PPSN 2016

Tutorial on Evolutionary Multiobjective Optimization

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updated slides will be available at
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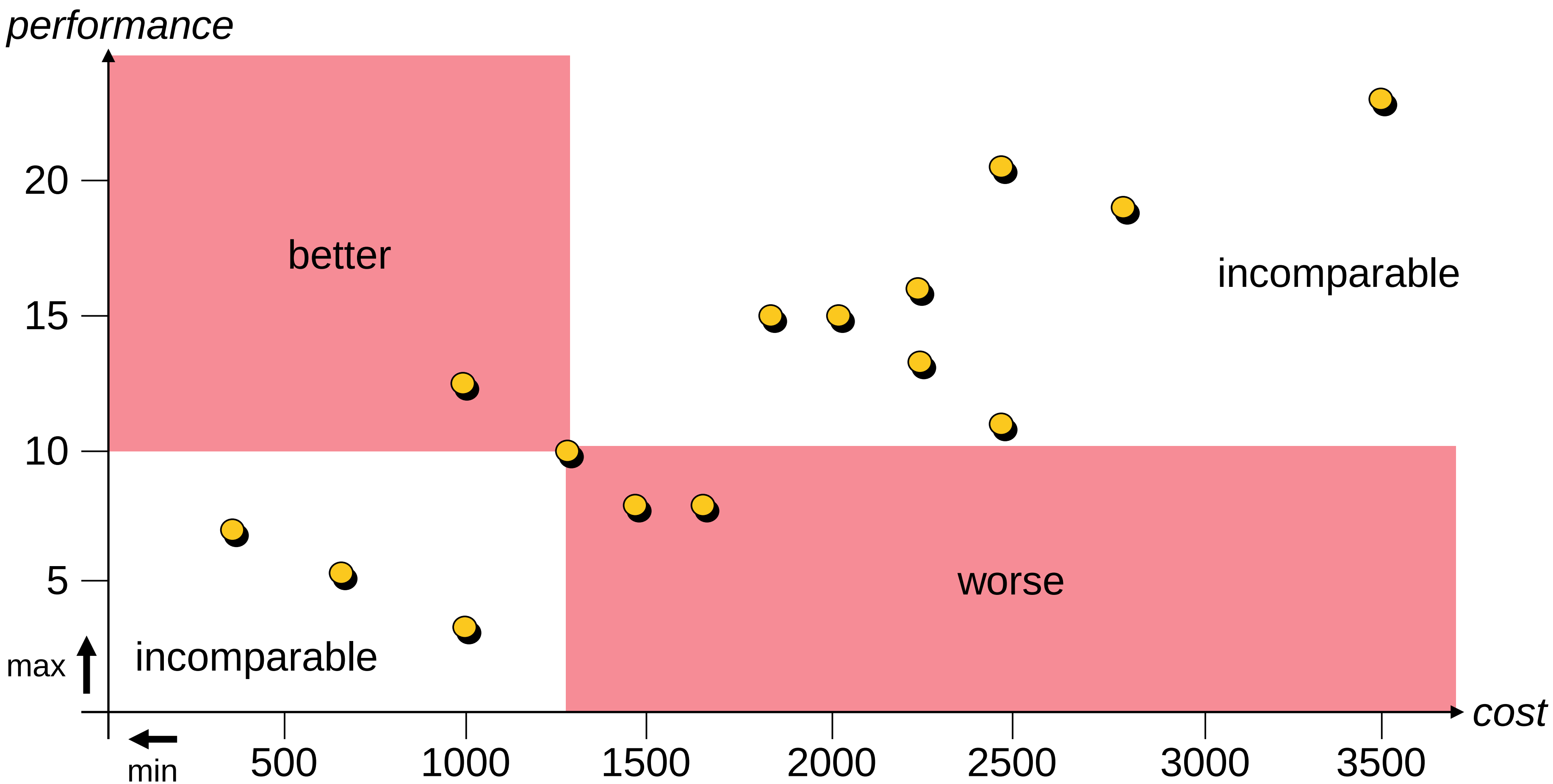
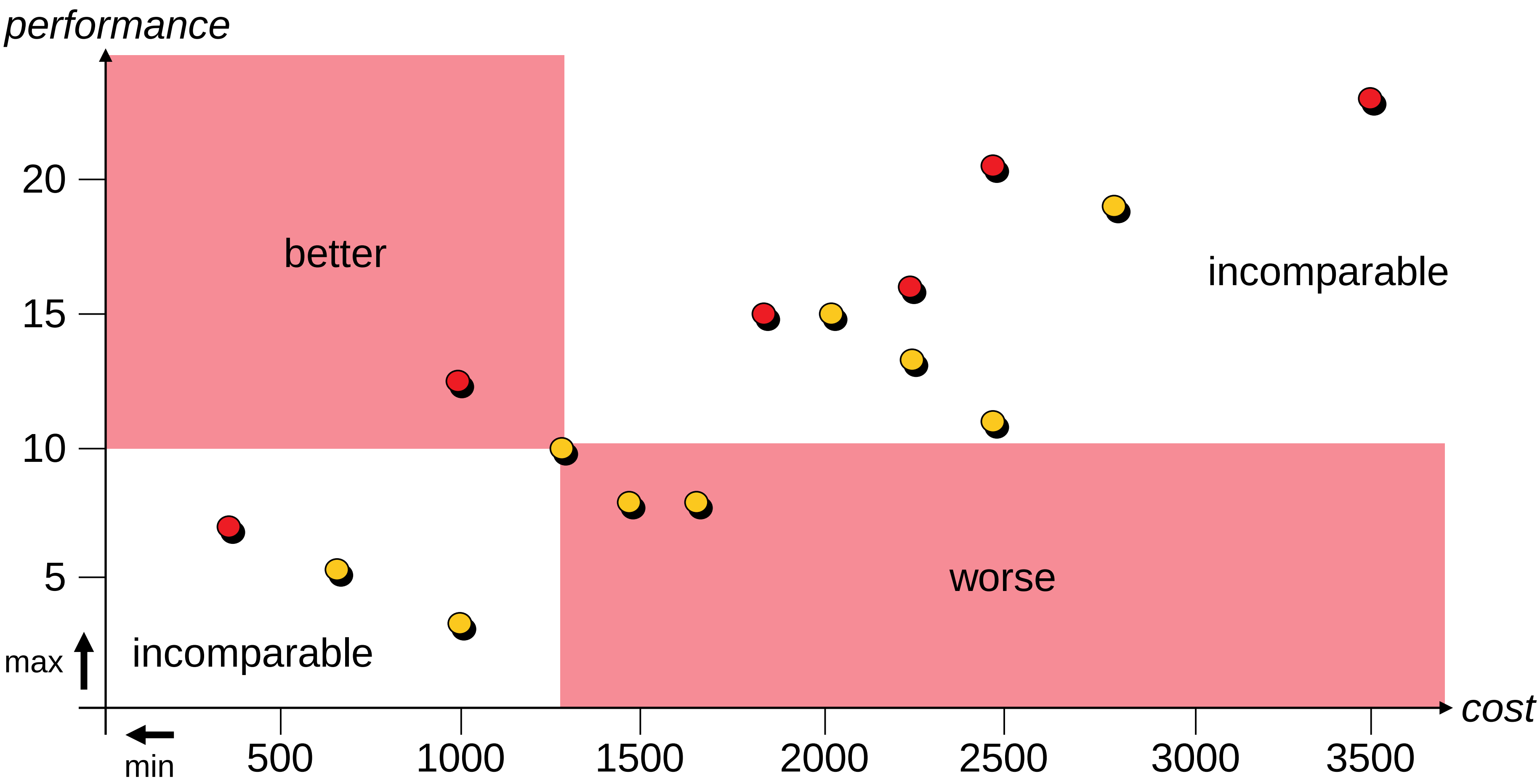
Multiobjective Optimization

