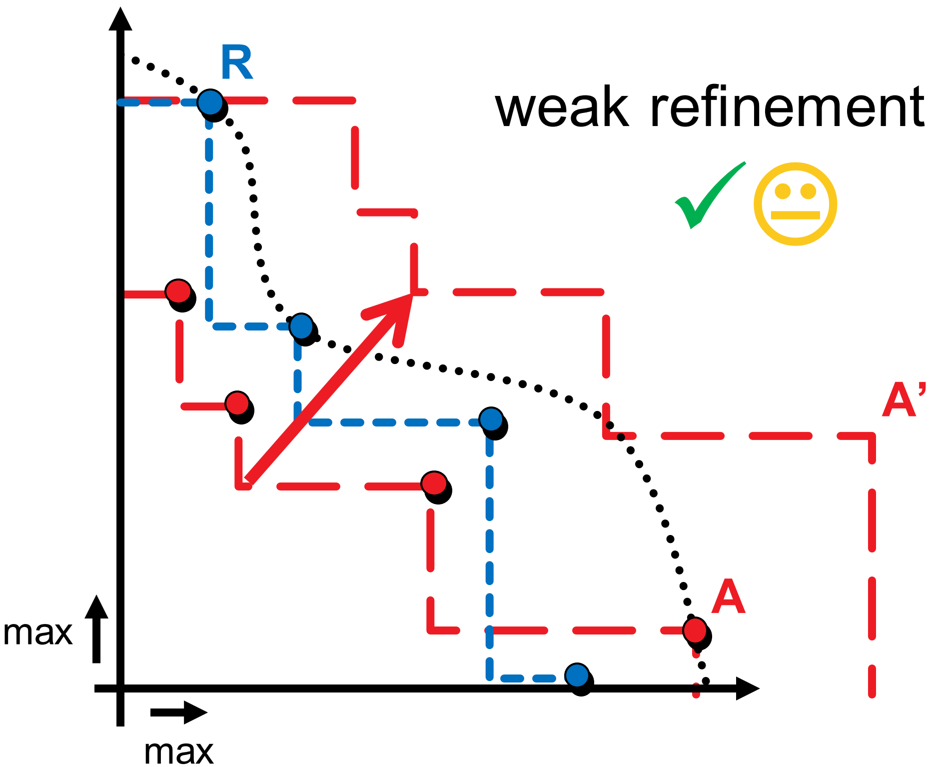
Example: Weak Refinement / No Refinement

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,R) \leq I(B,R)$$

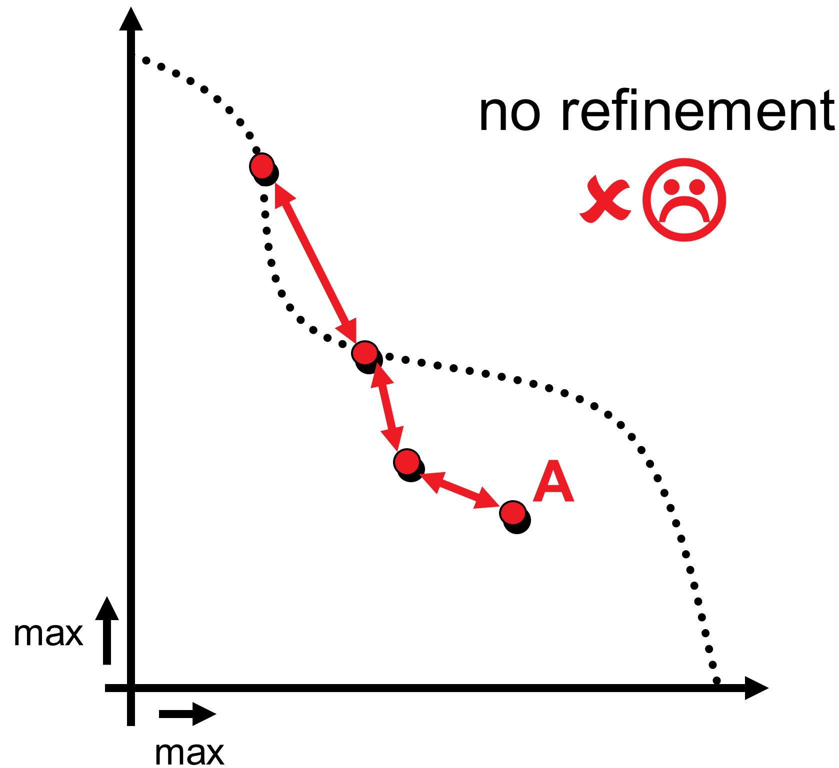
$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \leq I(B)$$

I(A,R) = how much needs A to be moved to weakly dominate R

I(A) = variance of pairwise distances



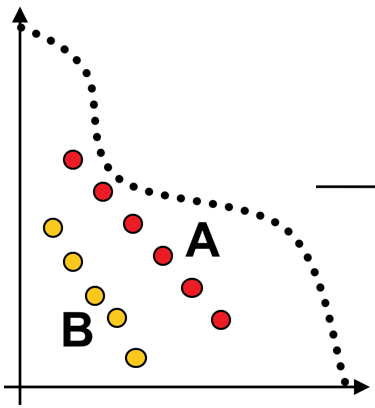
unary epsilon indicator



unary diversity indicator

Quality Indicator Approach

Goal: compare two Pareto set approximations A and B



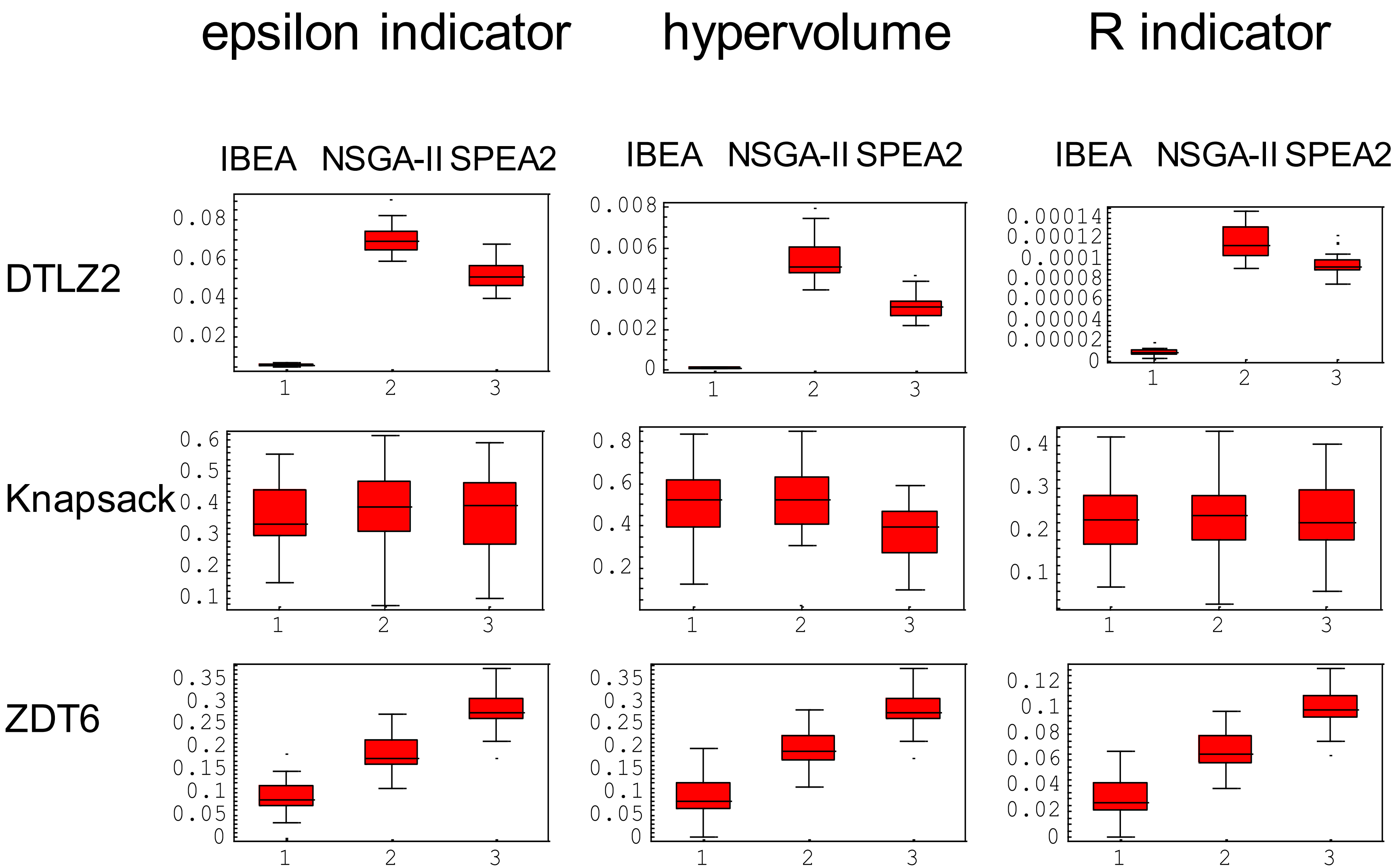
	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

→ "**A** better"

Comparison method C = quality measure(s) + Boolean function

$$A, B \xrightarrow[\textbf{reduction}]{\text{quality measure}} \mathbb{R}^n \xrightarrow[\textbf{interpretation}]{\text{Boolean function}} \text{statement}$$

Example: Box Plots



Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		~0 ☺
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

