Advanced Control

January 17, 2014
École Centrale Paris, Châtenay-Malabry, France

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Course Overview

Date		Topic
Fri, 10.1.2014	DB	Introduction to Control, Examples of Advanced Control
Fri, 17.1.2014	DB	Introduction to Fuzzy Logic, Introduction to Artificial Neural Networks
Fri, 24.1.2014	DB	Bio-inspired Optimization, discrete search spaces
Fri, 31.1.2014	AA	Continuous Optimization I
Fri, 7.2.2014	AA	Continuous Optimization II
break		
Fri, 28.2.2014	AA	The Traveling Salesperson Problem
Fr, 7.3.2014	DB	Controlling a Pole Cart
Fr, 14.3.2014		written exam (paper and computer)

all classes + exam at **8h00-11h15** (incl. a 15min break around 9h30) here in CTI-B3

Remark to last exercise

All information also available at

`http://researchers.lille.inria.fr/~brockhof/advancedcontrol/`

(exercise sheets, lecture slides, additional information, links, ...)

Introduction to Fuzzy Logic

Fuzzy Logic

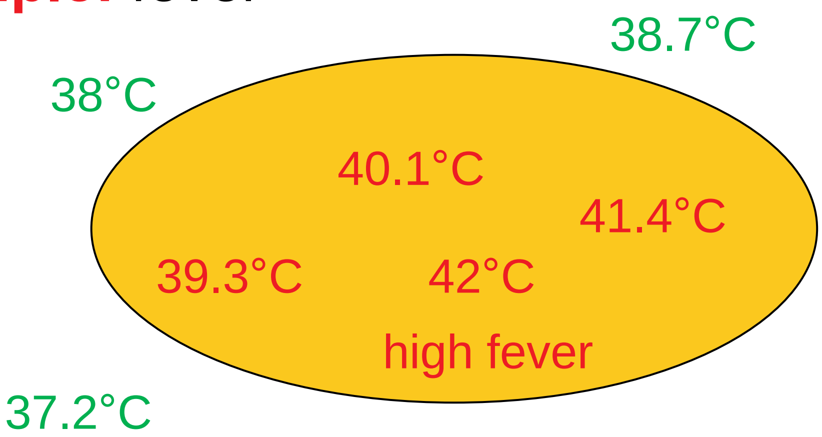
- introduced by Lotfi A. Zadeh at the University of California, Berkeley (*fuzzy sets* in 1965 and *fuzzy logic* in 1973)
- a mathematical tool to deal with uncertainties
- often described as “*computing with words*”[1]
  - e.g. {low, medium, high} instead of {0,1}
  - or “short” instead of “< 1 meter”

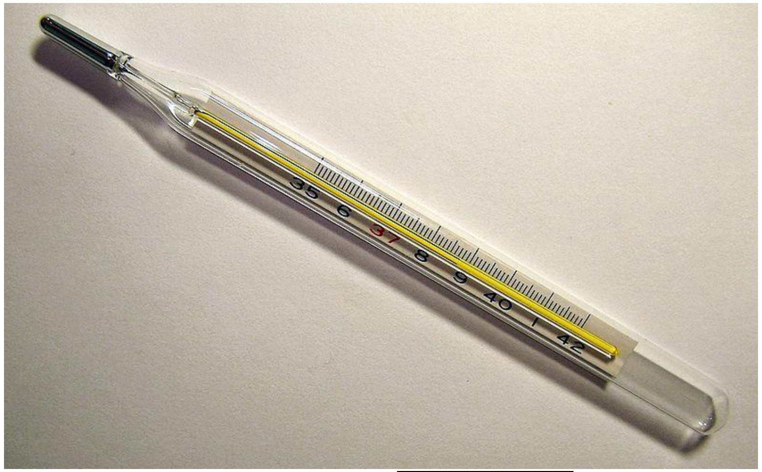
[1] L. A. Zadeh: Fuzzy logic = computing with words. In IEEE Transactions on Fuzzy Systems, 4(2), p. 103-111. 1996

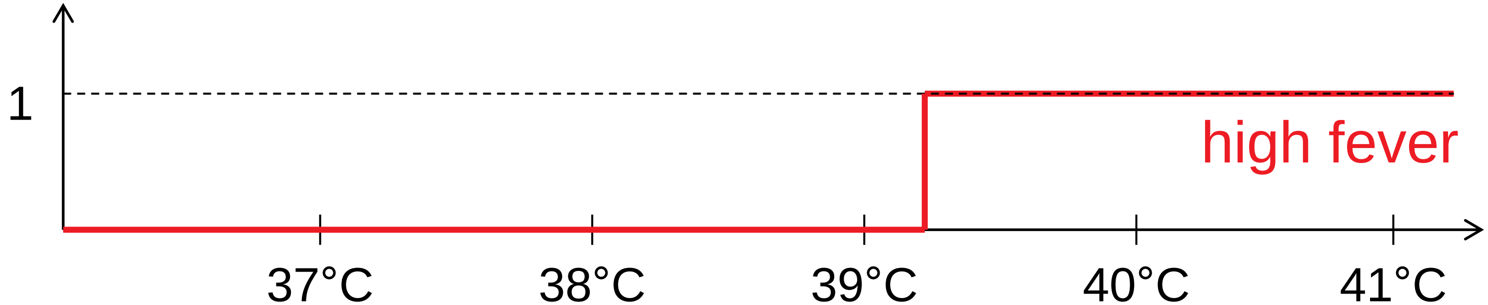
Idea of Membership Function

- standard sets: either a in A or a not in A
- fuzzy sets: a in A with probability p_a

Example: fever







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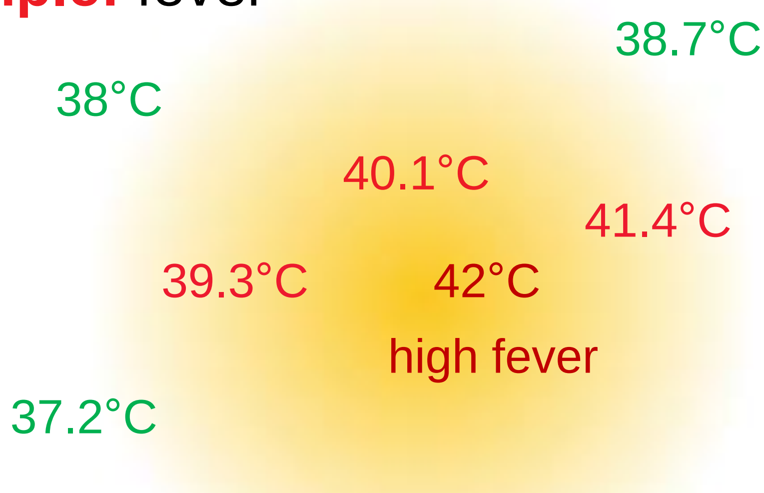
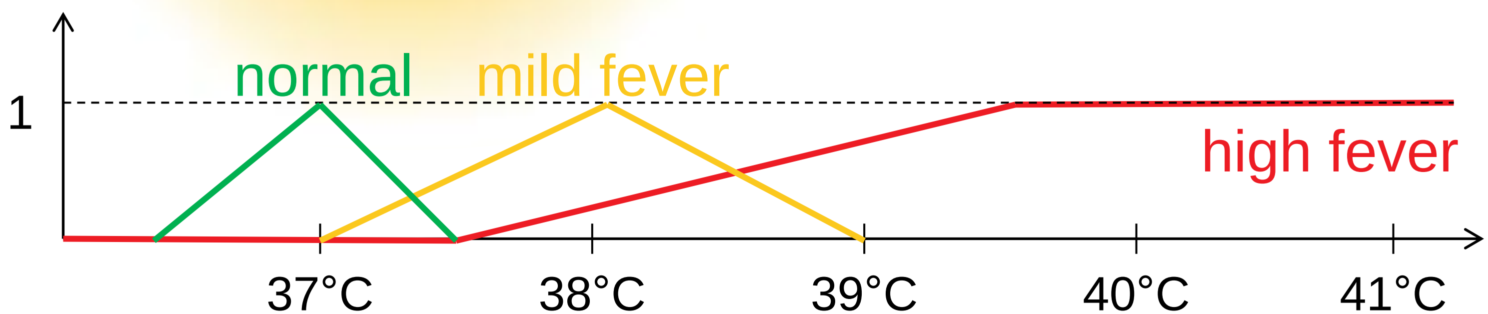
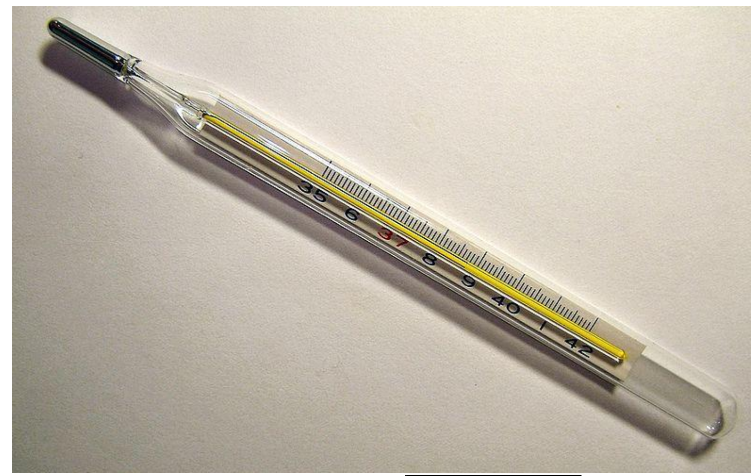
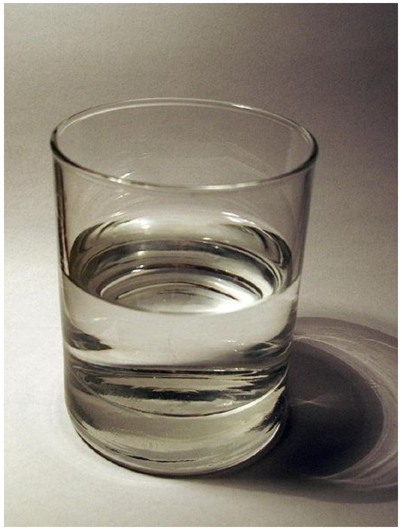


