

Intermediate Exam No. 1 — November 25th, 2021

Introduction to Optimization - MSc AIC - Univ. Paris-Saclay

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- (1) Explain in your own words what is the *curse of dimensionality* and which consequences it has in optimization.
- (2) Define the level sets of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$.

In the following, consider the function with two variables $f(x) = \frac{1}{2}x^\top Ax$ with $x = (x_1, x_2)^\top$ and $A = \begin{pmatrix} 16 & 0 \\ 0 & 1 \end{pmatrix}$.

- (3) Express the function as $f(x) = a_{11}x_1^2 + a_{22}x_2^2$ with a_{11} and a_{22} some coefficients to determine.
- (4) Compute $\nabla f(x)$, the gradient of the function in x .
- (5) Compute $\nabla^2 f(x)$, the Hessian matrix of f in x .
- (6) Compute the condition number of A . Is f a ill-conditioned problem? Justify your answer.
- (7) Plot the level sets of f .
- (8) Plot in addition (on the same drawing) in $x = (1, 1)$: $-\nabla f(x)$ and $-\left[\nabla^2 f(x)\right]^{-1}\nabla f(x)$.
- (9) Explain the general idea of a Quasi-Newton algorithm.