DTLZ2
R

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		1
SPEA2	1	~0 ☺	

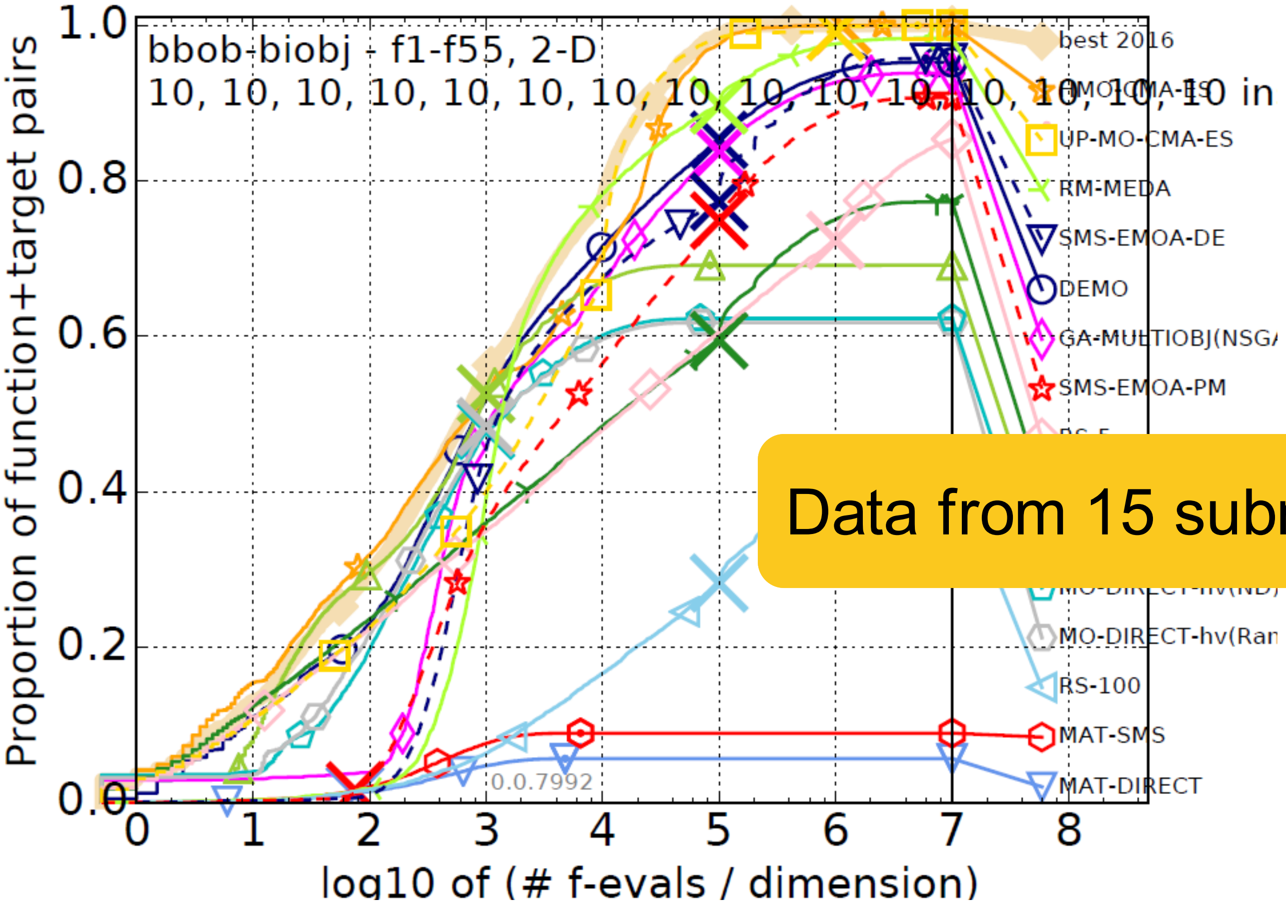
Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H_0 = No significance of any differences

Automated Benchmarking

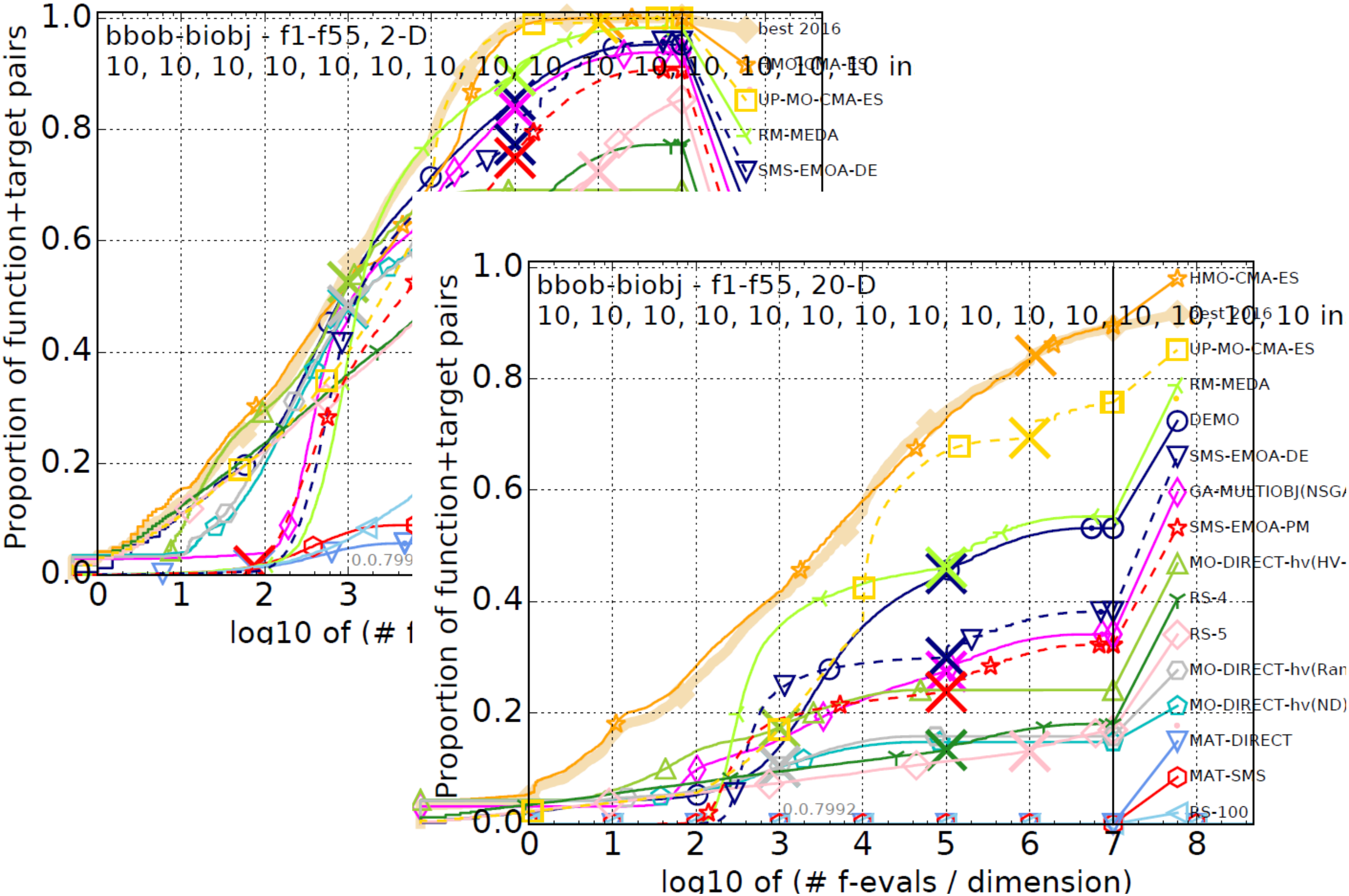
- State-of-the-art in single-objective optimization: Blackbox Optimization Benchmarking (BBOB) with COCO platform
  `https://github.com/numbbo/coco`
- In 2016: first release of a bi-objective test suite and corresponding BBOB-2016 workshop @ GECCO
- Focus on target-based runlengths
  - gives (nearly) anytime, interpretable results
  - defines problem=(test function instance, single-objective goal e.g. min. indicator difference to reference set, target precision)
  - reports average runtimes (aRT) to reach target precision
- COCO provides data profiles, scaling plots, scatter plots, tables, statistical tests, etc. automatically

Exemplary BBOB-2016 Results



Data from 15 submitted algorithms

Exemplary BBOB-2016 Results



The Big Picture

Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

Selected Advanced Concepts

- preference articulation
- visualization aspects

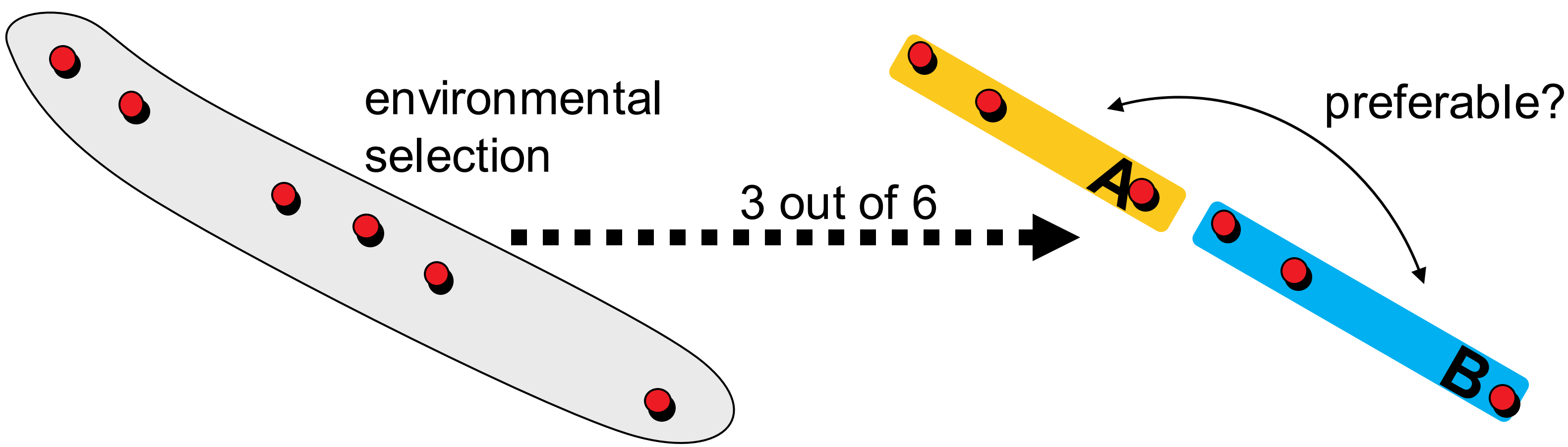
Articulating User Preferences During Search