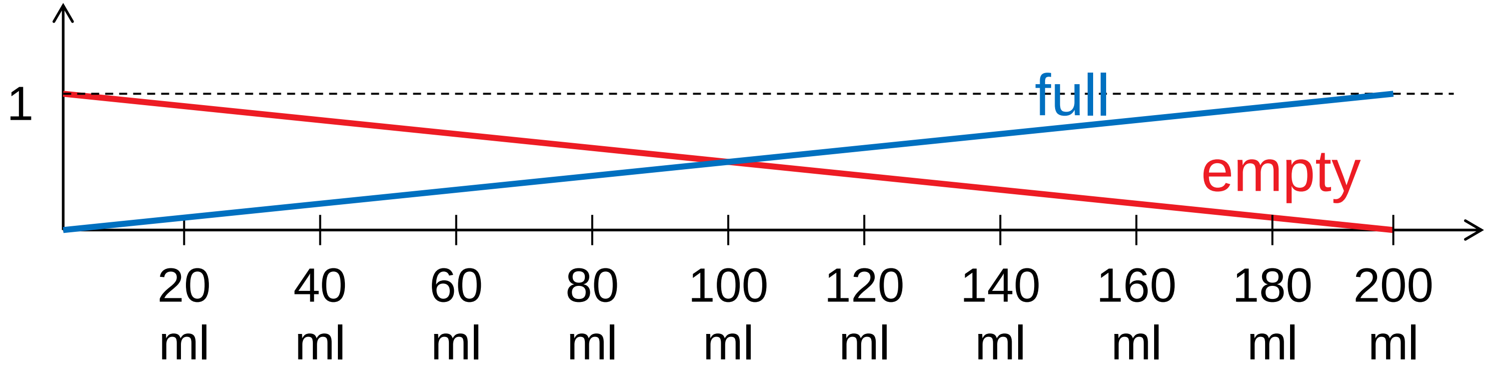


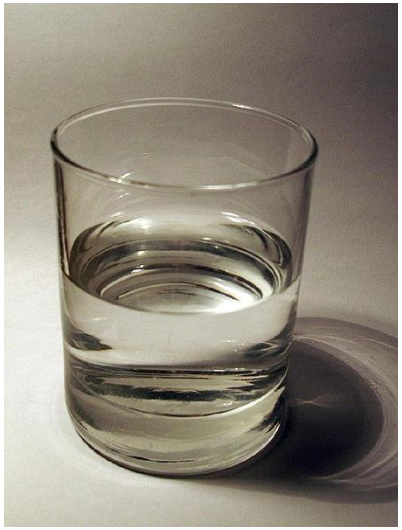


Idea of Membership Function



- 200ml glass with 100ml water: full or empty?
- standard logic: either full or empty
- fuzzy logic: glass can be full and empty!
  - 100ml: glass 50% full and 50% empty
  - 40ml: glass 20% full and 80% empty
  - but also more complex membership functions possible!





Jaques

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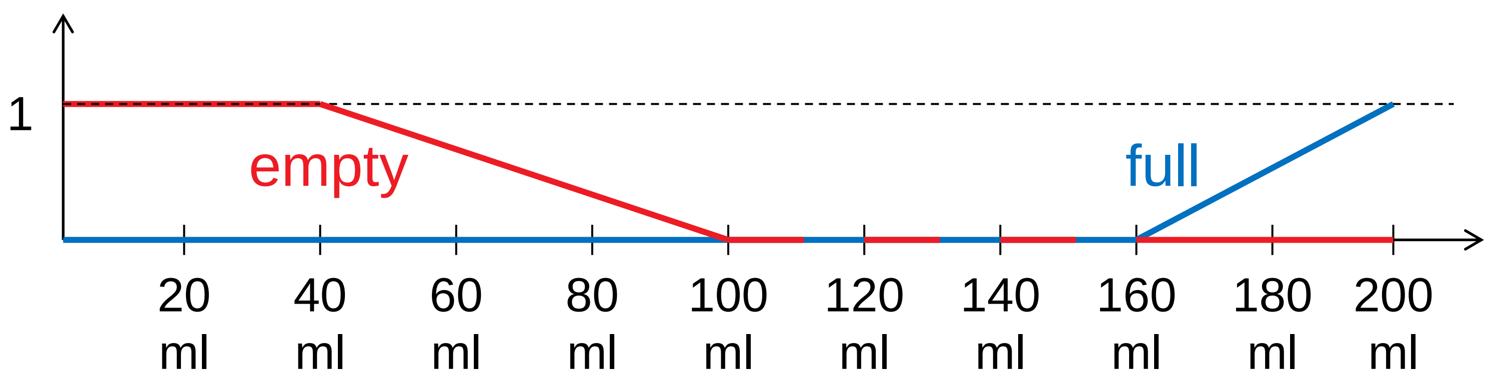
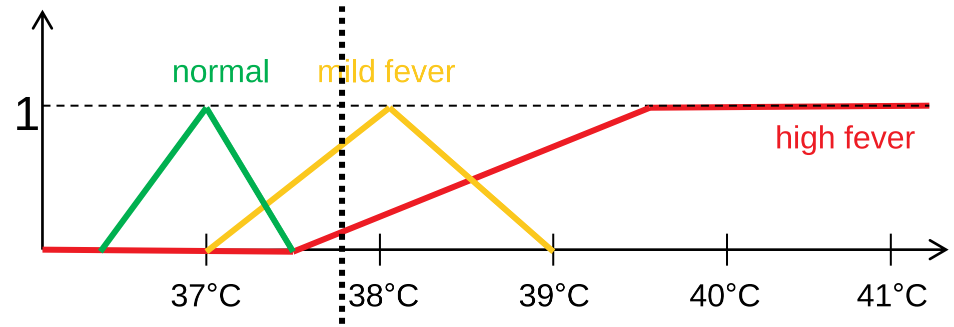


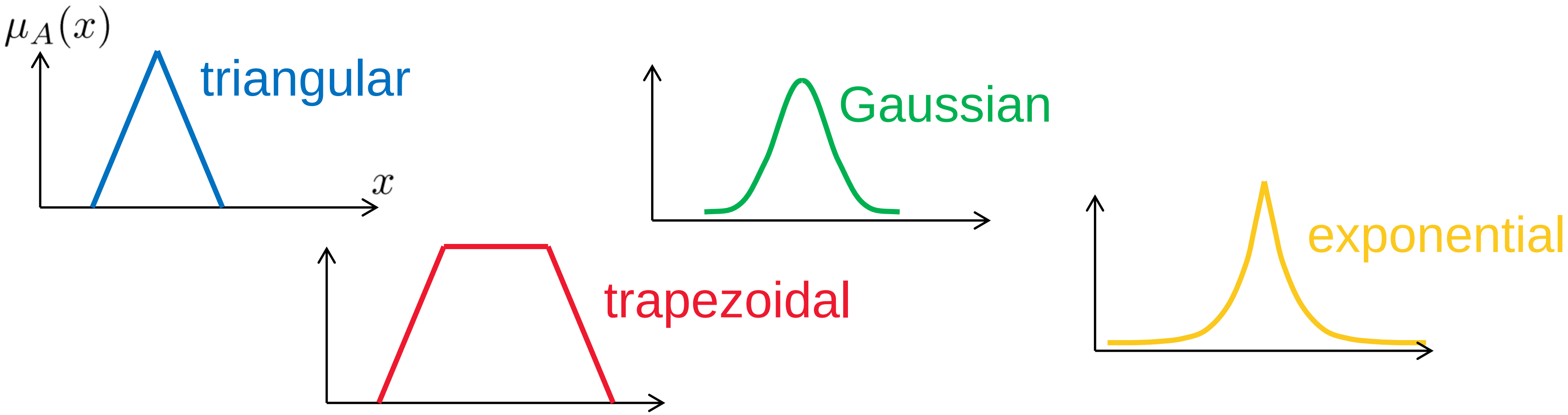
Fuzzification:

= transferring a real-valued variable into a fuzzy one



80% mild and 10% high fever

Several membership functions $\mu_A(x)$ known to do that:



In the end...

...everything is based on intuition (there are no strict rules)

Properties of Membership Functions

- μ_A is called normalized if its height is 1
- $\{x \mid \mu_A(x) > 0\}$ is called the support of μ_A
- $\{x \mid \mu_A(x) = 1\}$ is called the core of μ_A
- An α-cut of μ_A is the set $A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$
- If μ_A contains only one maximum, we call μ_A unimodal and A convex
- otherwise, μ_A is called multimodal and A nonconvex

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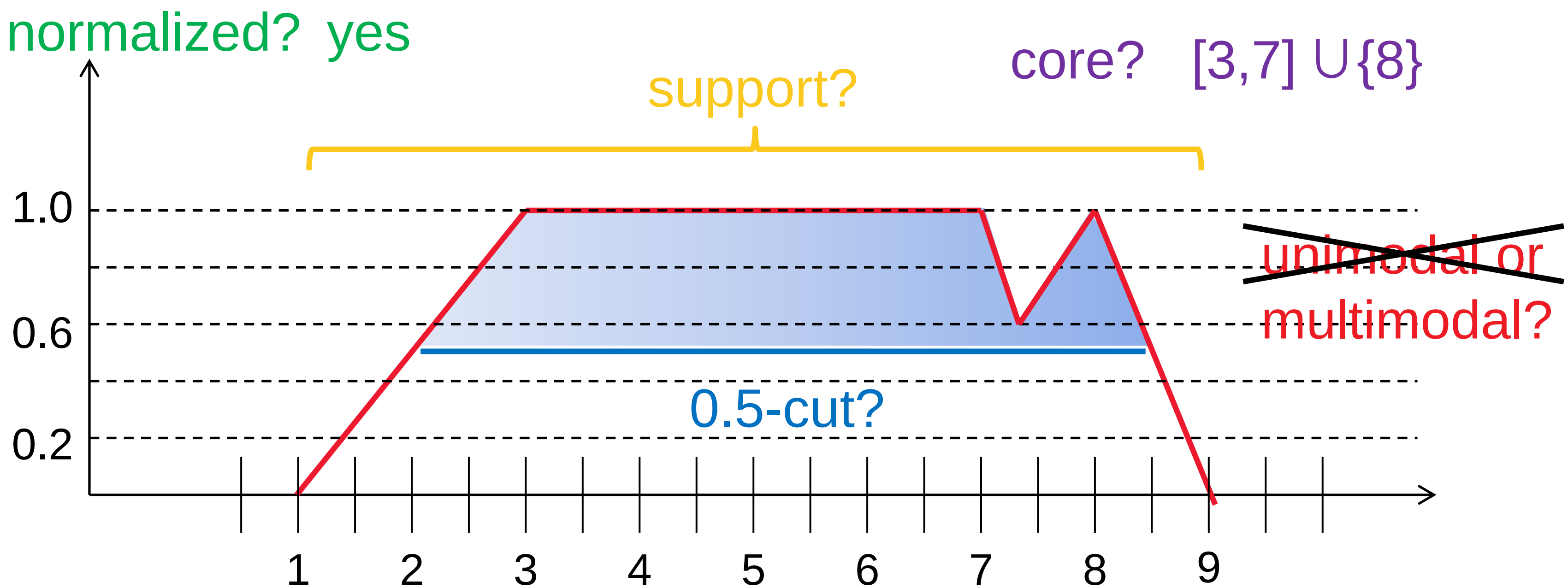
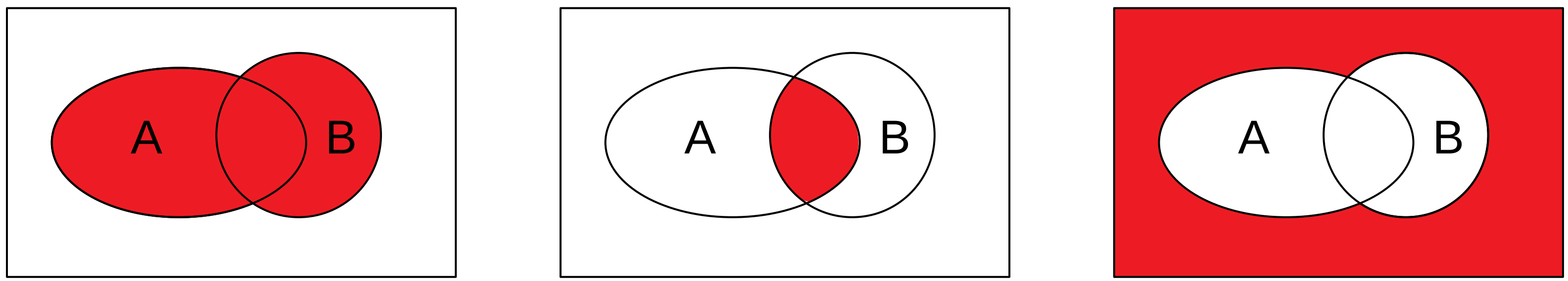