Multiple objectives that have to be optimized simultaneously



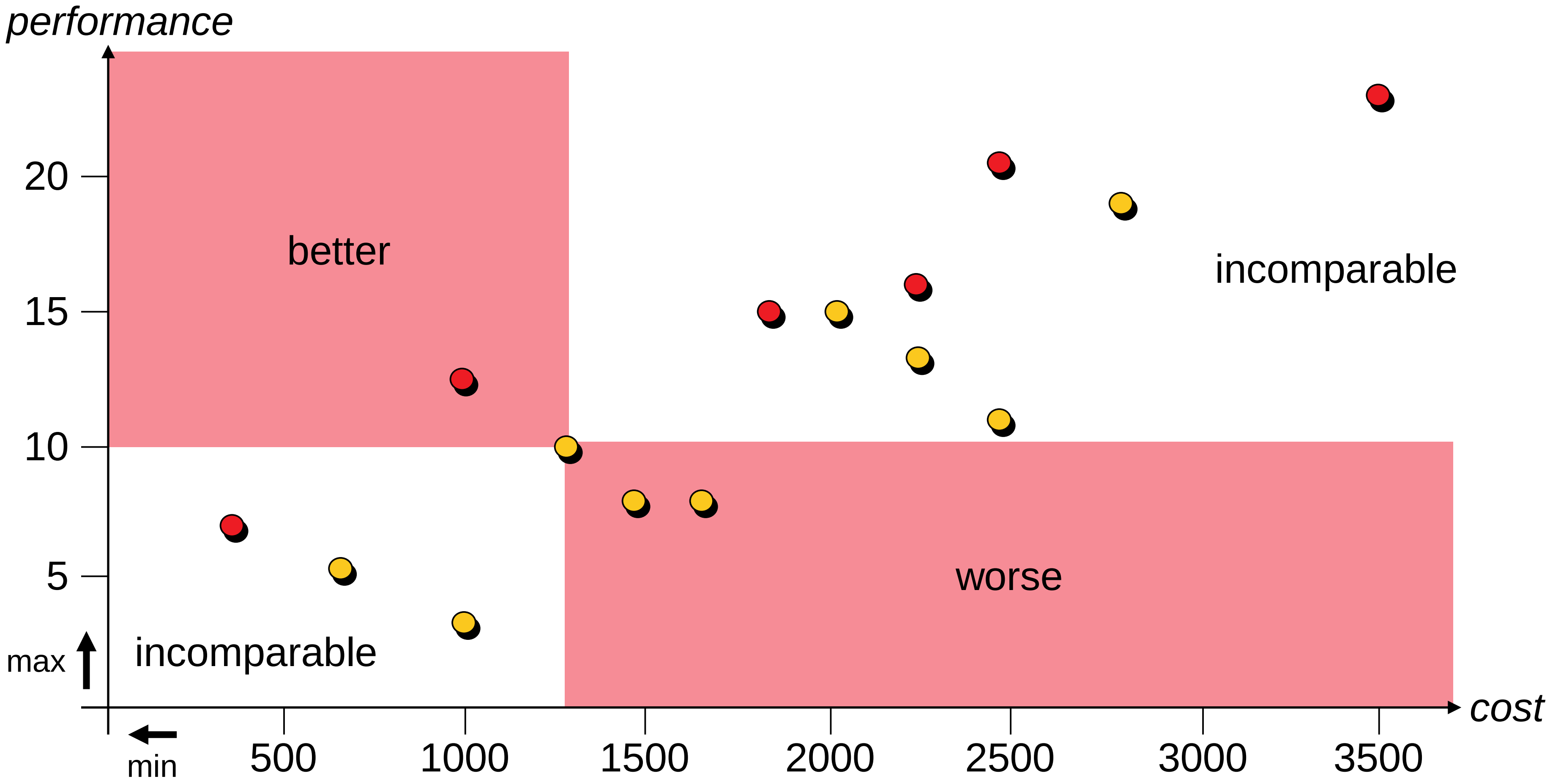
A Brief Introduction to Multiobjective Optimization

Observations: ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)



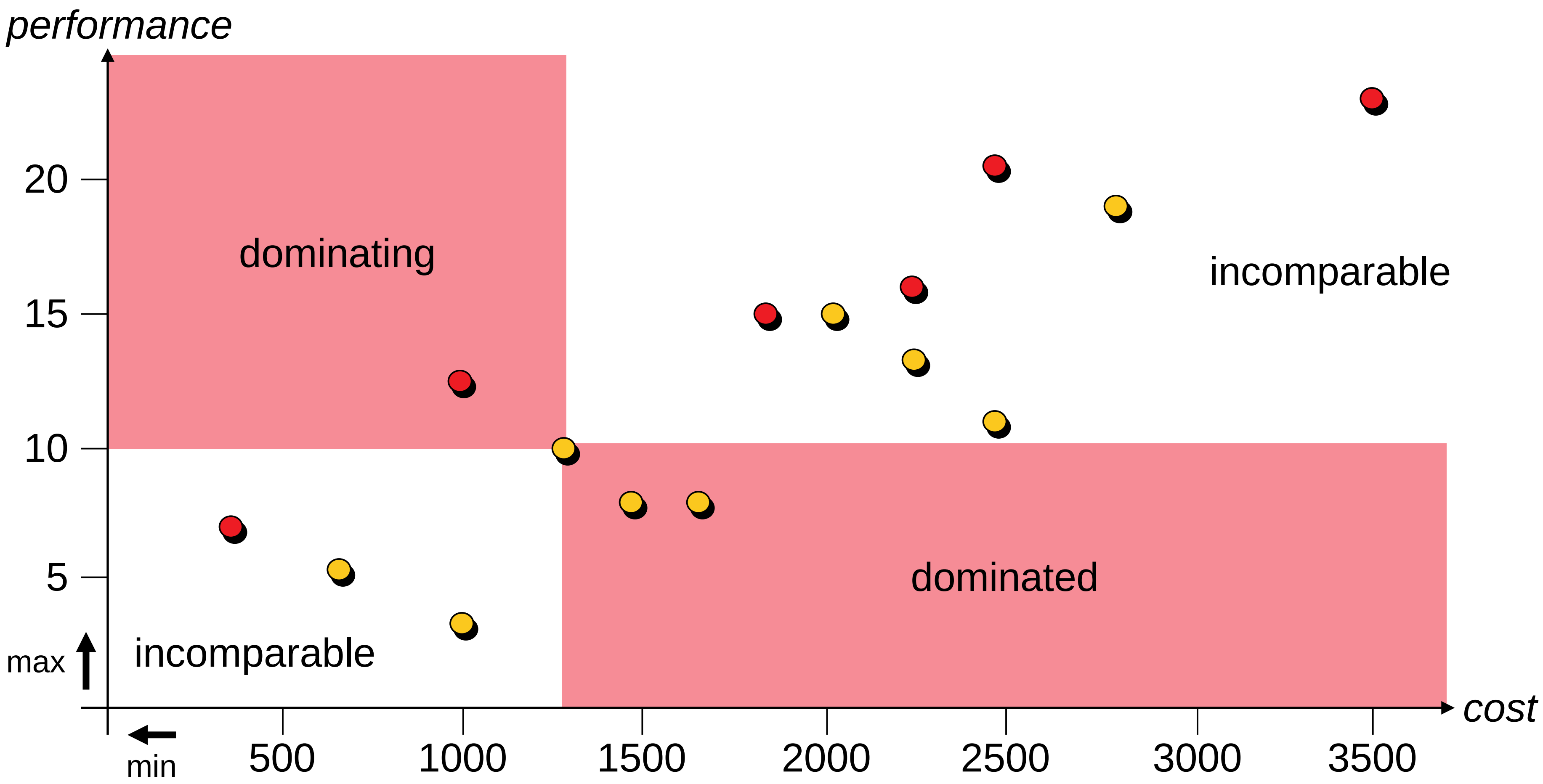
$$u \text{ weakly Pareto dominates } v \ (u \preccurlyeq_{par} v): \quad \forall 1 \leq i \leq k : f_i(u) \leq f_i(v)$$