What we thought: EMO is preference-less

Search before decision making: Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

[Zitzler 1999]

What we learnt: EMO just uses weaker preference information



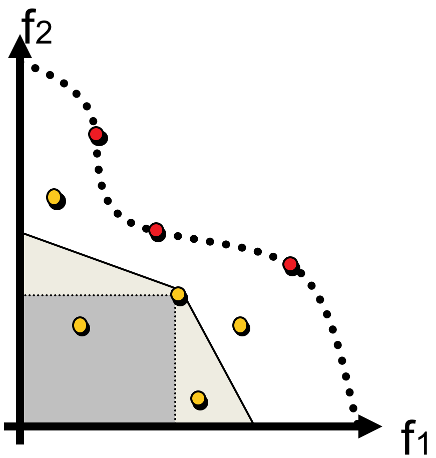
Nevertheless...

- the more (known) preferences incorporated the better
- in particular if search space is large

[Branke and Deb 2004] [Branke 2008] [Bechikh et al. 2015]

❶ Refine/modify dominance relation, e.g.:

- using goals, priorities, constraints
  [Fonseca and Fleming 1998a,b]
- using different types of dominance cones
  [Branke and Deb 2004]

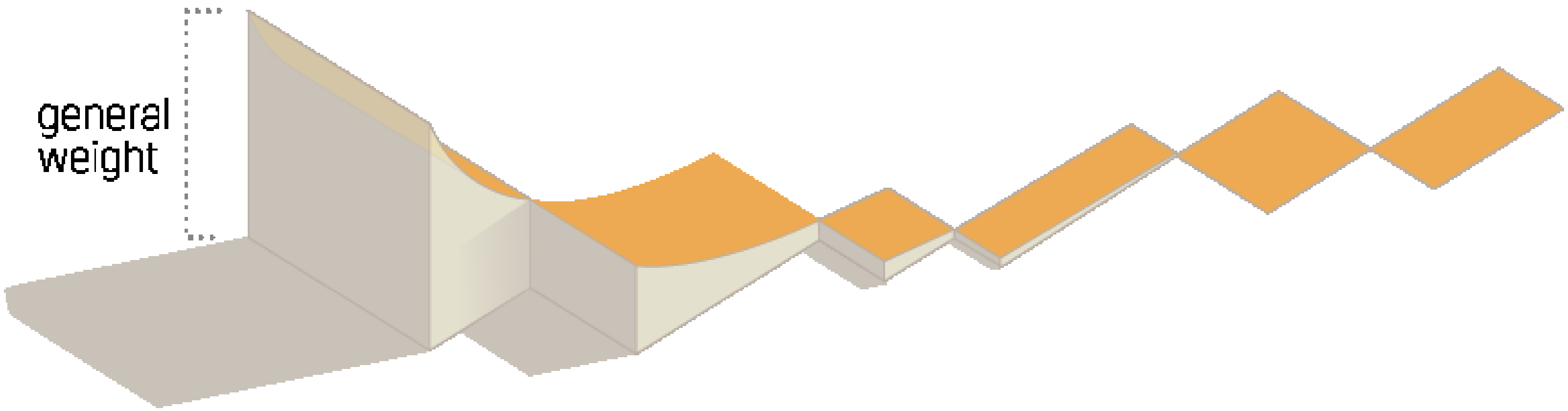


❷ Use quality indicators, e.g.:

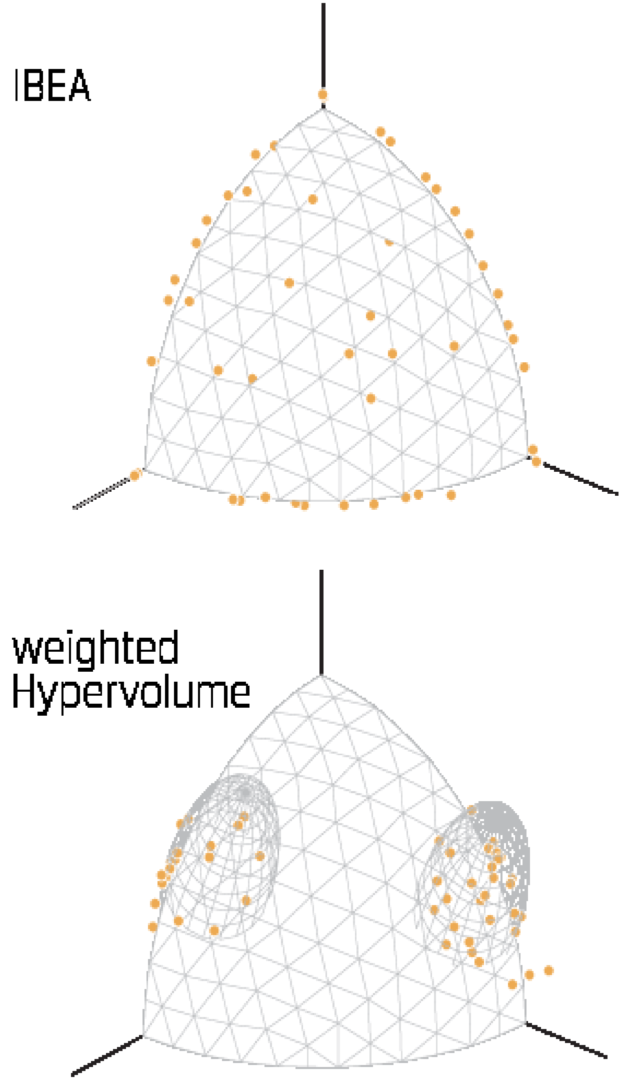
- based on reference points and directions [Deb and Sundar 2006, Deb and Kumar 2007]
- based on the hypervolume indicator
  [Brockhoff et al. 2013] [Wagner and Trautmann 2010]
- based on the R2 indicator [Trautmann et al. 2013]

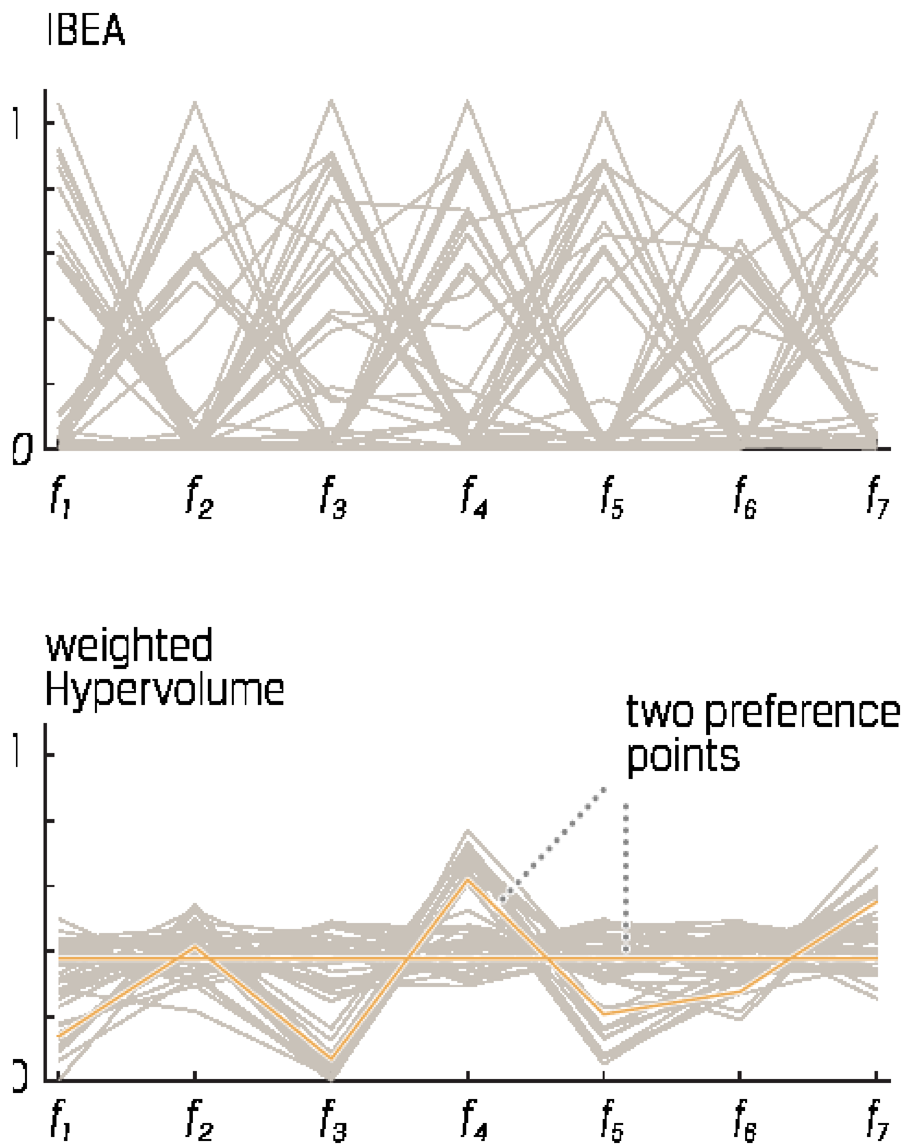
Example: Weighted Hypervolume Indicator

[Brockhoff et al. 2013]