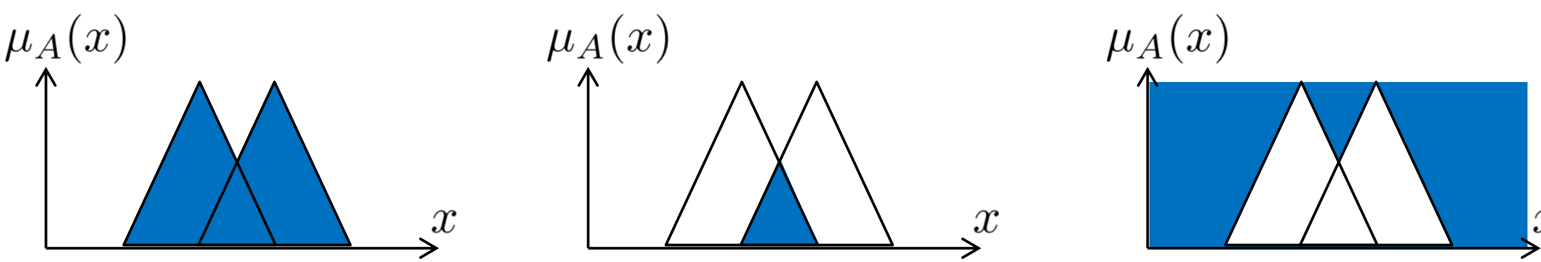
Operations on Fuzzy Sets

Union, intersection, and complement:

standard logic

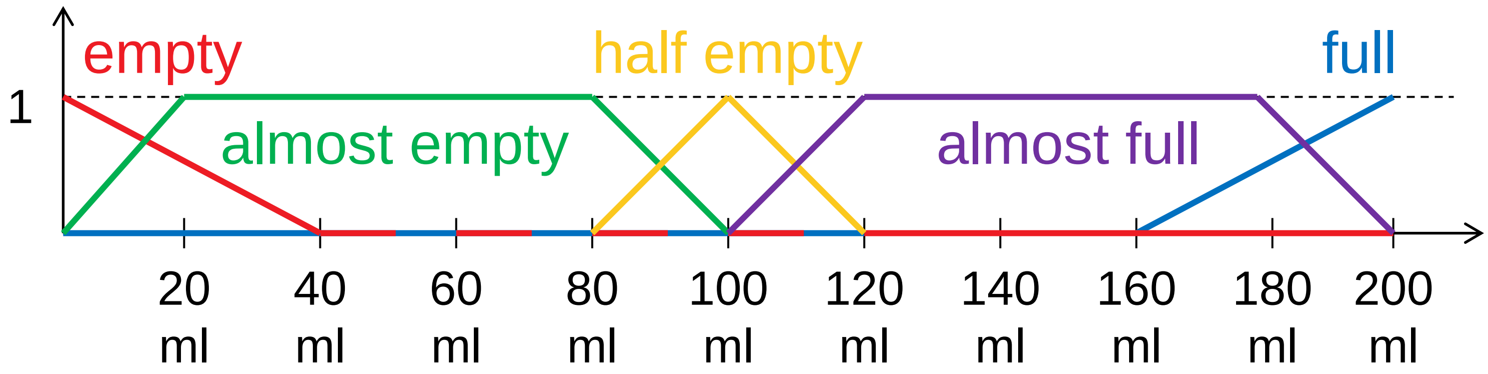


fuzzy logic



union = max | intersection = min | complement = 1-x

Defuzzifying



How do we get back "crisp" numbers (fuzzy set → real number)?

- there are many ways of doing it!

Maximum defuzzification: take x^* with $\forall x : \mu_A(x^*) \geq \mu_A(x)$

- simple but not accurate if μ_A multimodal

Centroid defuzzification: $x^* = \frac{\int \mu_A(x) x dx}{\int \mu_A(x) dx}$

- very accurate
- might be complicated to compute
- often used

Fuzzy Logic: Inferring Statements

Classical Logic:

- IF p THEN q
- equivalent to $\neg p \vee q$

	q = true	q = false
p = true	true	false
p = false	true	true

Fuzzy Logic:

- not so easy with fuzzy sets
  - interpretation as $\neg p \vee q$ results in some undesired effects
  - hence, rather “inference” than implication (for math. reasons)
- in general, implication is a function $\mu(x,y) = \Phi(\mu_A(x), \mu_B(y))$
- > 40 different implication rules proposed
- here, we consider only three (the easy and most used ones)

Fuzzy Logic: Inferring Statements

The sharp implication:

- $\mu(x,y) = \Phi(\mu_A(x), \mu_B(y)) = \begin{cases} 1 & \text{if } \mu_A(x) \leq \mu_B(y) \\ 0 & \text{else} \end{cases}$
- intuition: if X and Y are crisp sets, then $X \Rightarrow Y$ iff $X \subseteq Y$

	q=0	q=0.5	q=1
p=0	1	1	1
p=0.5	0	1	1
p=1	0	0	1

Mamdani's inference[1]:

membership function of implication:

$$\mu(x,y) = \Phi(\mu_A(x), \mu_B(y)) = \min(\mu_A(x), \mu_B(y))$$

	q=0	q=0.5	q=1
p=0	0	0	0
p=0.5	0	0.5	0.5
p=1	0	0.5	1

only ¼ of corner values equal to 2-valued logic!

inference, no implication

[1] E. H. Mamdani. "Application of fuzzy logic to approximate reasoning using linguistic synthesis". IEEE Transactions on Computers, C-26(12):1182–1191, December 1977.

Fuzzy Logic: Inferring Statements

Larsen Product implication[1]:

membership function of implication:

$$\mu(x,y) = \Phi(\mu_A(x), \mu_B(y)) = \mu_A(x) \cdot \mu_B(y)$$

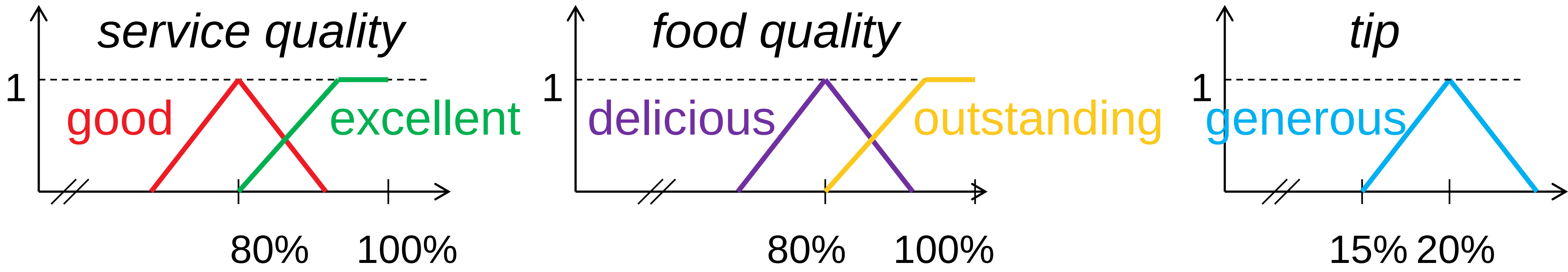
again: only ¼ of corner values equal 2-valued logic!

inference, no implication

	q=0	q=0.5	q=1
p=0	0	0	0
p=0.5	0	0.25	0.5
p=1	0	0.5	1

[1] P. M. Larsen, "Industrial Applications of Fuzzy Logic Control", International Journal of Man-Machine Studies, Vol. 12, No. 1, 1980, pp. 3-10.

Example



IF service is excellent AND food is delicious THEN tip is generous

What happens for different service and food qualities?

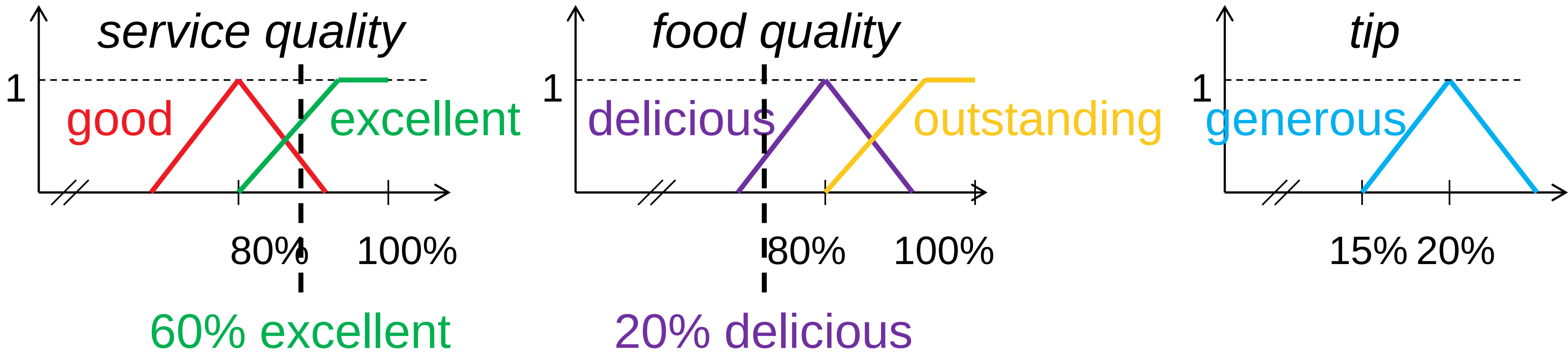
❶ fuzzify inputs

❷ compute value of left-hand side

❸ then apply above rule (e.g. wrt. Mamdani’s rule)