$$u \text{ Pareto dominates } v \ (u \prec_{par} v): \quad u \preccurlyeq_{par} v \ \wedge \ v \not\preccurlyeq_{par} u$$

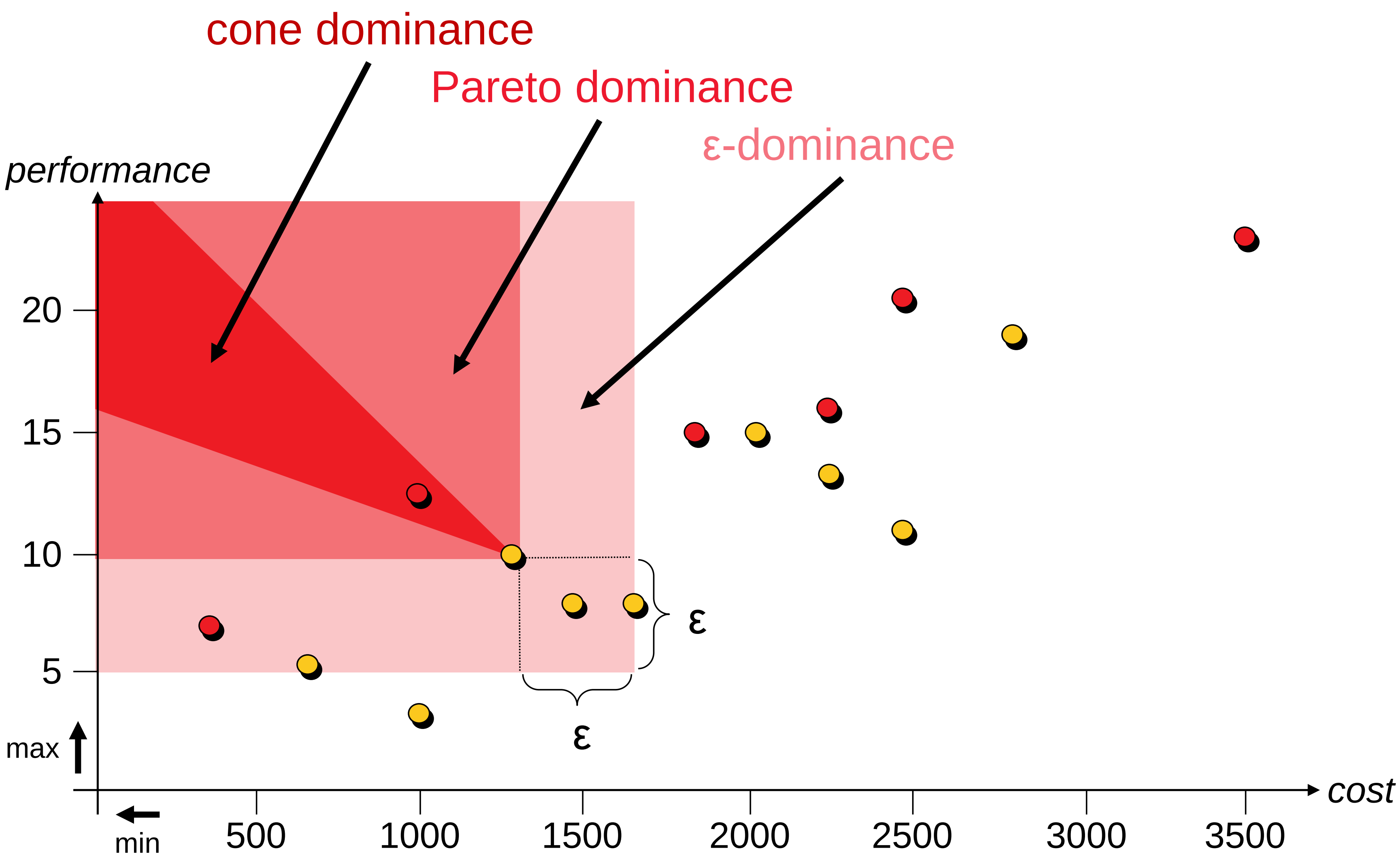


$$u \ \textit{weakly Pareto dominates } v \ (u \leqslant_{par} v): \quad \forall 1 \leq i \leq k : f_i(u) \leq f_i(v)$$