Weighted Hypervolume in Practice



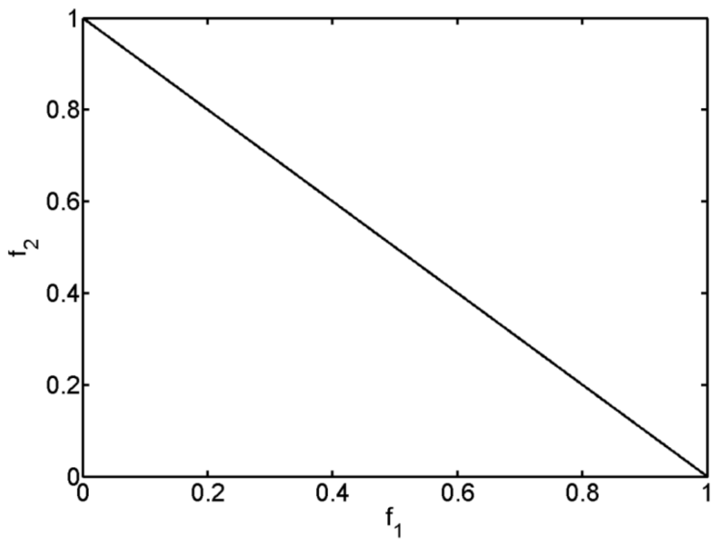


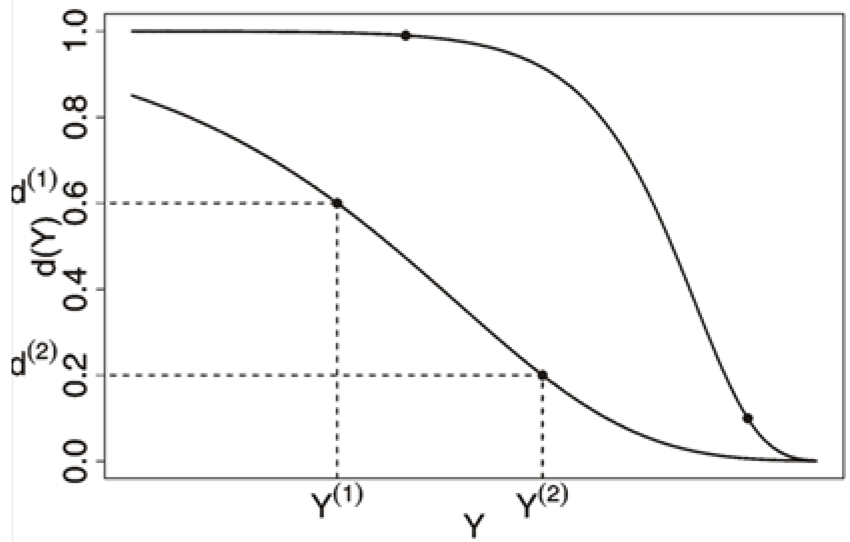
[Auger et al. 2009b]

Example: Desirability Function (DF)-SMS-EMOA

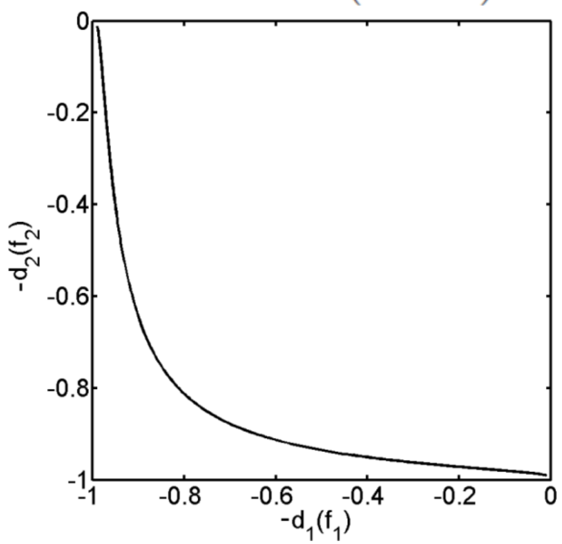
[Wagner and Trautmann 2010]

Shape of the untransformed Pareto front

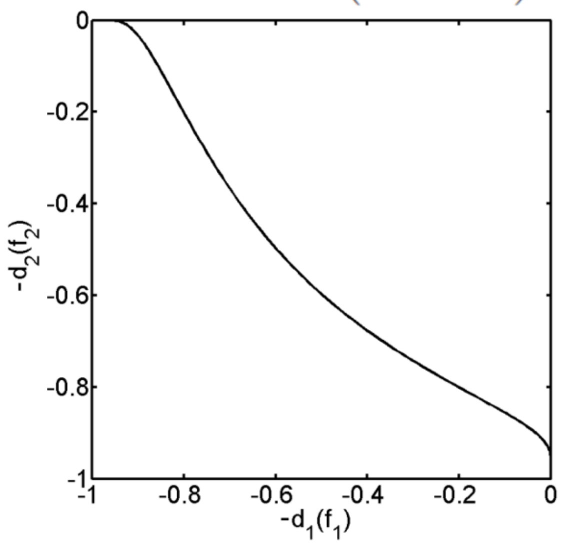




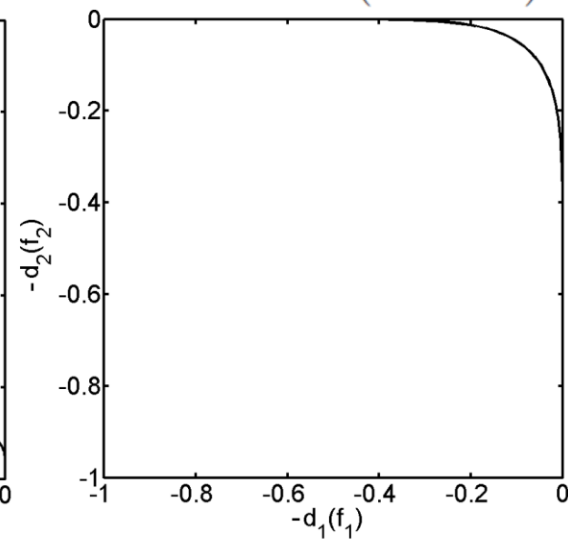
Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 1 & 0.01 \end{pmatrix}$



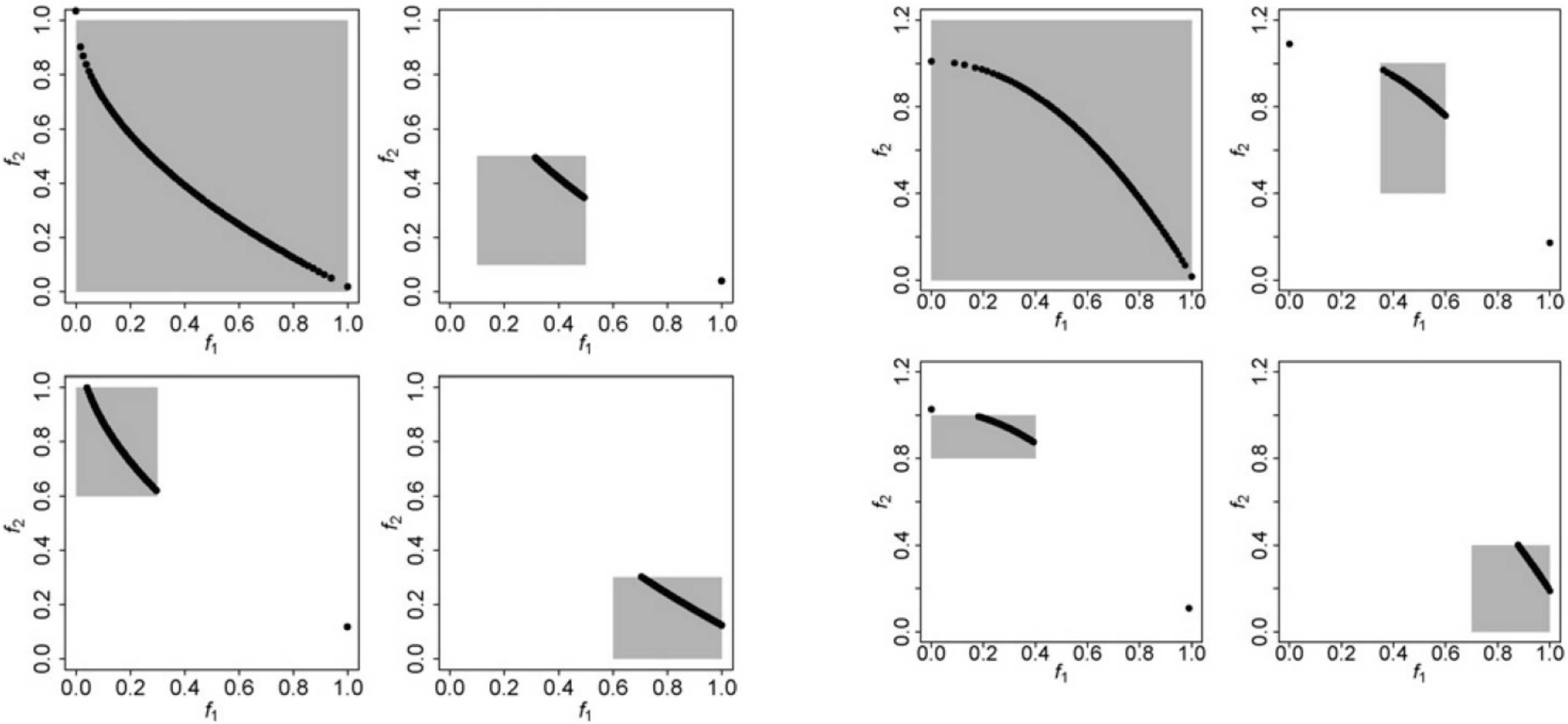
Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 0.75 & 0.01 \end{pmatrix}$



Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 0.55 & 0.01 \end{pmatrix}$



DF-SMS-EMOA in Practice

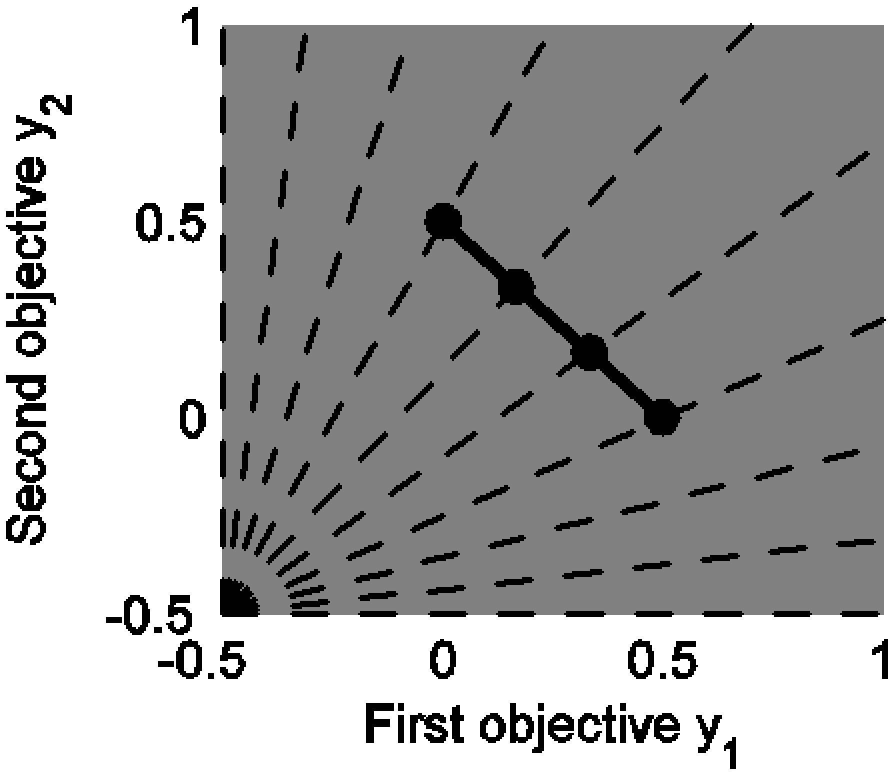


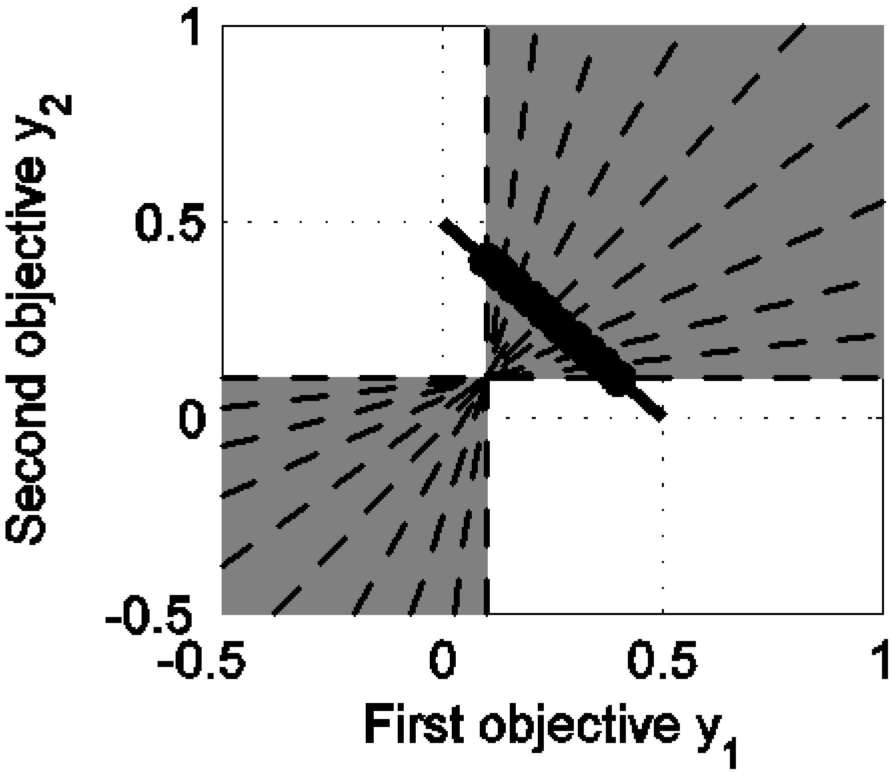
Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Position of ideal point



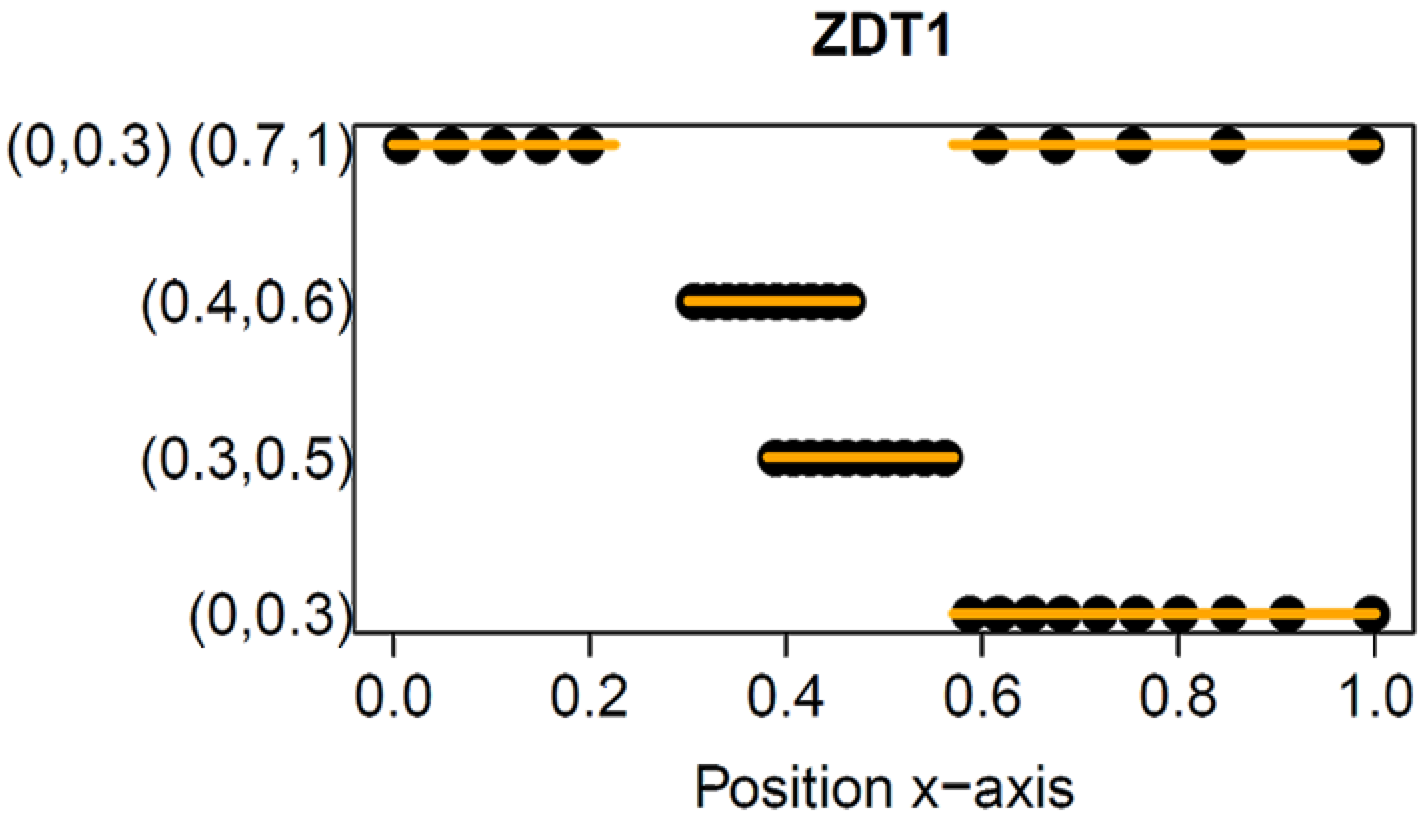


Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Restriction of the weight space