❹ use defuzzification rule (e.g. centroid)

Example



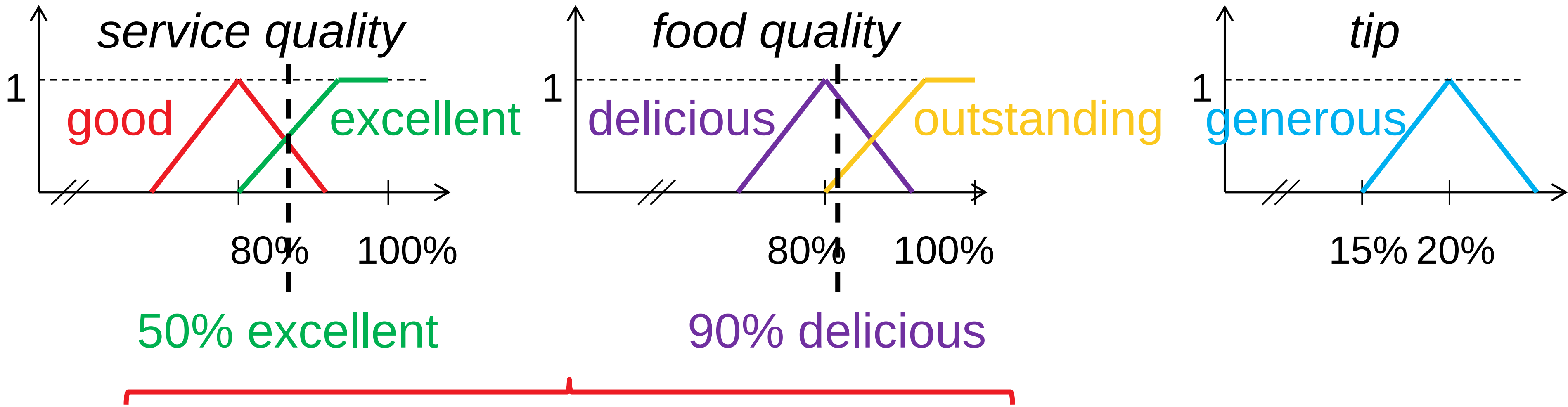
IF service is excellent AND food is delicious THEN tip is generous

What happens for different service and food qualities?

➊ fuzzify:

- 60% excellent AND 20% delicious

Example

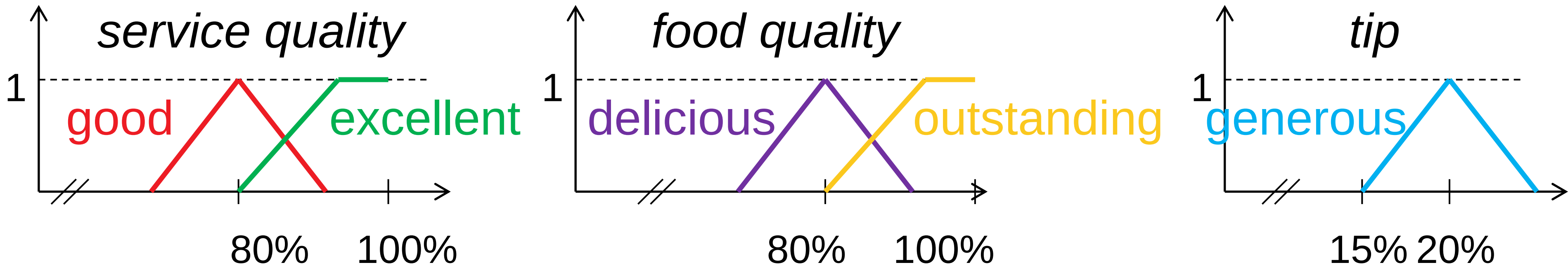


IF service is excellent AND food is delicious THEN tip is generous

What happens for different service and food qualities?

❶ fuzzify:
- 60% excellent AND 20% delicious
- 50% excellent AND 90% delicious

❷ compute value of left-hand: here “AND = min.”
- 20%
- 50%

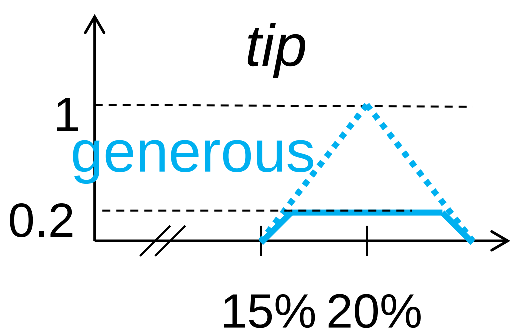


IF service is excellent AND food is delicious THEN tip is generous

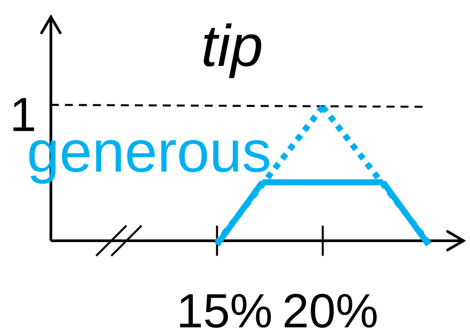
What happens for different service and food qualities?

❸ apply Mamdani's rule: $\mu(x,y) = \min(\mu_A(x), \mu_B(y))$

$\mu_A(x) = 20\%$



$\mu_A(x) = 50\%$



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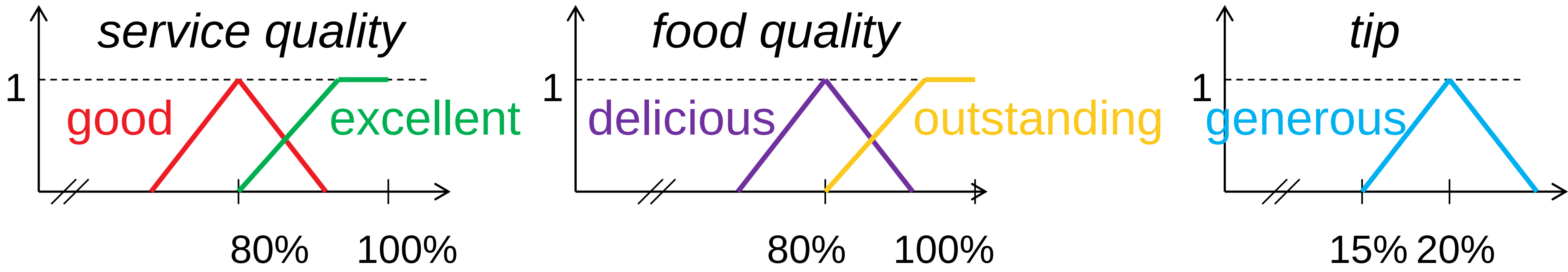
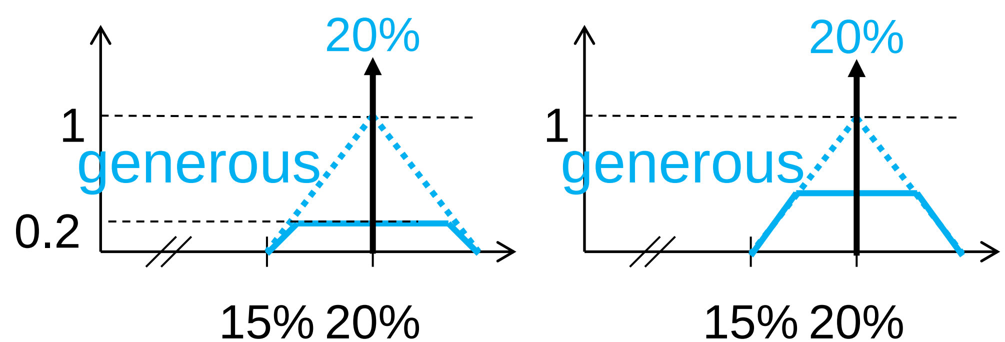


IF service is excellent AND food is delicious THEN tip is generous

What happens for different service and food qualities?

❹ use defuzzification rule (e.g. centroid)

here: same result, but also only 1 rule applied



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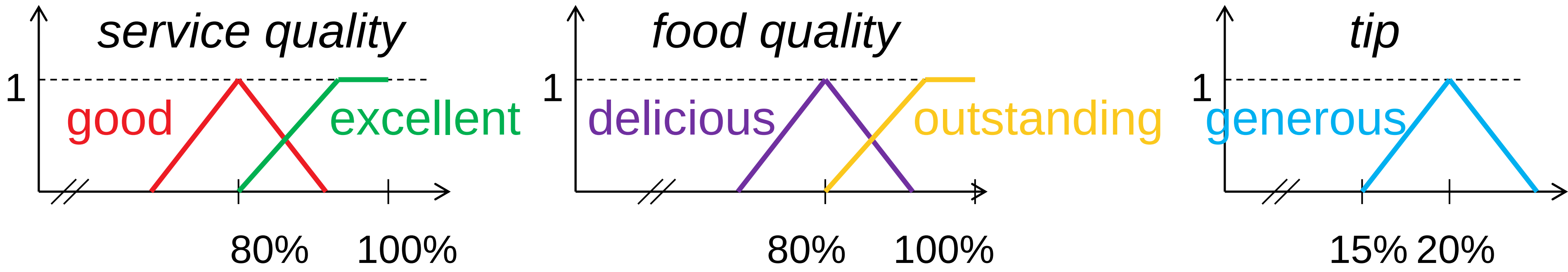
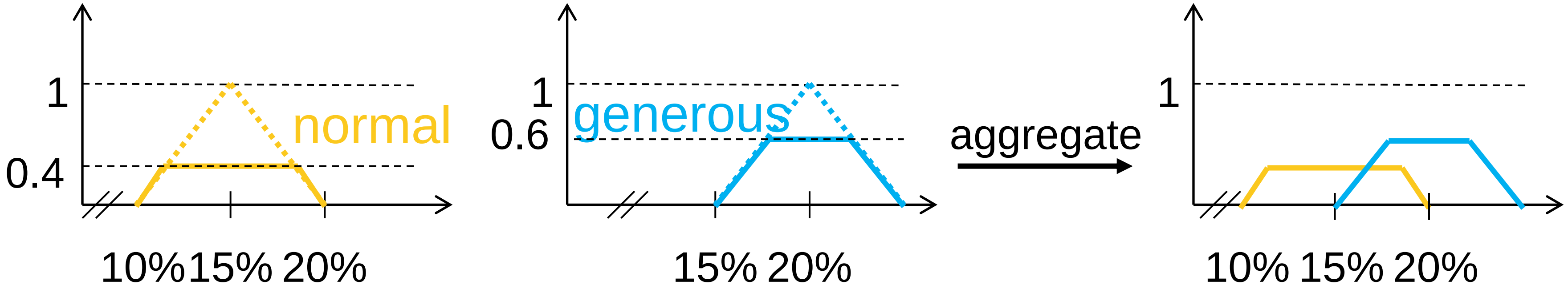


IF service is normal AND food is normal THEN tip is normal
IF service is excellent AND food is delicious THEN tip is generous

Multiple rules

- ❸ a) apply all inference rules
- ❸ b) aggregate resulting membership functions (e.g. with max.)