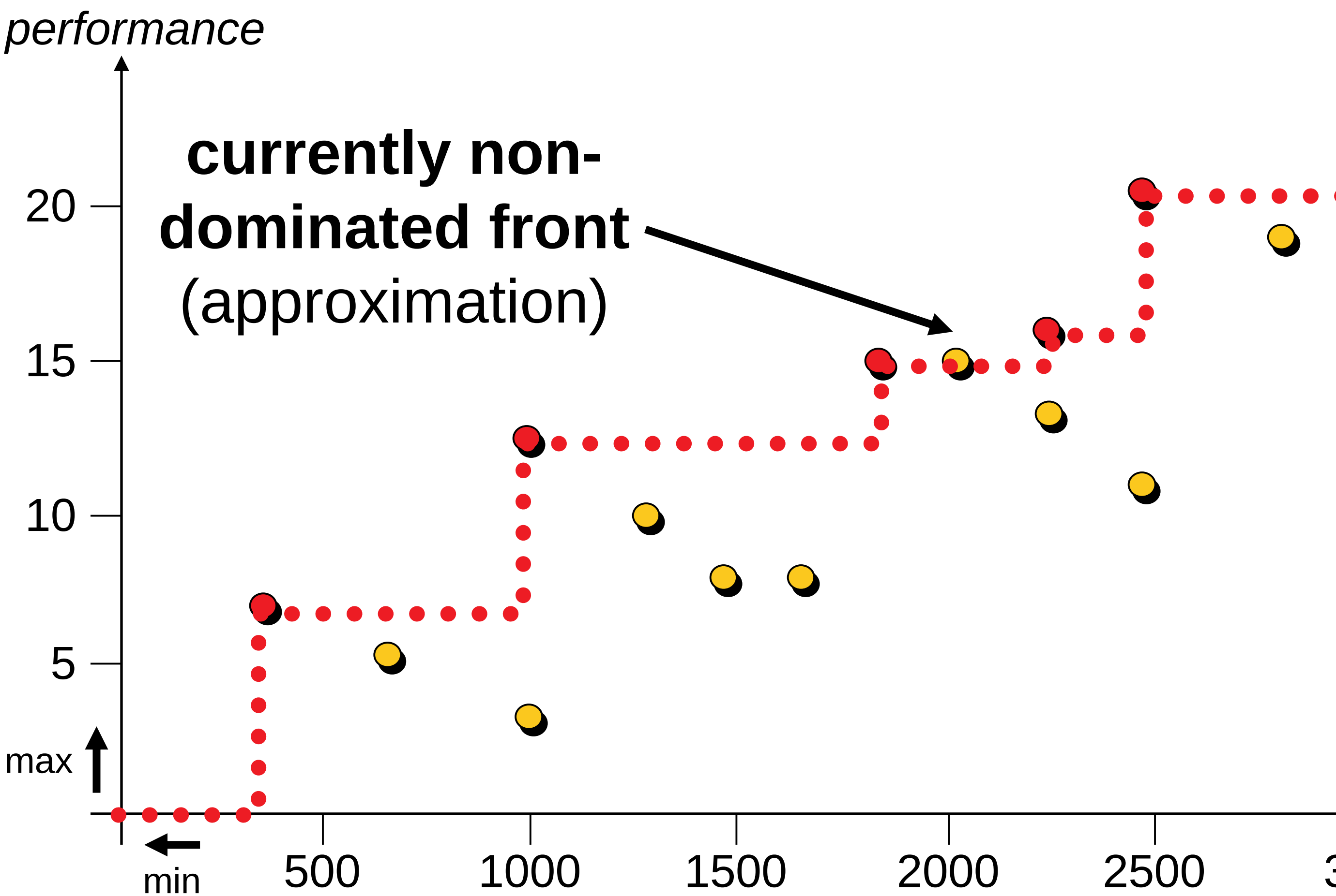
$$u \ \textit{Pareto dominates } v \ (u <_{par} v): \quad u \leqslant_{par} v \ \wedge \ v \nleqslant_{par} u$$

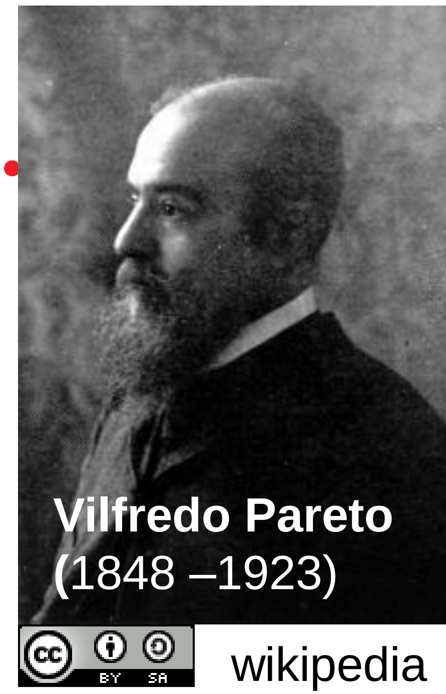


A Brief Introduction to Multiobjective Optimization



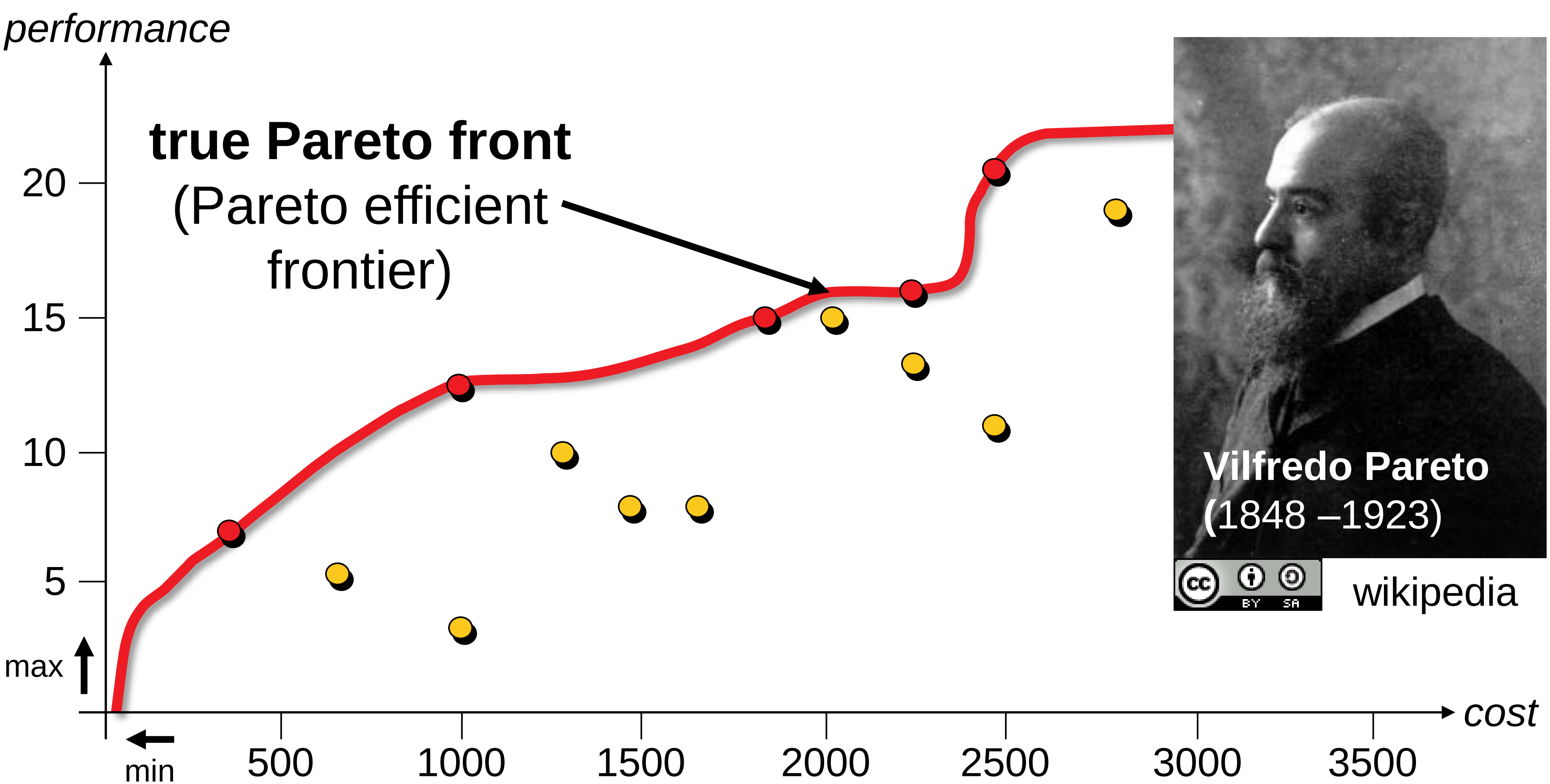
Pareto set: set of all non-dominated solutions (decision space)
Pareto front: its image in the objective space