Interactive Approaches

Successive Preference Articulation = Interactive EMO

- recent interest of both EMO and MCDM community
- important in practice

Examples

- first interactive EMO: [Tanino et al. 1993]
- good overview: [Jaszkiewicz and Branke 2008]
- more recent work: [Brockhoff et al. 2014] [Branke et al. 2014]

Issues/Open Questions

- realistic scenarios/ value functions
- evaluation of interactive algorithms [López-Ibáñez and Knowles 2015]

Overview

The Big Picture

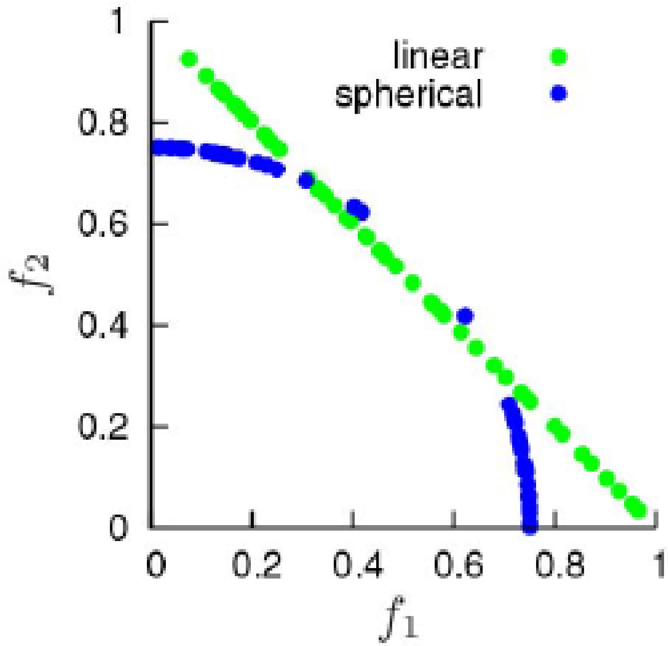
Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

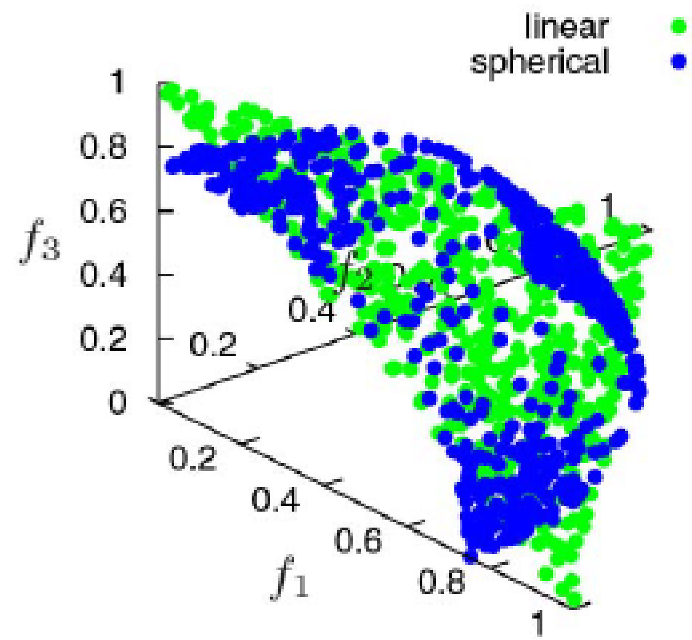
Selected Advanced Concepts

- preference articulation
- visualization aspects

Visualization is Difficult for Many Objectives



2 objective functions



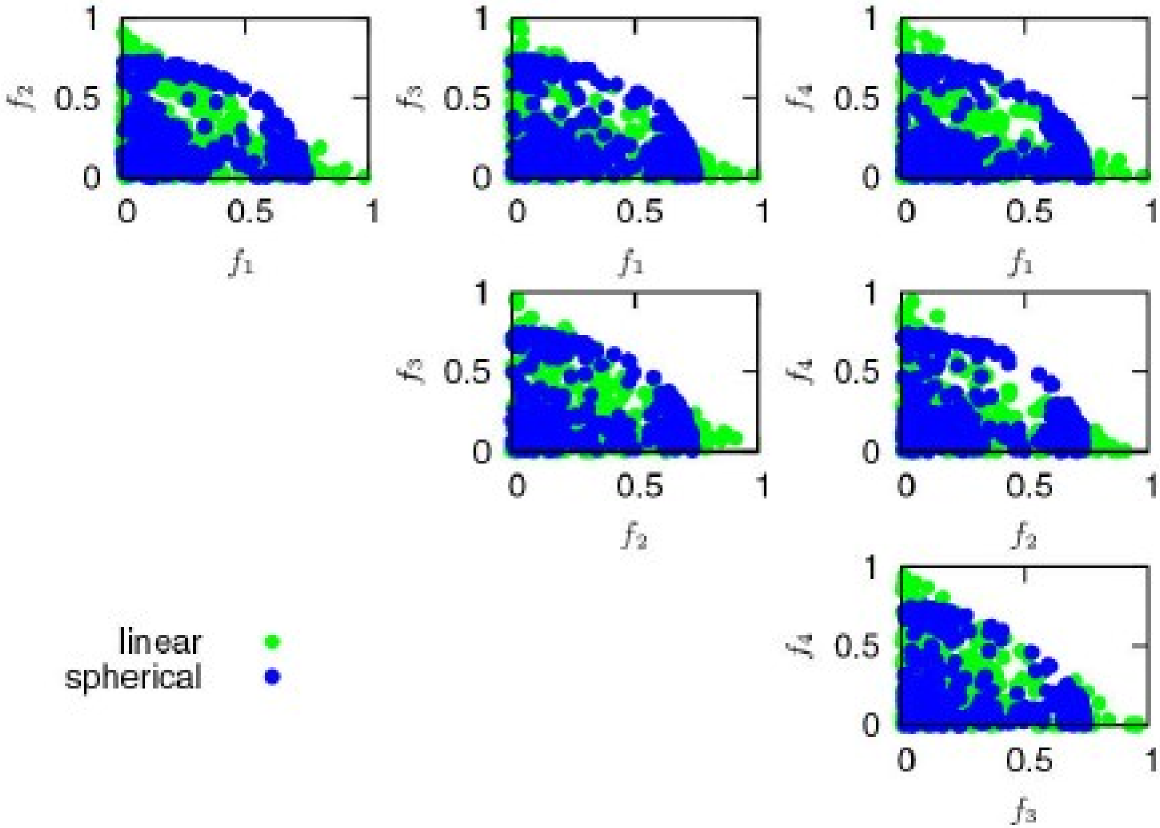
3 objective functions

>3 objective functions?

These and the following plots are taken from

Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

Scatter Plots for all Objective Combinations



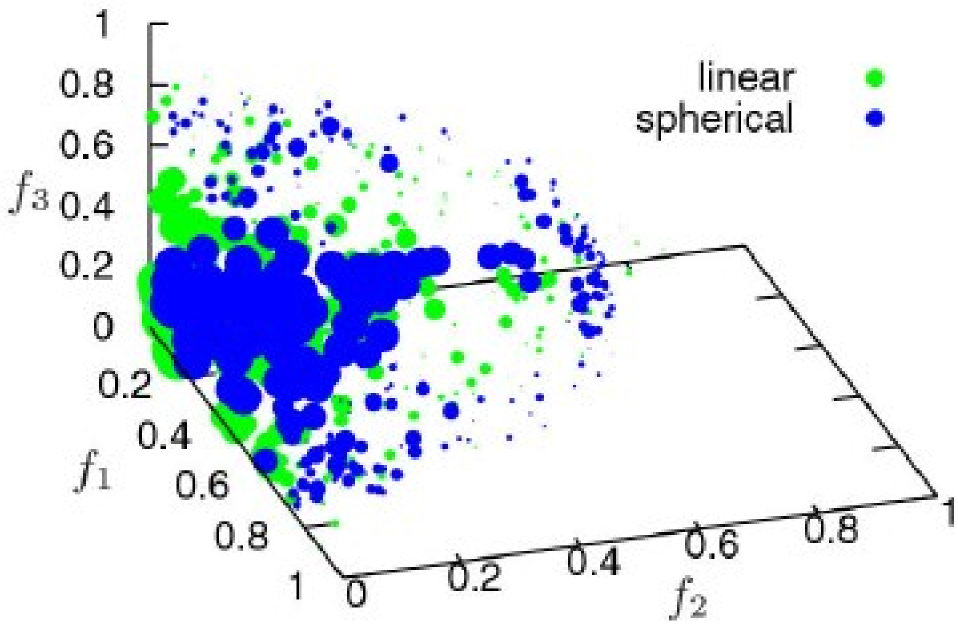
These and the following plots are taken from

Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

Bubble Chart

Bubble chart:

size of bubble = forth objective



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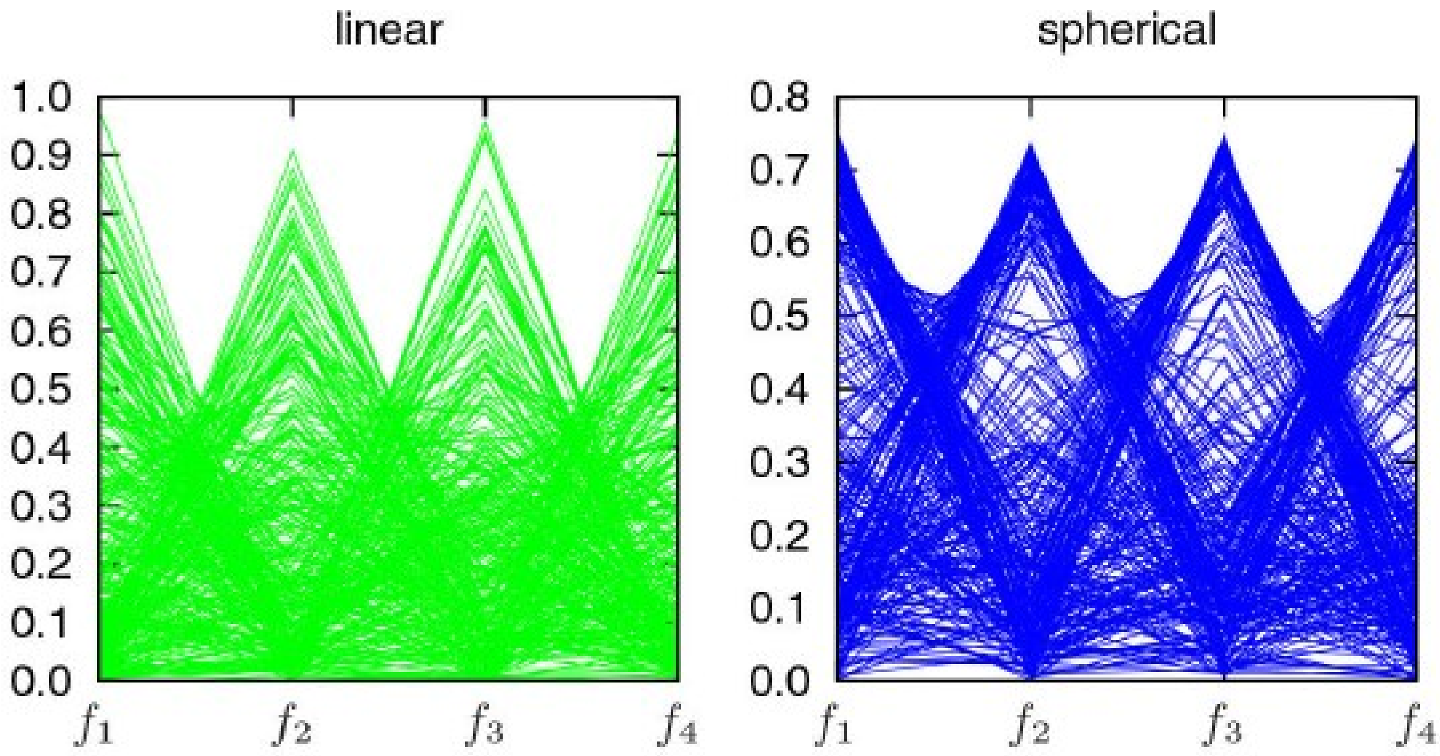
Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

Parallel Coordinates



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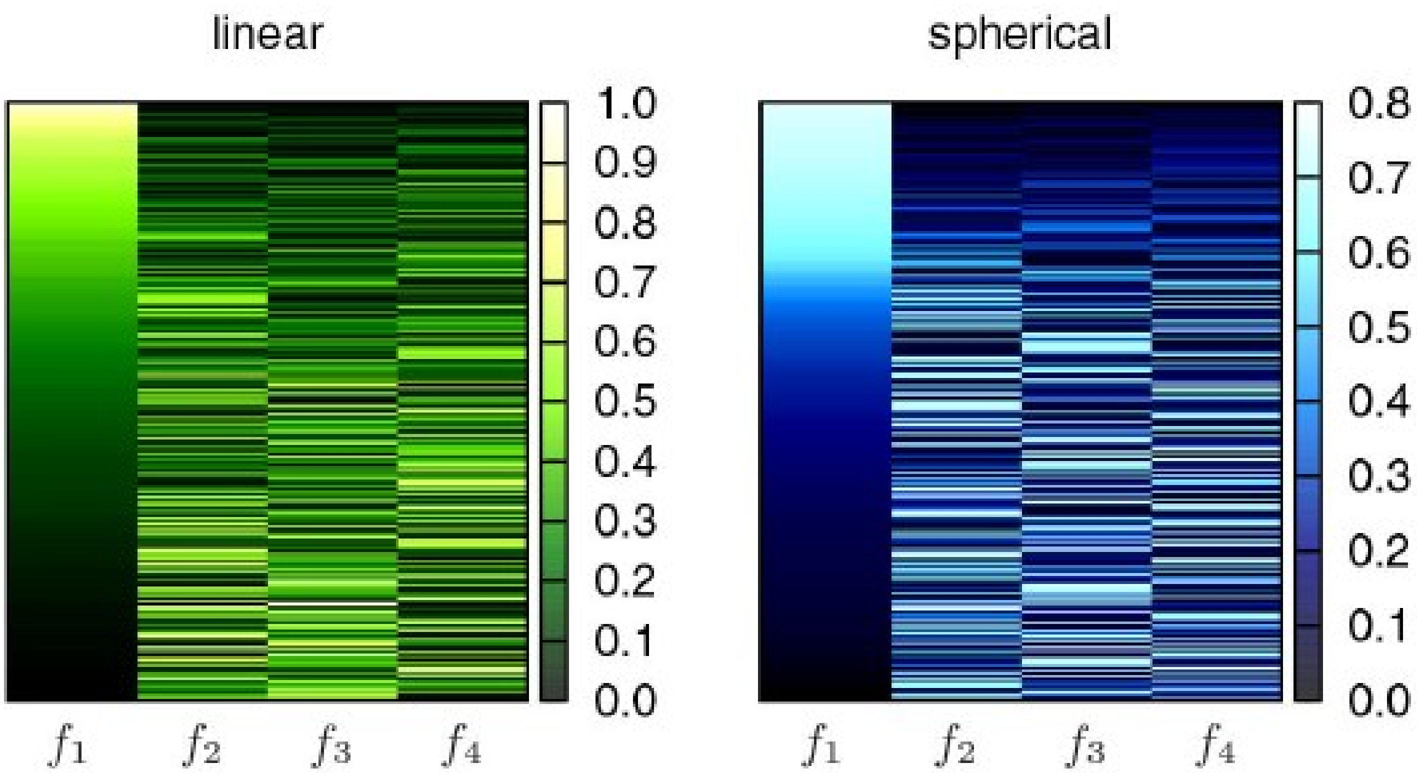
Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

Heat Maps

and many more...



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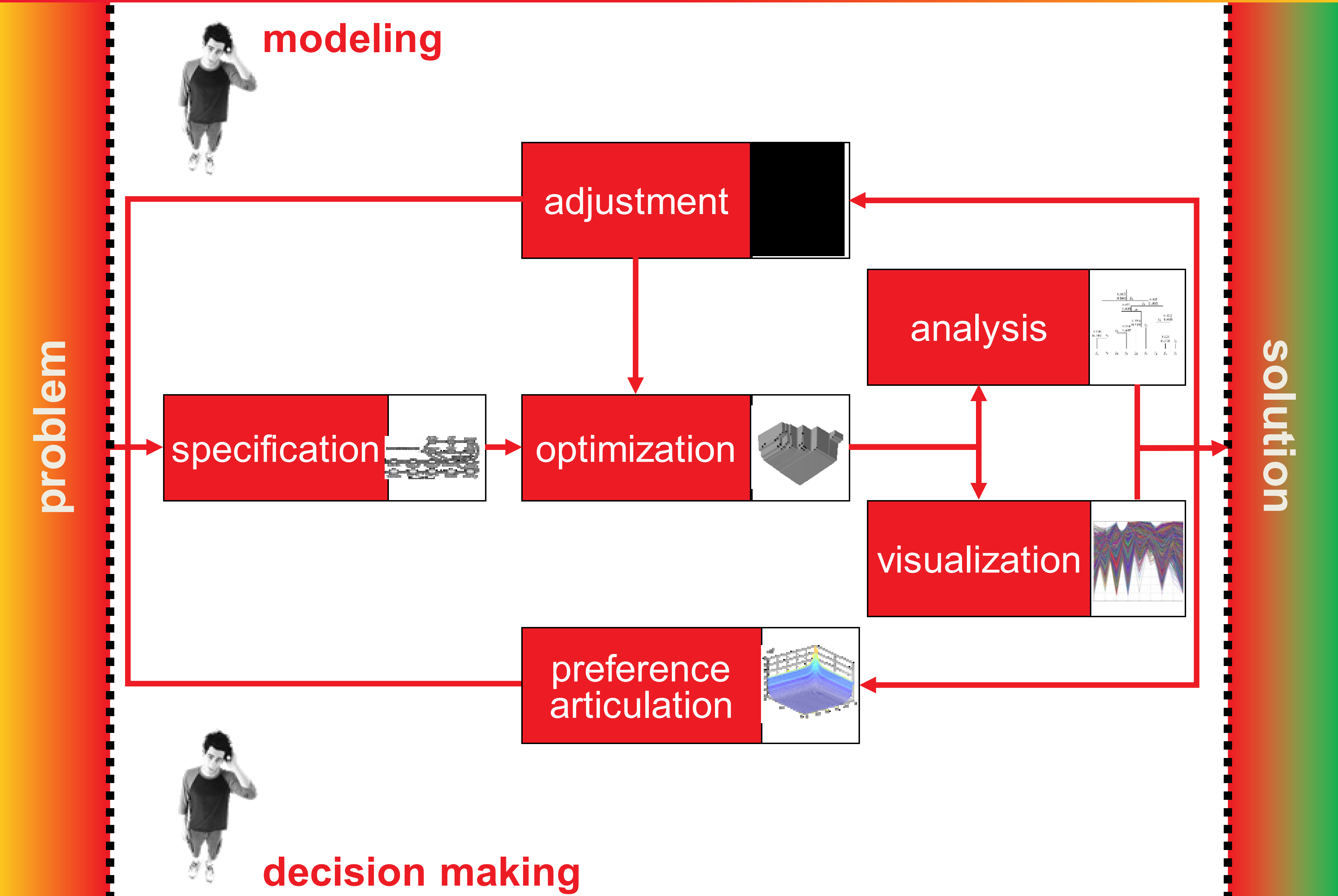
Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