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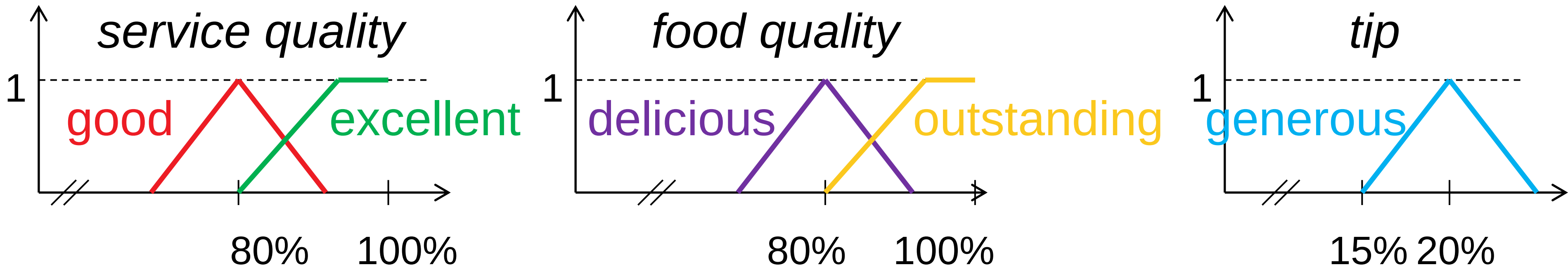

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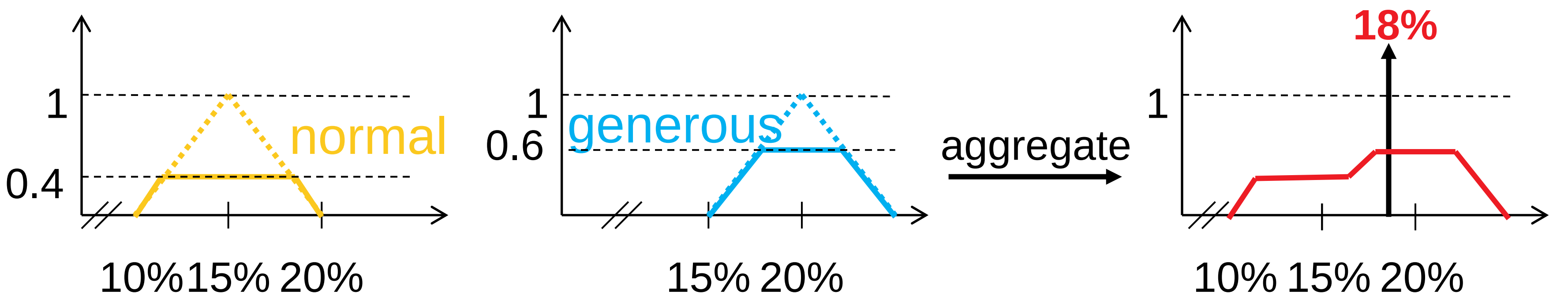

Fuzzy Control

"Classical" control:

- mathematical ("crisp") formulations
- based on mathematical models, especially ODEs
- e.g. "210°C < TEMP < 220°C"

Fuzzy control:

- design formalized by words
- based on experience of the designer
- e.g. "IF (process is too cool) AND (process is getting colder) THEN (add heat to the process)" or "IF (process is too hot) AND (process is heating rapidly) THEN (cool the process quickly)"

A Simple Rule Matrix

Back to the water tap problem from last week:

- imagine measurements of temperature and water flow (e.g. per second) and the controllable inputs "hot water" and "cold water"
- further assume the inputs are fuzzified as {too cold, fine, too hot} (for the temperature) and {not enough, fine, too much} (for the water flow)

Then, a 3x3 rule matrix can show the responses:

	too cold	fine	too hot
not enough			
fine			
too much			

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Then, a 3x3 rule matrix can show the responses:

	too cold	fine	too hot
not enough	increase hot	increase hot & cold	increase cold
fine	decrease cold & increase hot	do nothing	increase cold & decrease hot
too much	decrease cold	decrease hot & cold	decrease hot

e.g. IF temperature is fine AND water flow is not enough THEN increase both cold and hot water

Another Rule Matrix

Example: electric heater

- given: goal temperature T_{opt}
- measured: temperature T and temperature change dT/dt
- controlled inputs: heat (heating on) and cool (fan on)
- fuzzify: $T-T_{opt}$ and $d(T-T_{opt})/dt$ in {negative, zero, positive}

		temperature: $T-T_{opt}$		
		negative	zero	positive
temperature change: $d(T-T_{opt})/dt$	negative			
	zero			
	positive			

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		temperature: $T-T_{opt}$		
		negative	**zero**	**positive**
temperature change: $d(T-T_{opt})/dt$	**negative**	heat	heat	cool
	zero	heat	do nothing	cool
	positive	heat	cool	cool

Remarks on Rule Matrices

- nothing fancy, but assisting to not forget a rule
- not much helpful if >2 input variables
- not always necessary to define output for all input combinations
- not usable if rules are not of the form “IF a AND b THEN c”
- odd number of rows and columns often helpful (to have a “zero” state with no change)

Again: What if a fuzzified “crisp” input value fire >1 rule?

  - then: aggregation (union, max) of output membership functions

How to Design a Fuzzy Controller

1) Define control objectives and criteria

   What am I trying to control? What do I have to do to control the system? What kind of response do I need? What are the possible (probable) system failure modes?

2) Determine input/output relationships and choose the variables.
3) Break the control problem down into a series of IF X AND Y THEN Z rules (or similar) that define the desired system output response for given system input conditions.

   ! If possible, use at least one variable and its time derivative.

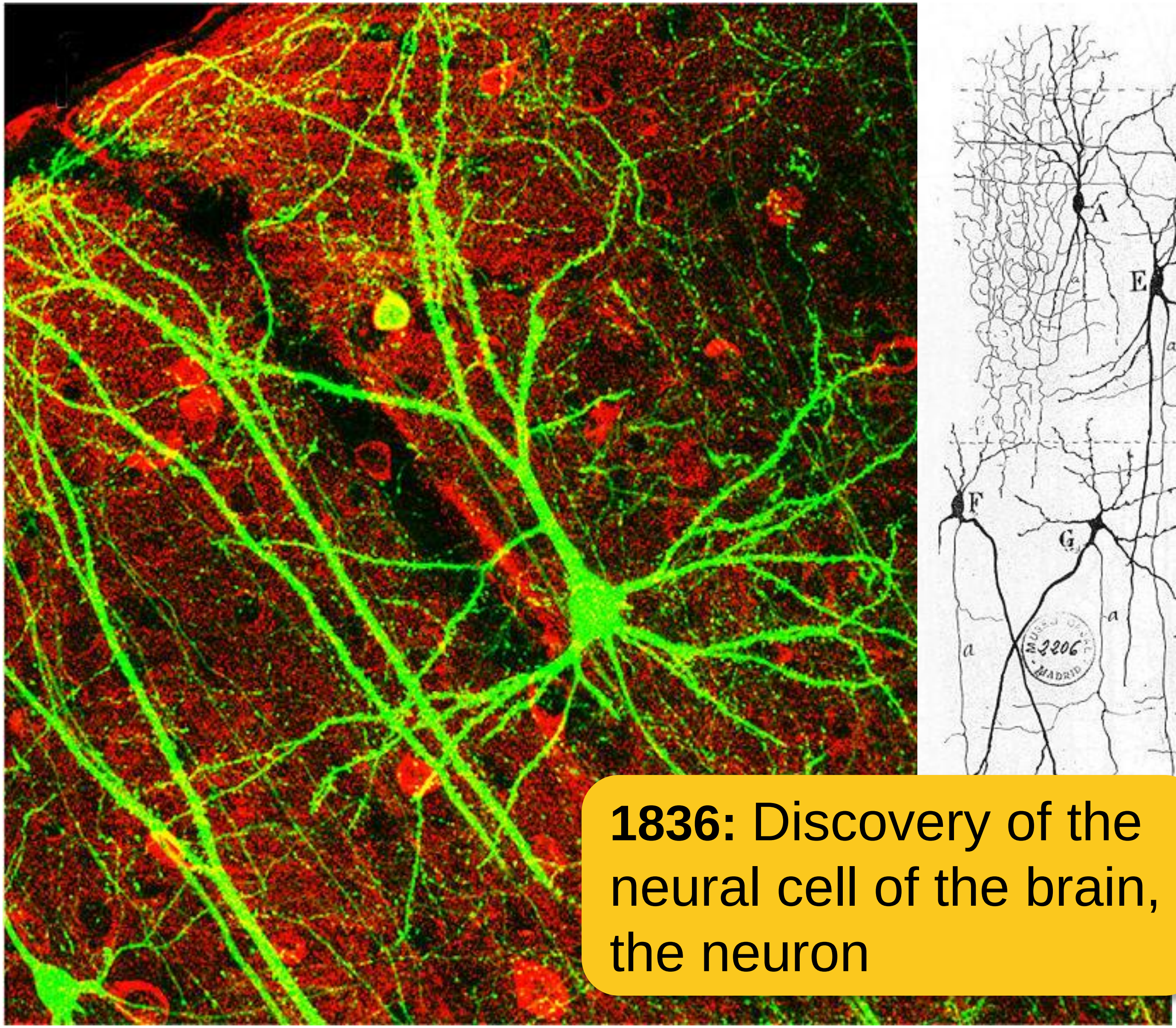
4) Create Fuzzy Logic membership functions and decide on inference rules that define the meaning (values) of the Input/Output terms used in your rules.
5) Implement the system in software (or hardware).
6) Test, evaluate, and tune the rules and membership functions, until satisfactory results are obtained.