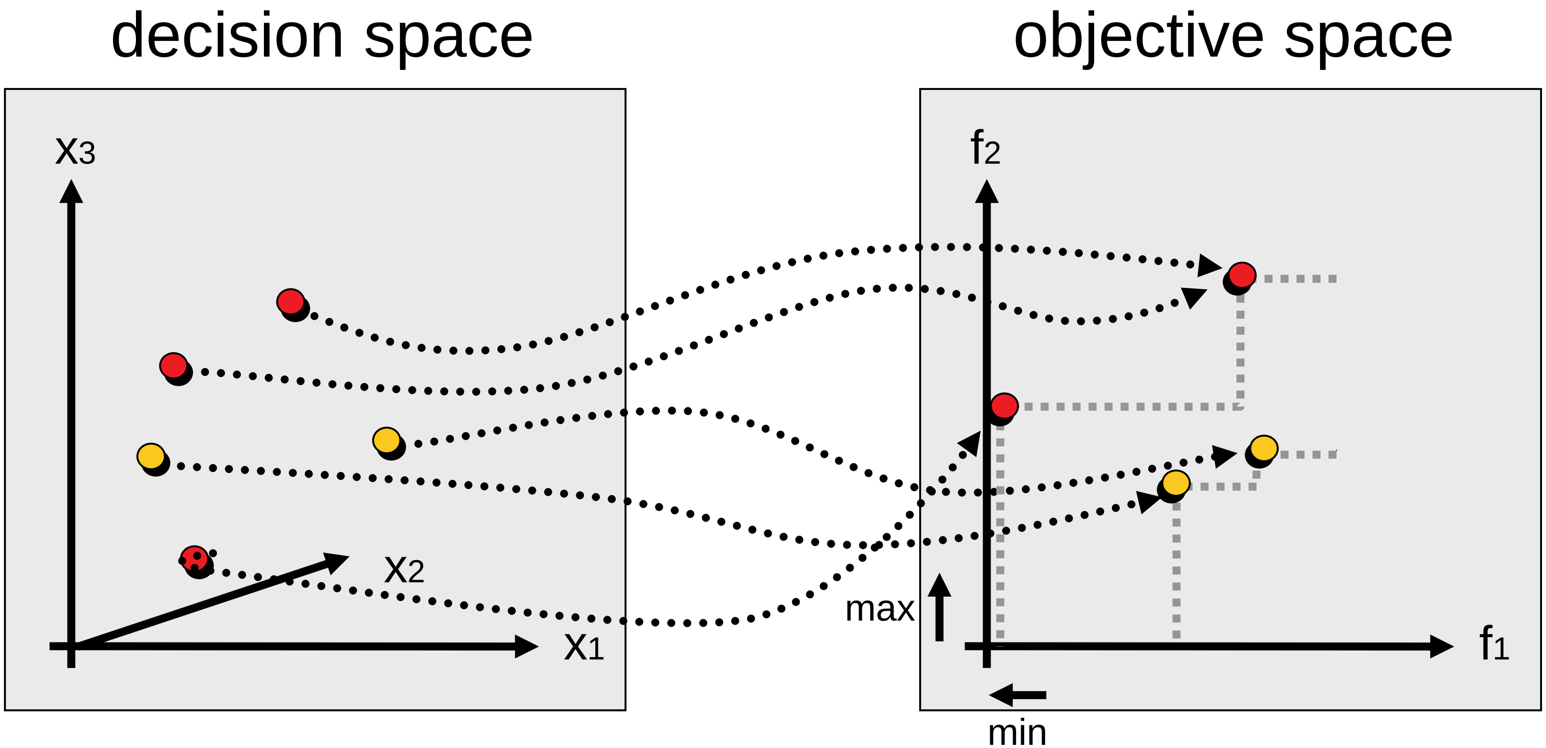




wikipedia

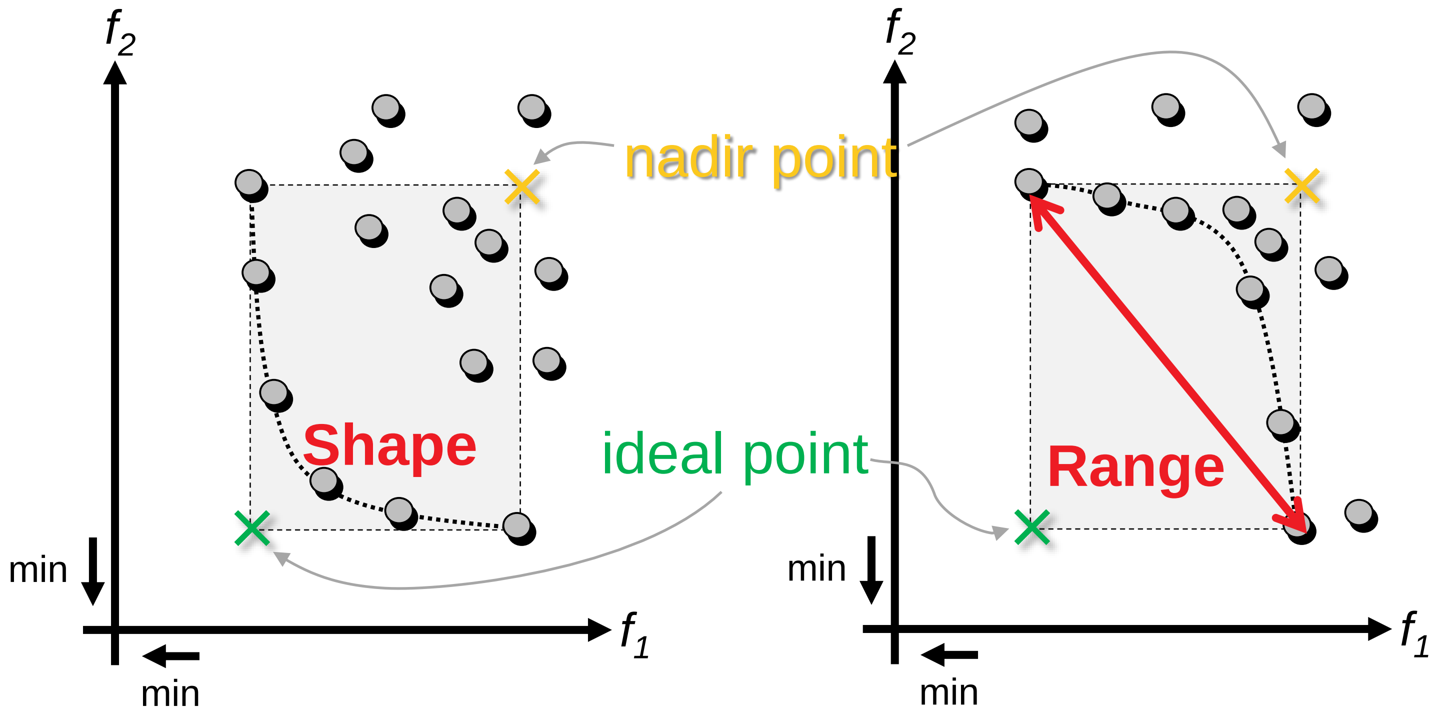
Pareto set: set of all non-dominated solutions (decision space)
Pareto front: its image in the objective space





