Home Exercise 6: Randomized Search Heuristics

Algorithms and Complexity lecture
at CentraleSupelec/ESSEC

Dimo Brockhoff
firstname.lastname@inria.fr

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Abstract

Please send your solutions by email to Dimo Brockhoff (preferably in PDF format) with a clear indication of your full name until the submission deadline on November 4, 2019 (a Monday). Groups of up to 4 students are explicitly allowed and even encouraged. In the case of group submissions, please make sure that you submit maximally four times with the same partner!

1 Mutation operators for the Knapsack Problem (4 points)

Assume, you want to implement an evolutionary algorithm to solve the knapsack problem (do you remember the "robbery" exercise?). A bitstring of length n is thereby interpreted as the choice of items: a 0 at position x_i means that item i is not chosen, a 1 at the same position interpreted as "item i is chosen".

Do you think that relying solely on 1-bit flips is a reasonable operator for the knapsack problem? What do you expect will happen in particular in the end of the optimization when you only use 1-bit flips? What do you suggest to circumvent this behavior?

2 Roulette wheel selection (6 points)

a) Given the fitness function $f(x) = x^2$ on a single variable $x \in \mathbb{R}$, calculate the probability of selecting the individuals a with $x = 1$, b with $x = 2$, and c with $x = 3$, using the roulette wheel selection from the lecture.

b) Calculate the probability of selecting the same individuals when the fitness function is $g(x) = f(x) + 100$.

c) Calculate the probabilities of selecting each solution from the population $\{a, b, c\}$ (with the same above solutions $x = 1$ (a), $x = 2$ (b), and $x = 3$ (c) and for both fitness functions $f(x)$ and $g(x)$) in a single binary tournament. A single binary tournament means, we draw two solutions uniformly at random from the population (with replacement) and pick the better one.

d) Which of the selection operators do you favor? Why?

3 Pure Random Search (10 points)

The first stochastic optimization algorithm, introduced before any genetic algorithm (GA) or evolution strategy (ES), is the so-called pure random search, or PRS for short[1]. Assuming a bounded search domain, the algorithm consists in sampling independently points *uniformly* distributed in the search space as follows:

[1]The historical papers proposing the use of the PRS are:

- ⋆ S.H. Brooks: "Discussion of random methods for locating surface maxima". Operations Research 6 (1958), pp. 244–251.
- ⋆ L.A. Rastrigin: "The convergence of the random search method in the extremal control of a many-parameter system". Automation and Remote Control 24 (1963), pp. 1337–1342.

Pure Random Search to maximize $f : \{0,1\}^n \to \mathbb{R}$

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Initialize uniformly at random x ∈ Ω = {0,1}^n
while not terminate
     Sample x' uniformly at random in Ω
     if f(x') ≥ f(x)
          x = x'
return x
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In the following, we consider the optimization of the following two functions, both defined on bitstrings of length n:

Example 1 *For* $\boldsymbol{x} \in \{0,1\}^n$*, the function* OM *is defined as*

$$f_{\mathrm{OM}}(\boldsymbol{x}) = \sum_{i=1}^{n} \boldsymbol{x}_i$$

Example 2 *For* $\boldsymbol{x} \in \{0,1\}^n$*, the function* LO *is defined as*

$$f_{\mathrm{LO}}(\boldsymbol{x}) = \sum_{i=1}^{n} \prod_{j=1}^{i} \boldsymbol{x}_j$$

a) Describe in words what the functions f_{OM} and f_{LO} compute.

b) What are the maxima of the two functions? What are the values of f_{OM} and of f_{LO} at their optima?

Note: in a blackbox scenario, the time for an algorithm to reach the optimum is measured by counting the number of function evaluations needed to reach the optimum (aka the hitting time of the optimum) and not by the real wall-clock time. In other words, we assume that the internal operations are generally negligible compared to the cost of evaluating the objective function.

c) Compute the expected time (in number of function evaluations) to reach the optimum as a function of the search space dimension n. Hint: show that the time to reach the optimum follows a geometric distribution with a parameter to determine.