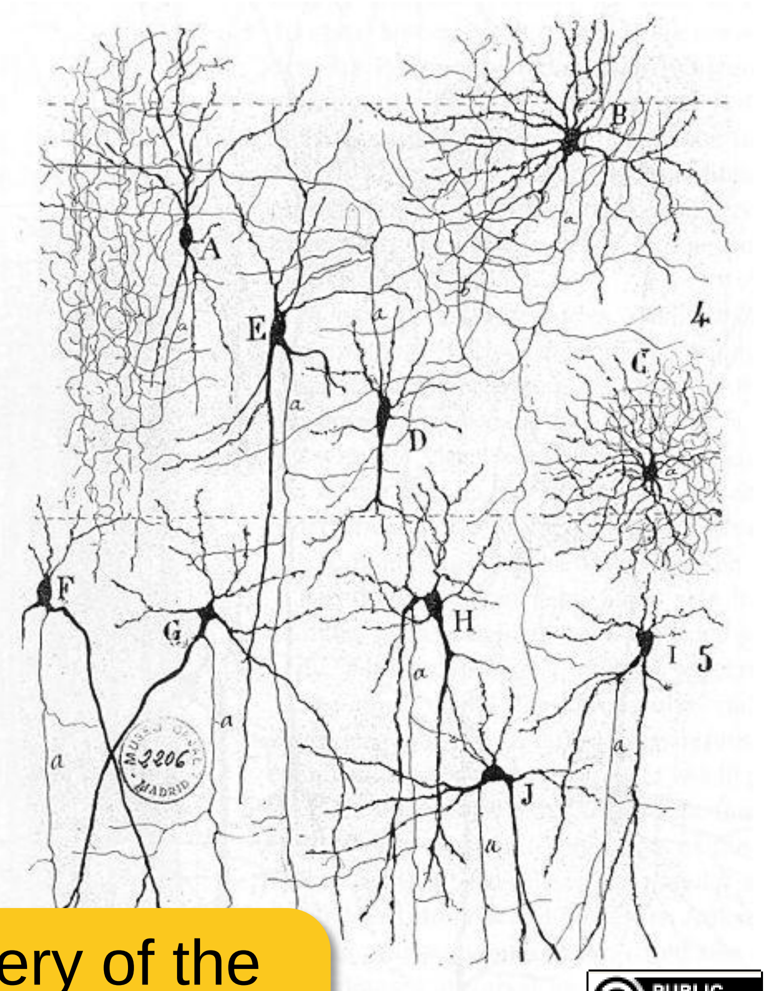
according to the Fuzzy Logic Tutorial by Steven D. Kaehler
http://www.seattlerobotics.org/encoder/mar98/fuz/flindex.html

Exercise:
A Fuzzy Controller for the Pole Balancing Problem

Artificial Neural Networks

The Biological Neuron

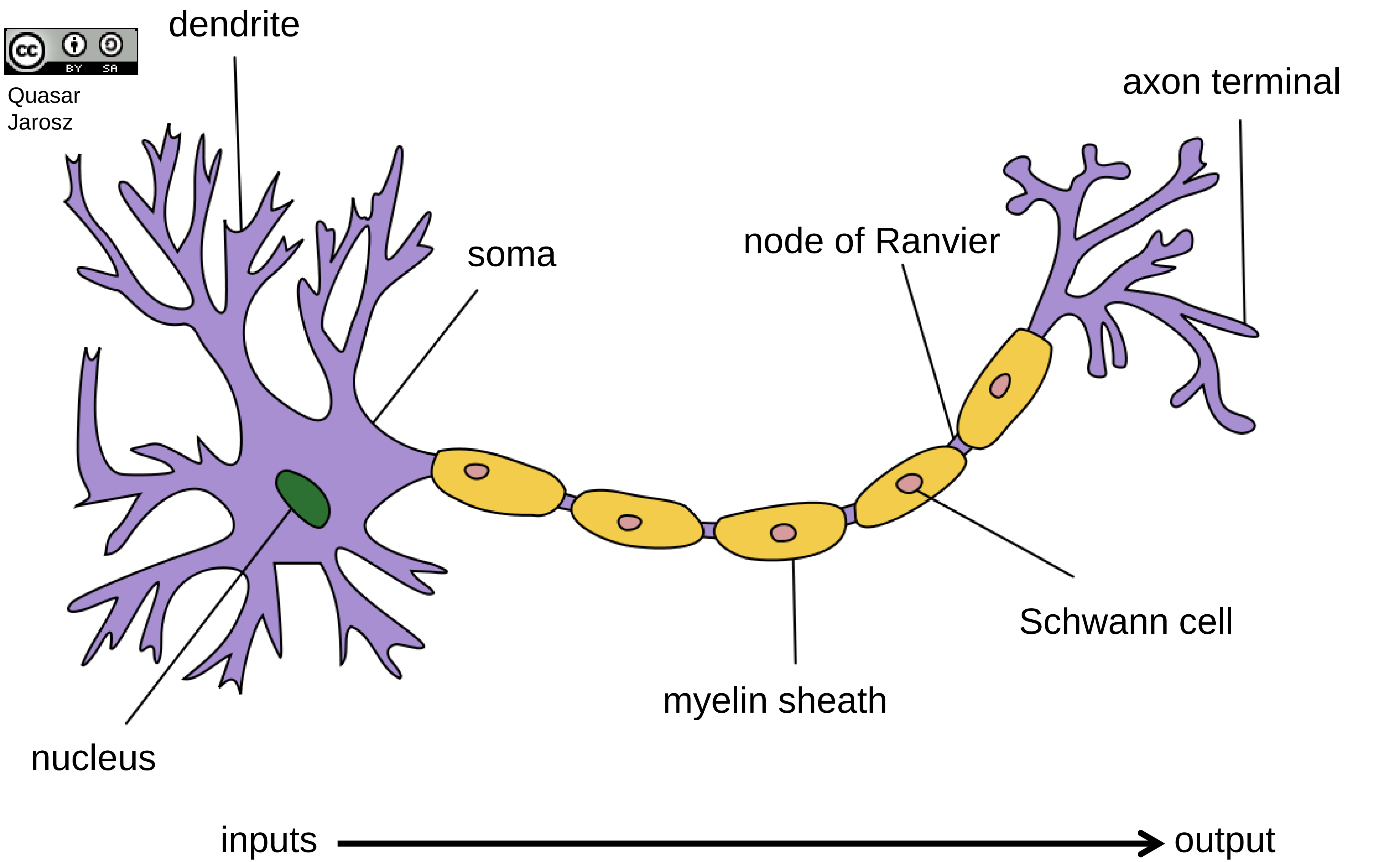




1836: Discovery of the neural cell of the brain, the neuron

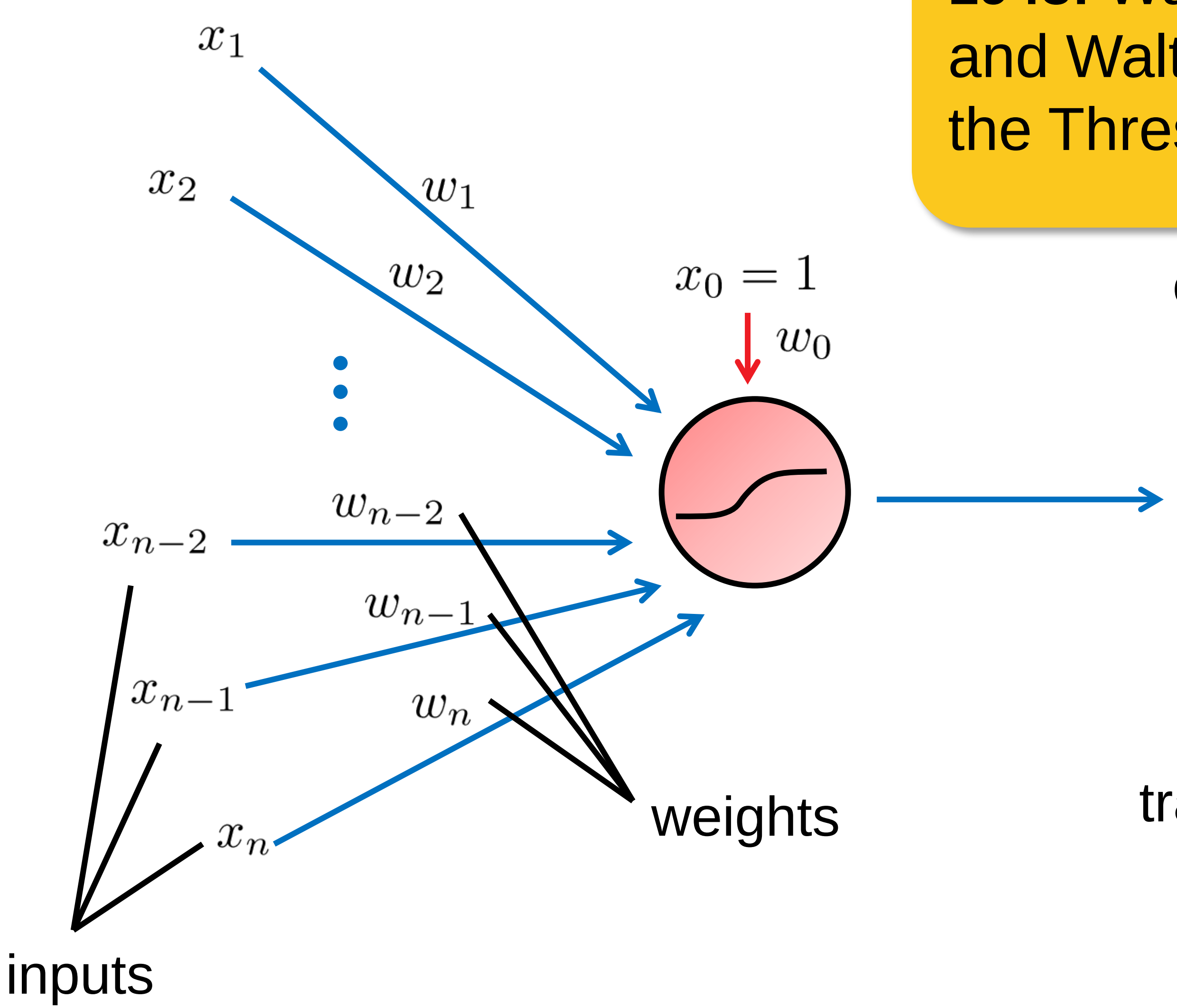
 W.-C. A. Lee, H. Huang, G. Feng, J. R. Sanes, E. N. Brown, P. T. So, E. Nedivi