solution of Pareto-optimal set — vector of Pareto-optimal front

non-optimal decision vector — non-optimal objective vector



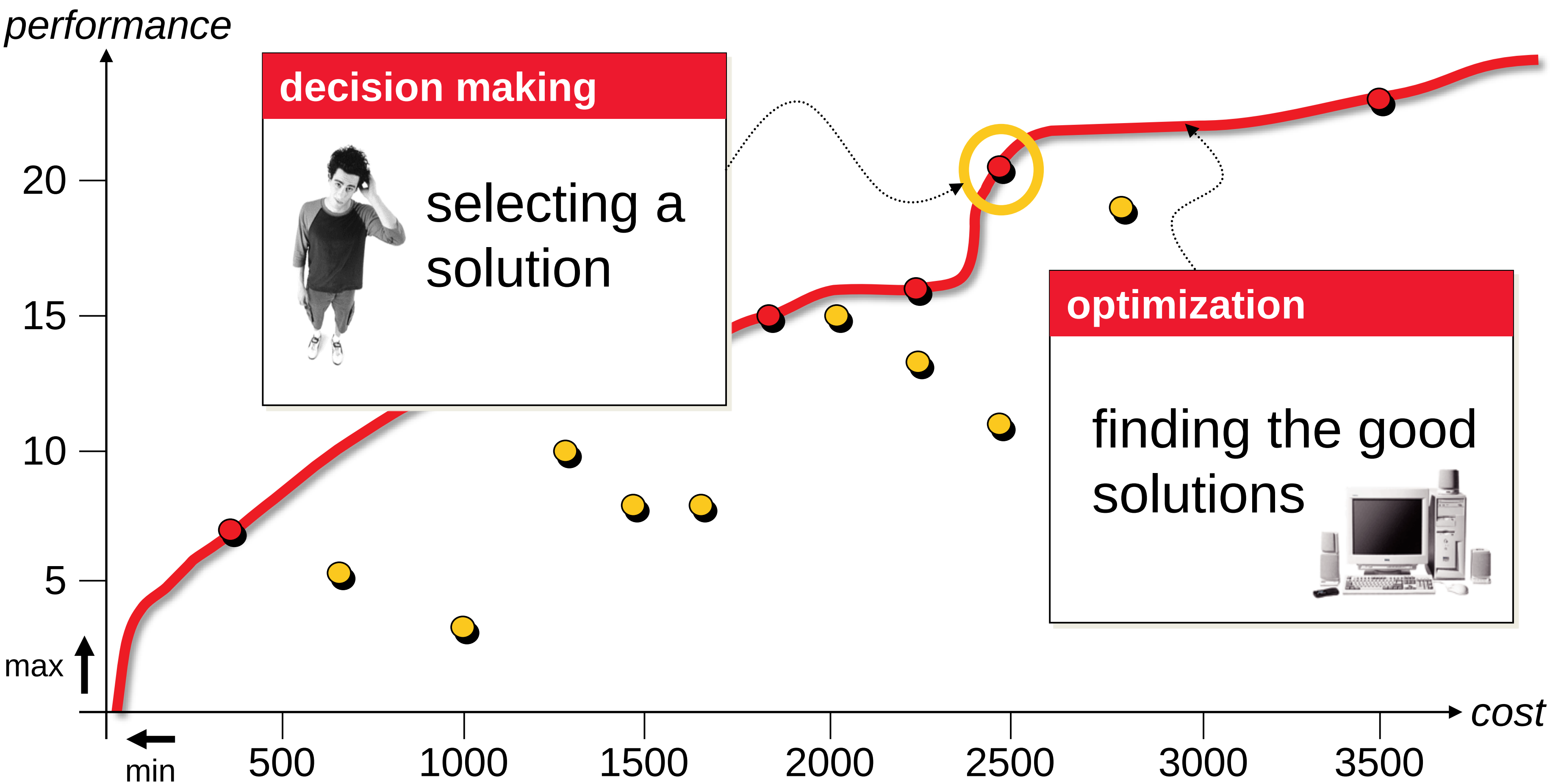
ideal point: best values
nadir point: worst values