Conclusions: EMO as Interactive Decision Support



The EMO Community

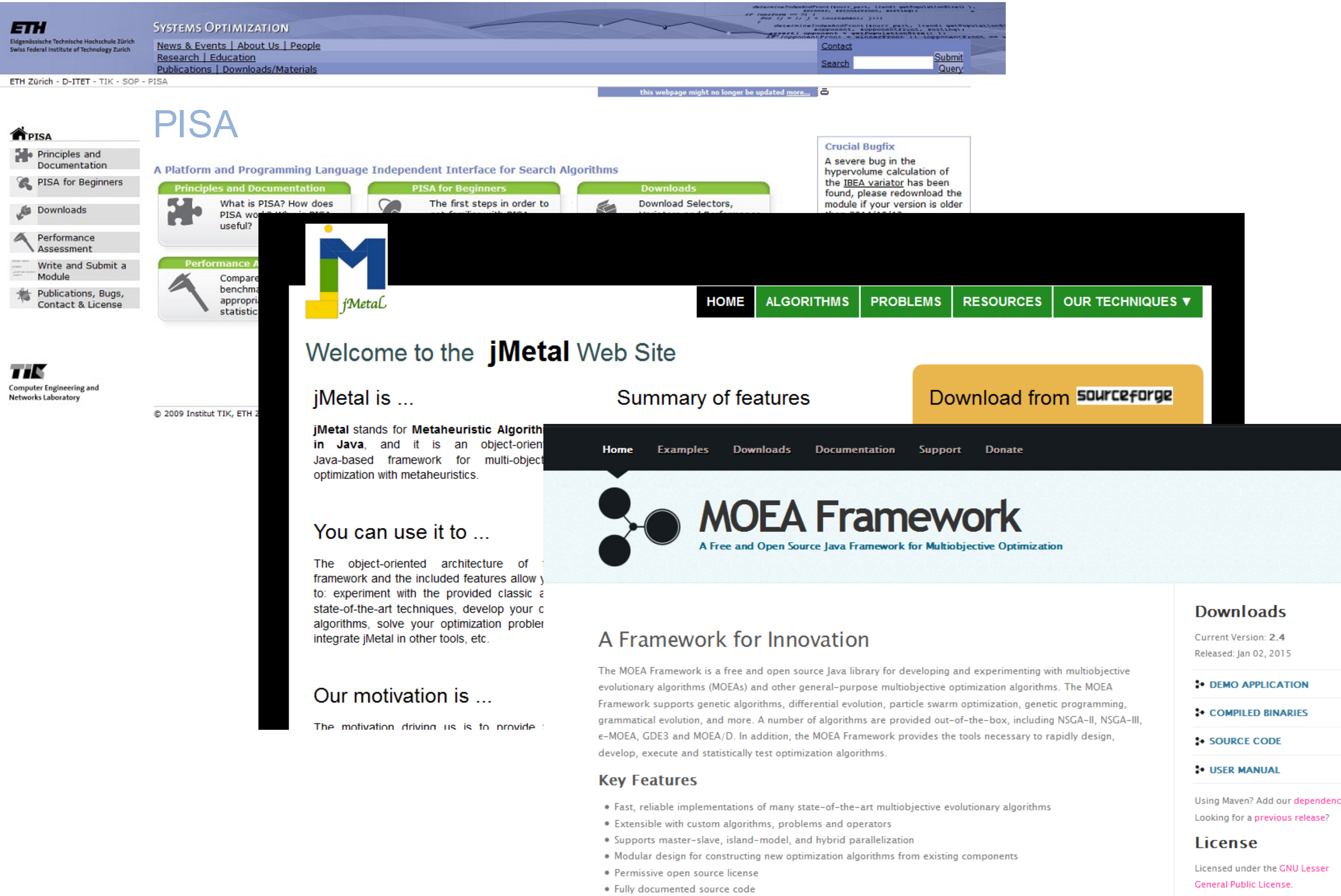
Links:

- EMO mailing list: *https://lists.dei.uc.pt/mailman/listinfo/emo-list*
- MCDM mailing list: *http://lists.jyu.fi/mailman/listinfo/mcdm-discussion*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.dep.uminho.pt/EMO2015/*

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms***
  Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [(still) many open questions!]
- and more…

Software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MOEA Framework
A Free and Open Source Java Framework for Multiobjective Optimization
A Framework for Innovation
The MOEA Framework is a free and open source Java library for developing and experimenting with multiobjective evolutionary algorithms (MOEAs) and other general-purpose multiobjective optimization algorithms. The MOEA Framework supports genetic algorithms, differential evolution, particle swarm optimization, genetic programming, grammatical evolution, and more. A number of algorithms are provided out-of-the-box, including NSGA-II, NSGA-III, ε-MOEA, GDE3 and MOEA/D. In addition, the MOEA Framework provides the tools necessary to rapidly design, develop, execute and statistically test optimization algorithms.
Key Features
Fast, reliable implementations of many state-of-the-art multiobjective evolutionary algorithms
Extensible with custom algorithms, problems and operators
Supports master-slave, island-model, and hybrid parallelization
Modular design for constructing new optimization algorithms from existing components
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Released: Jan 02, 2015
DEMO APPLICATION
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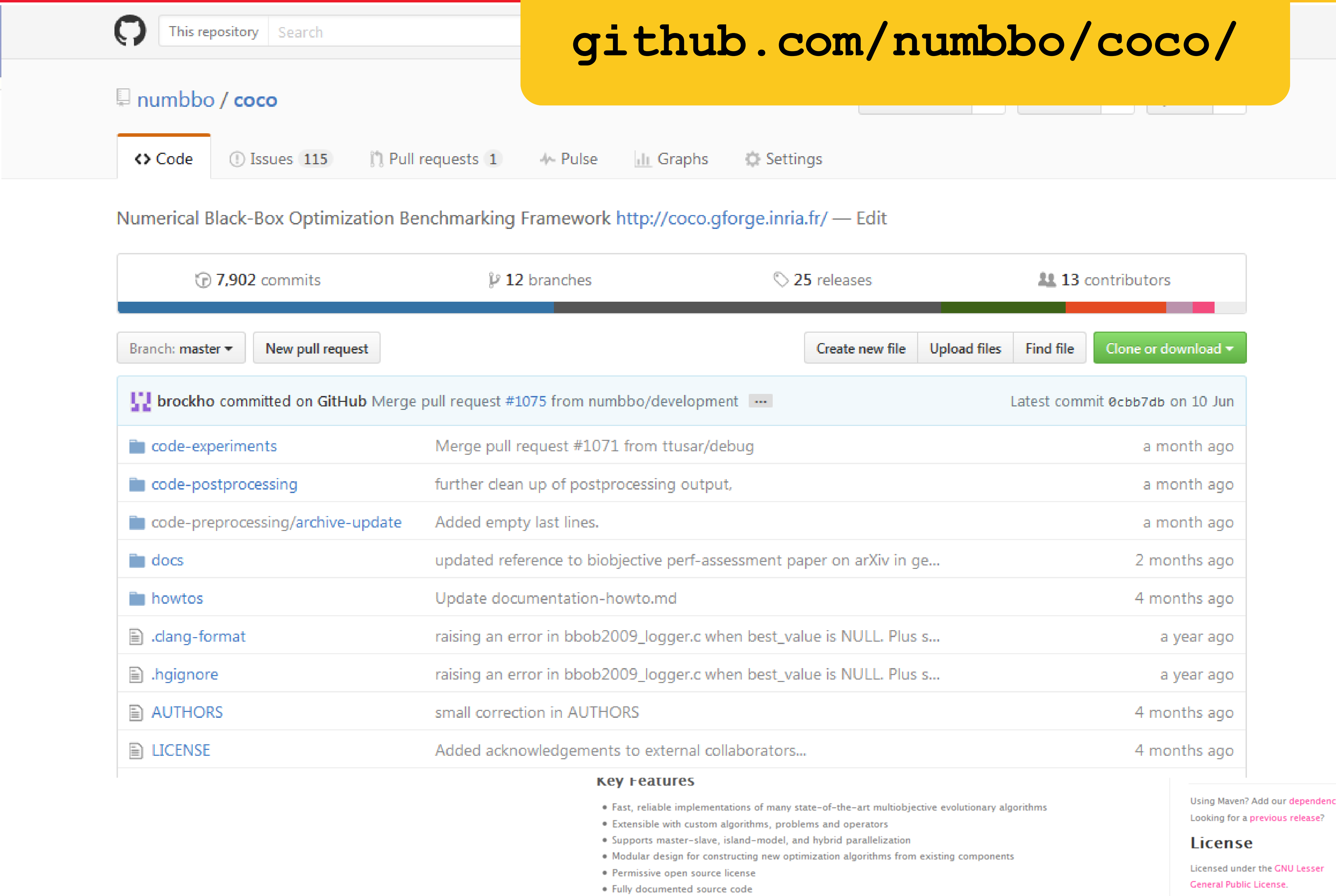
Software

github.com/numbbo/coco/



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