The Biological Neuron



An Artificial Neuron

1943: Warren McCulloch and Walter Pitts propose the Threshold Logic Unit



output

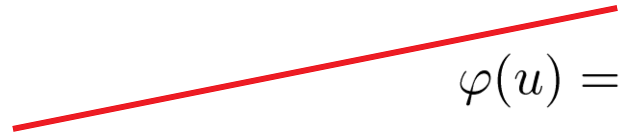
$$y = \varphi\left(\sum_{i=\cancel{1}0}^{n} w_i x_i\right)$$

transfer function

Types of Transfer Functions

$$y = \varphi\Big(\underbrace{\sum_{i=1}^{n} w_i x_i}_{u}\Big)$$

linear

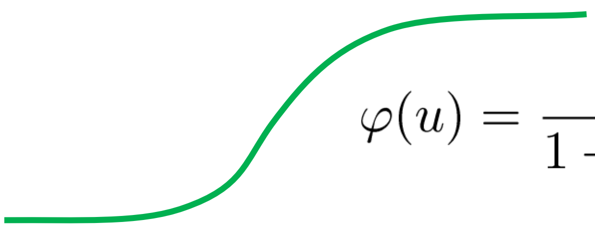


$$\varphi(u) = u + x_0$$

step

$$\varphi(u) = \begin{cases} 1 & \text{if } u \geq \theta \\ 0 & \text{if } u < \theta \end{cases}$$

sigmoidal

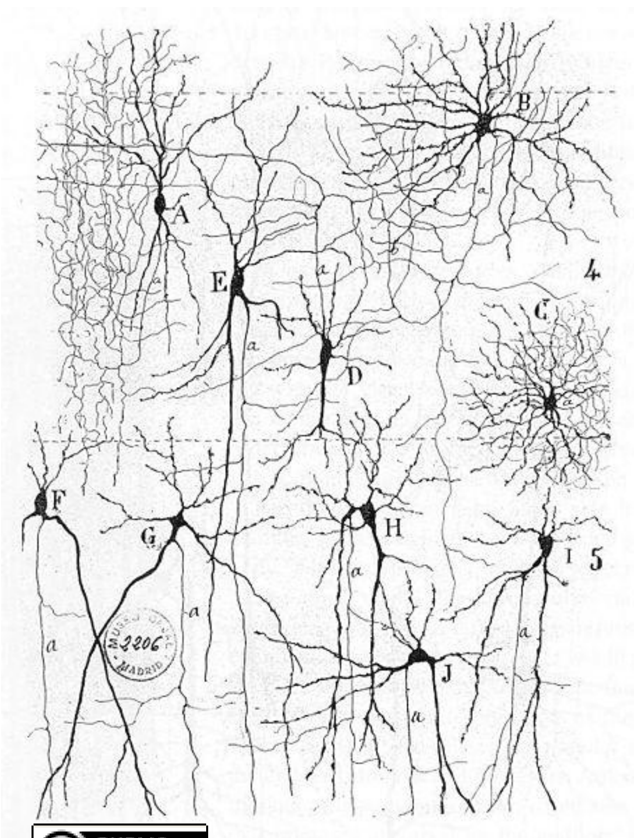


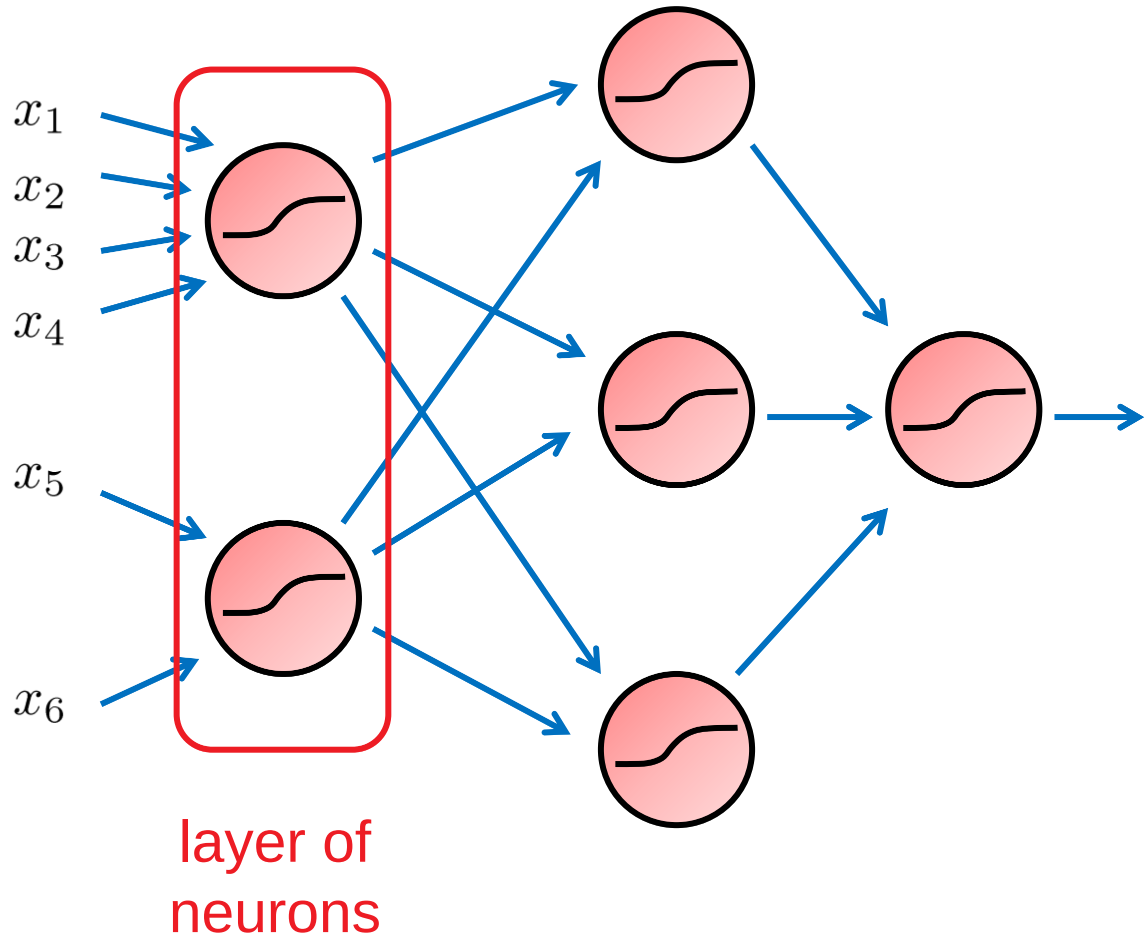
$$\varphi(u) = \frac{1}{1 + e^{-cu}}$$

advantage: differentiable

Combining Artificial Neurons

Artificial Neural Networks (ANNs) = a network of artificial neurons





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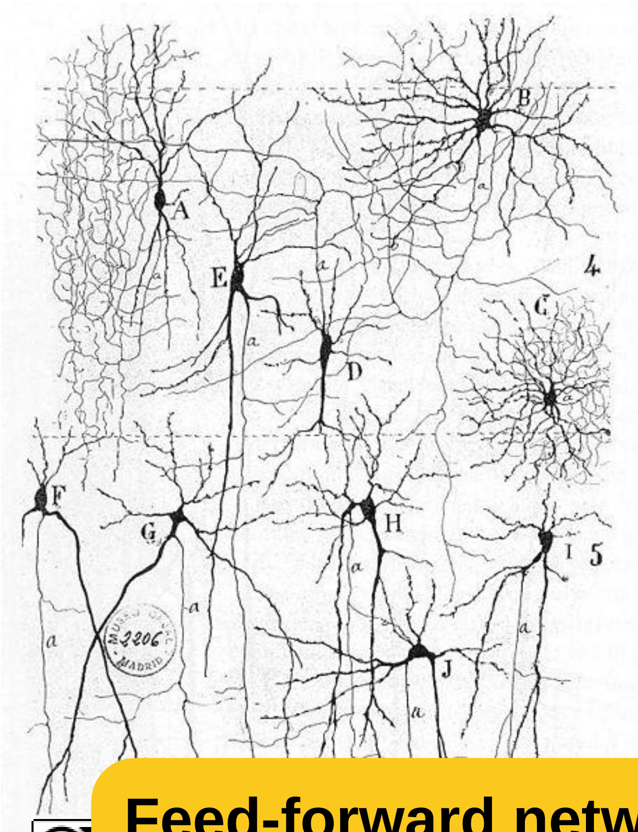
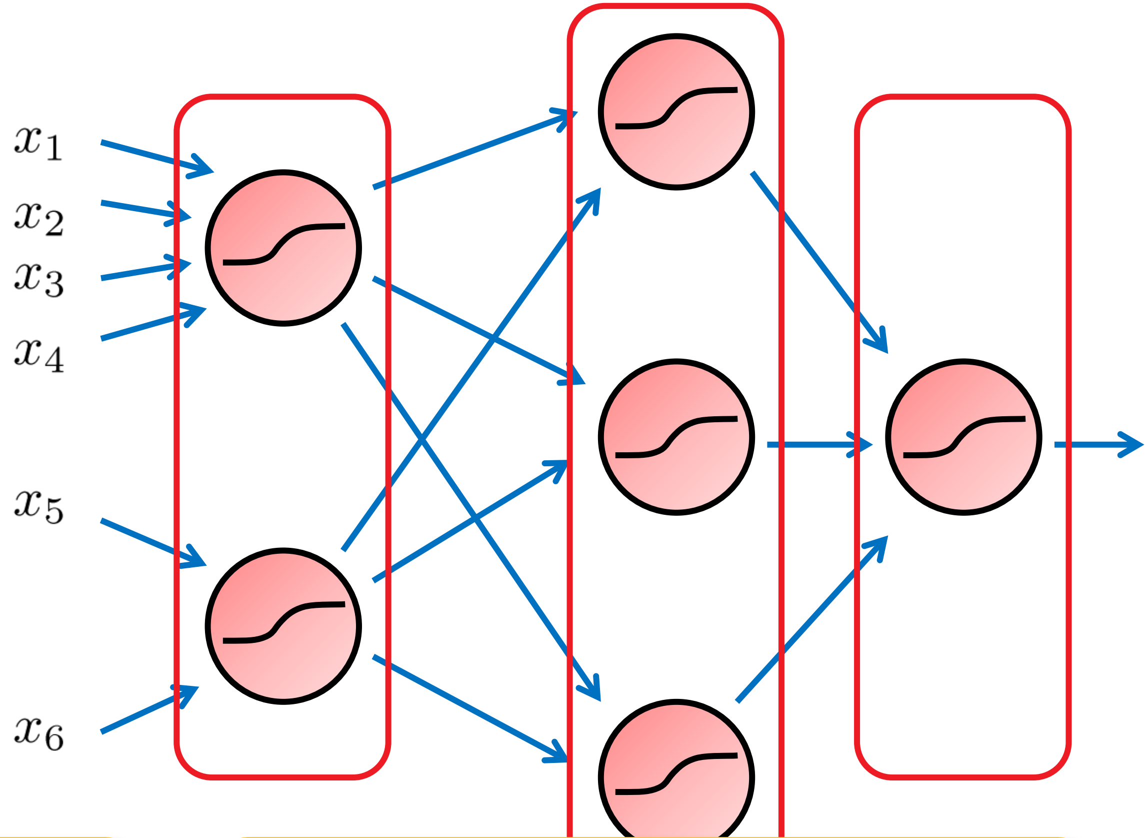




Feed-forward network:
no “backwards” flow of information

Linear transfer functions:
multi-layer networks can be simulated by a single-layer ANN