} obtained for *Pareto-optimal* points

Optimization vs. Decision Making

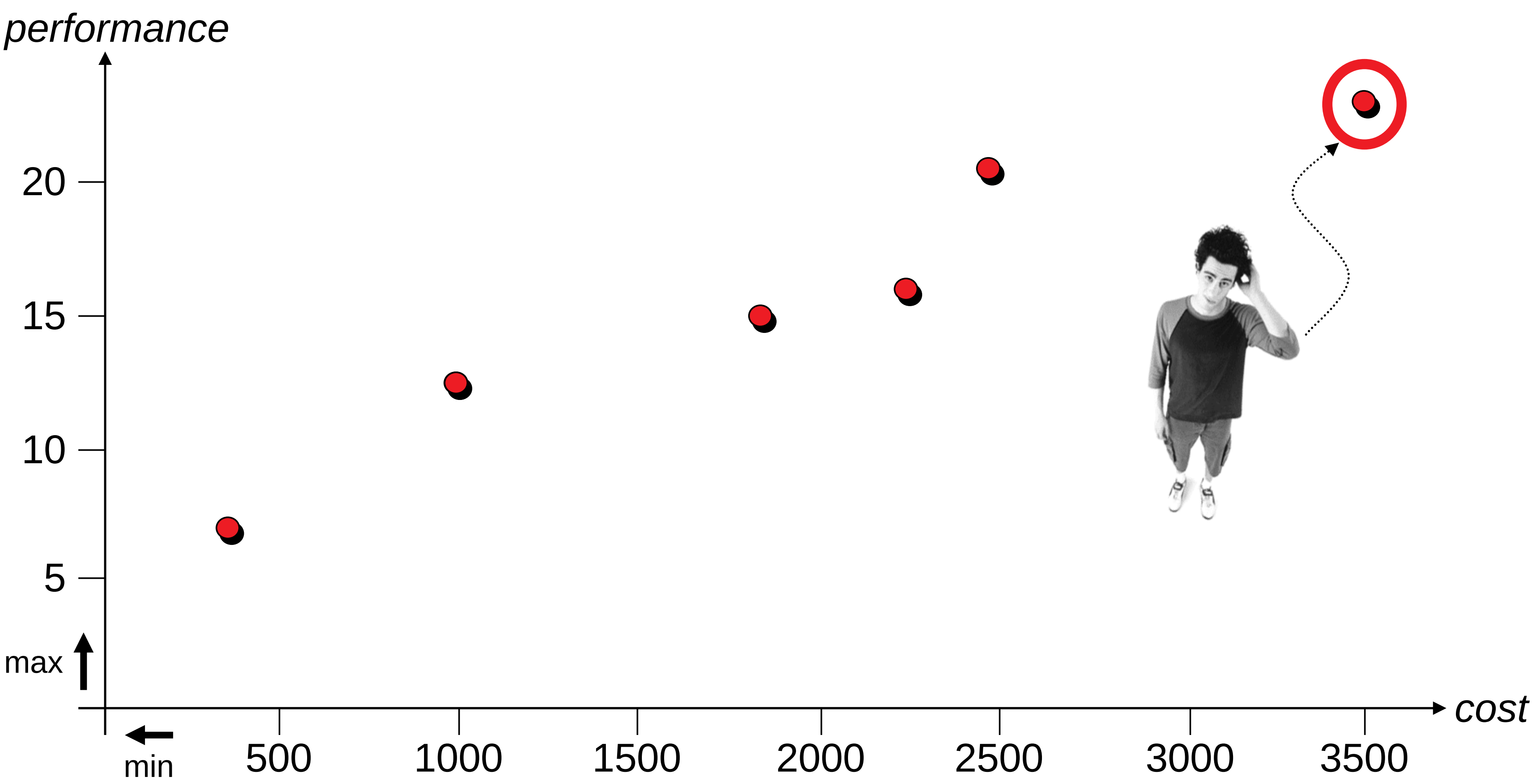
Multiobjective Optimization

combination of optimization of a set and a decision for a solution



Possible Approaches:

➊ **ranking:** performance more important than cost

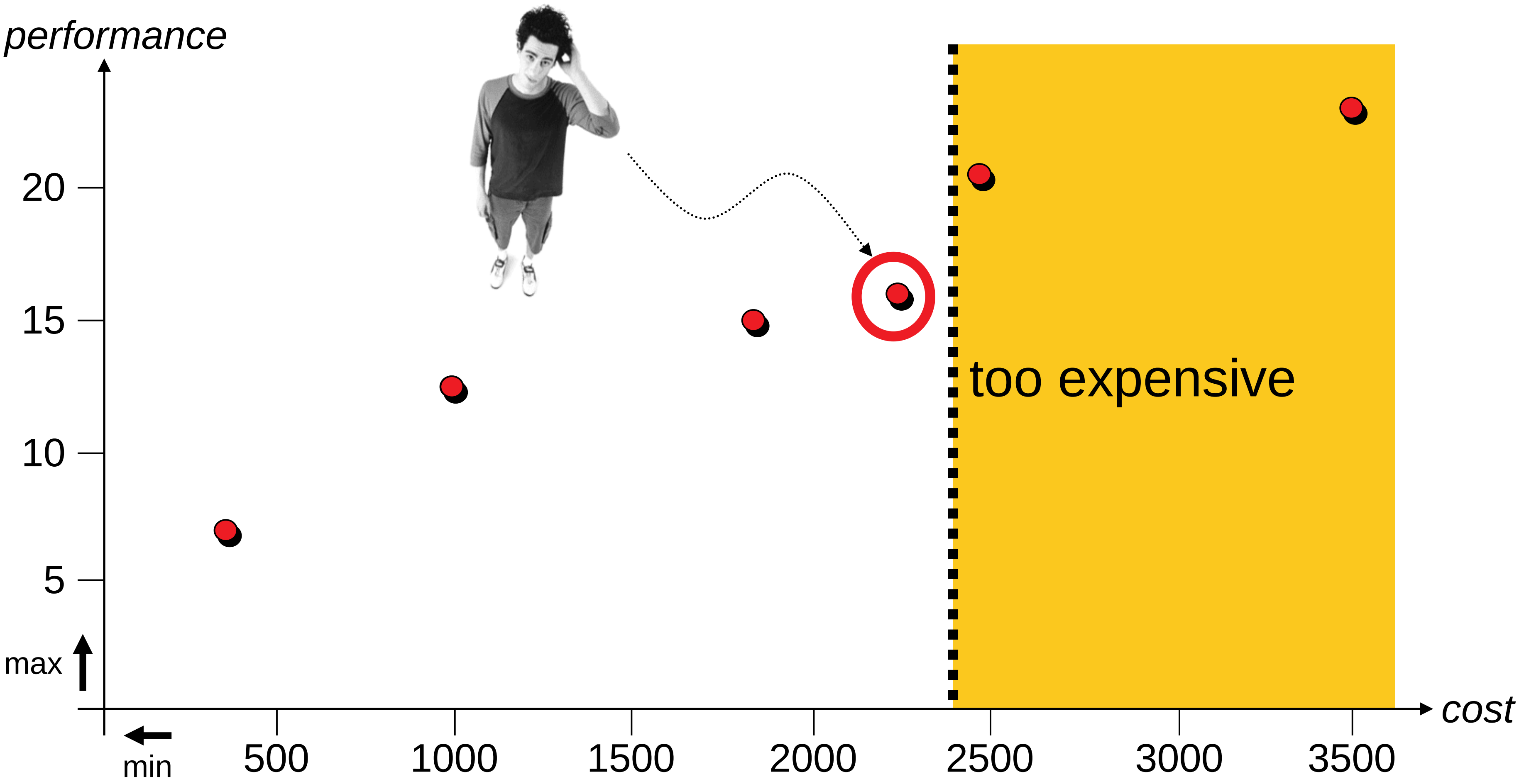


Selecting a Solution: Examples

Possible Approaches:

❶ ranking: performance more important than cost

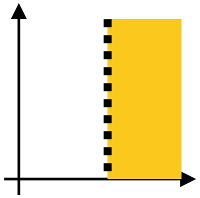
❷ **constraints:** cost must not exceed 2400