Optimizing Weights in Order to Optimize Output

Supervised learning scenario:

- neural network with n inputs and m outputs
- given a set of training data $(\vec{x}_1, \vec{d}_1), \dots, (\vec{x}_p, \vec{d}_p)$
- what are "optimal" weights such that

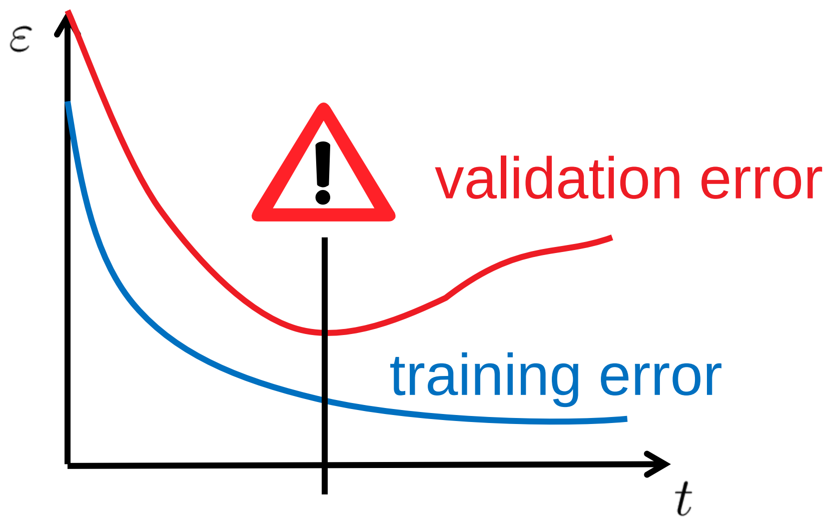
$$E(\vec{w}) = \sum_{j=1}^{p} \sum_{k=1}^{m} ||y_{j,k} - d_{j,k}||^2 = \sum_{j=1}^{p} \sum_{k=1}^{m} ||\varphi_k(\vec{x}_j, \vec{w}) - d_{j,k}||^2$$

is minimal?

Testing and Training in Supervised Learning

training data set vs. testing data set

training error vs. validation error



Generalization vs. Overfitting

- generalization behaviour desired
- overfitting especially when not much training data available

Gradient Descent to Optimize

Optimization: $$\min_{x \in \mathbb{R}^n} f(x)$$

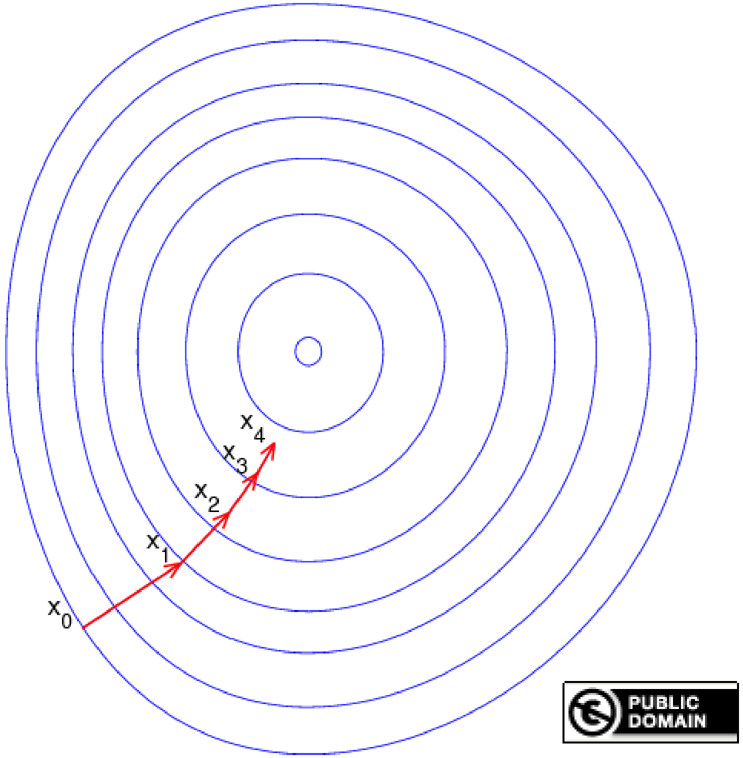
Gradient Descent Algorithm

initialize $x_0 \in \mathbb{R}^n$

At each iteration t :

- compute gradient ∇f
- $x_{t+1} = x_t - \gamma \nabla f(x_t)$

learning rate



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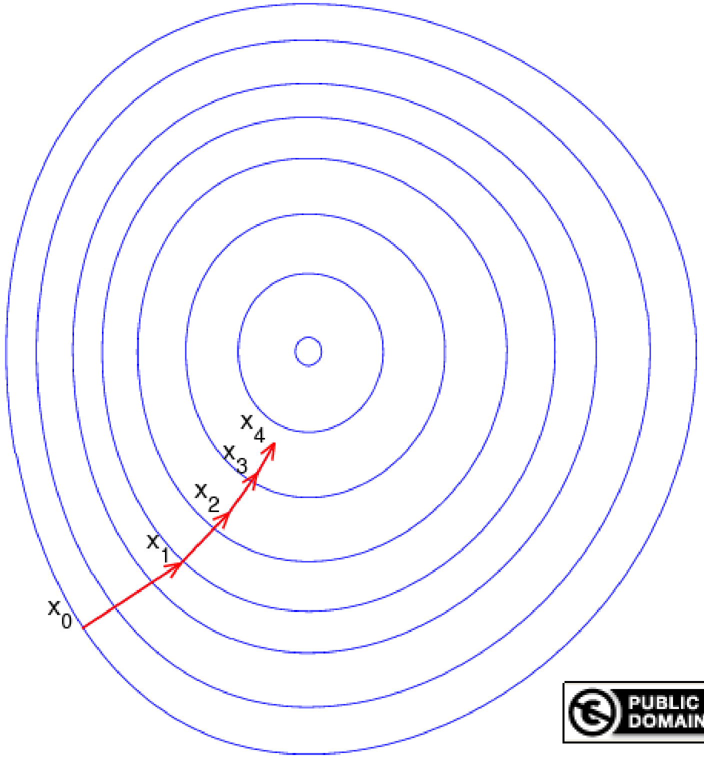

! can be slow close to optimum

other algorithms might be favorable

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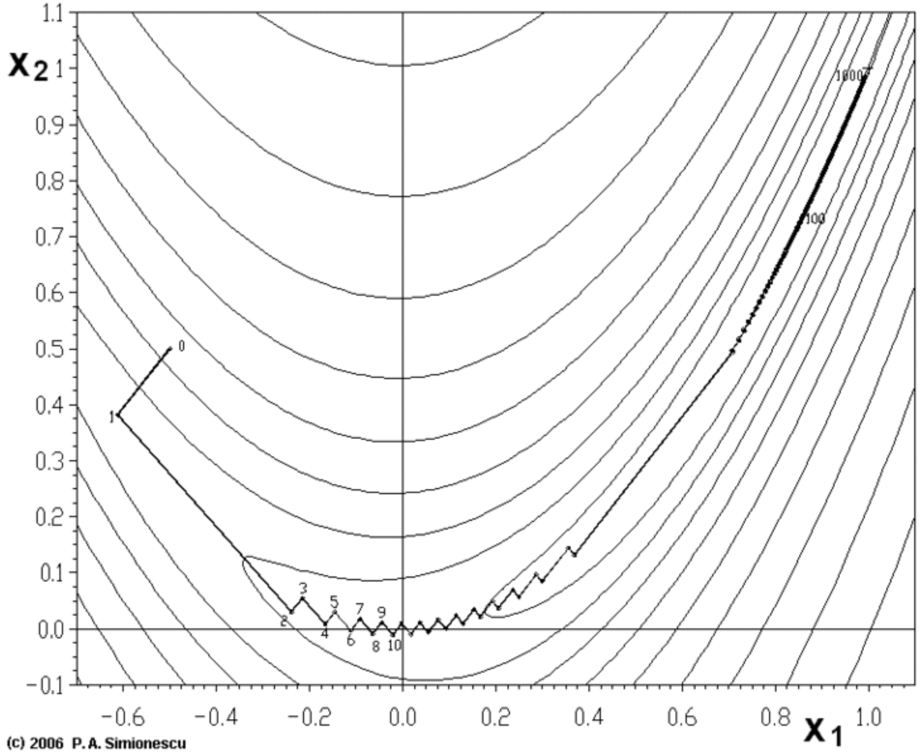

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learning rate

! can be slow close to optimum

other algorithms might be favorable

(keyword: natural gradient)

Example:
Rosenbrock function

Optimizing Weights in a Layered Network

How to choose the weights in a multi-layered ANN?

Why not optimize weights directly?

$$E(\vec{w}) = \sum_{j=1}^{p}\sum_{k=1}^{m} ||y_{j,k} - d_{j,k}||^2 = \sum_{j=1}^{p}\sum_{k=1}^{m} ||\varphi(\vec{x}_j, \vec{w}) - d_{j,k}||^2$$

$$\vec{w} = \vec{w} - \nabla\left(\sum_{j=1}^{p}\sum_{k=1}^{m} ||\varphi(\vec{x}_p, \vec{w}) - d_{j,k}||^2\right)$$

since complicated*, better:

- gradient descent after each training sample

  = stochastic gradient descent (SGD, online gradient descent)

  $$w = w - \nabla\left(\sum_{k=1}^{m} ||\varphi(\vec{x}_j, \vec{w}) - d_{j,k}||^2\right)$$

- descent steps can be performed multiple times over the training set (e.g. with random shuffling)

* complicated: difficult analytically, numerically expensive

Optimizing Weights in a Layered Network

The Backpropagation Algorithm

- introduced around 1970, it gave rise to a renaissance of ANNs
- "backwards propagation of errors"
- mainly useful for feed-forward networks
- all transfer functions must be differentiable

Main Idea:

for each training sample:

- compute output of ANN, given the current weights
- compute gradient wrt. weight *on each node* from the output layer *backwards* to the input layer
- update the weights according to gradient descent

→ an efficient stochastic gradient descent by updating all weights at once in a smart way

Optimizing Weights in a Layered Network

Notes:

- stochastic gradient descent converges to local minimum
- random initial values, restarts
- more about optimization algorithms within the next weeks

Applications of Neural Networks

Many application areas: e.g.

- identification problems
  - face recognition
  - medical diagnoses
  - character recognition in mobile devices
- predictions/forecasting
  - stock market
  - electronic nose
- control

Ralf Pfeifer

At the end of the course:
exercise using ANNs for
the pole balancing problem

Conclusions

I hope it became clear...

...how to build a fuzzy controller (at least in principle)

...what artificial neural networks are

...and that designing a good controller is not always easy

In the next weeks...

...we will see how to actually optimize with randomized search heuristics