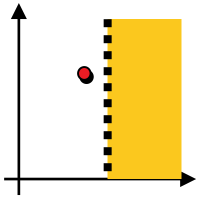
When to Make the Decision

Before Optimization:

rank objectives,
define constraints,…



search for one
(good) solution



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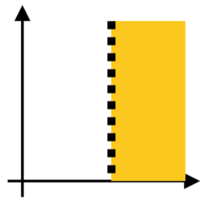

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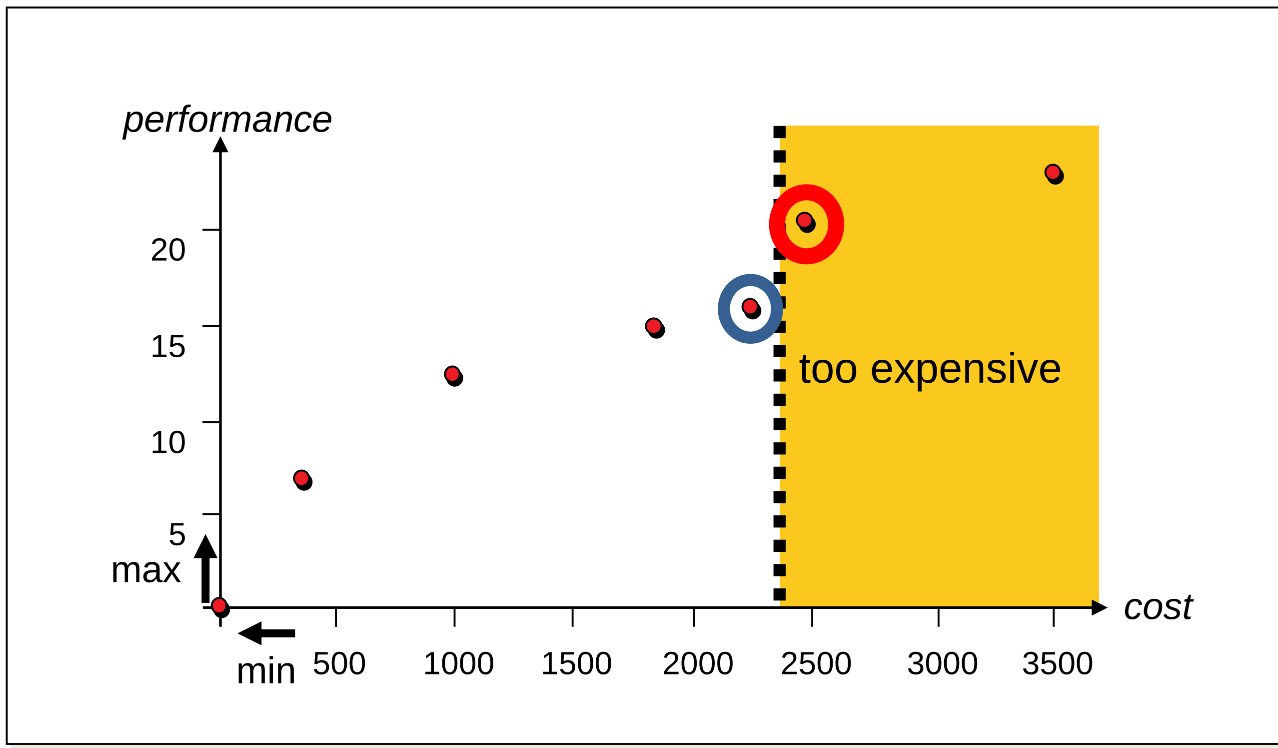

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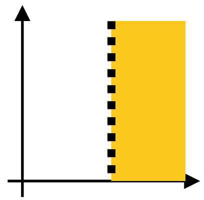

search for one
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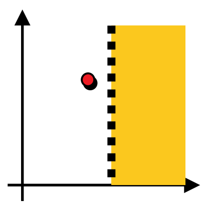
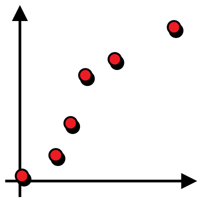
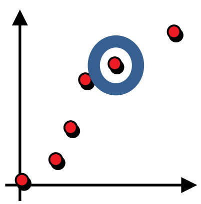


After Optimization:

search for a set of
(good) solutions



select one solution
considering
constraints, etc.



When to Make the Decision

Before Optimization:

rank objectives,
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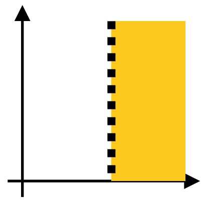

search for one
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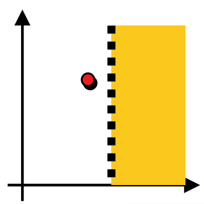

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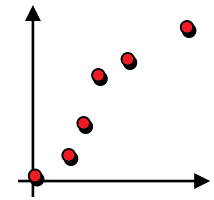

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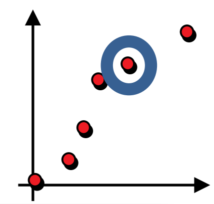


Focus: learning about a problem

- trade-off surface
- interactions among criteria
- structural information
- also: interactive optimization

Two Communities...

MCDM International Society on Multiple Criteria Decision Making

- established field (beginning in 1950s/1960s)
- bi-annual conferences since 1975
- background in economics, math, management and social sciences
- focus on optimization and decision making

emo

- quite young field (first papers in mid 1980s)
- bi-annual conference since 2001
- background in computer science, applied math and engineering
- focus on optimization algorithms

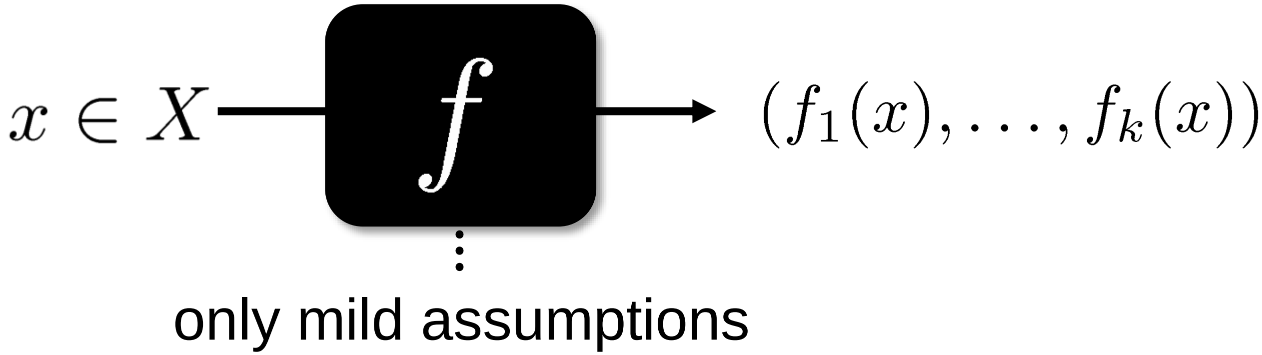
...Slowly Merge Into One

MCDM International Society on Multiple Criteria Decision Making

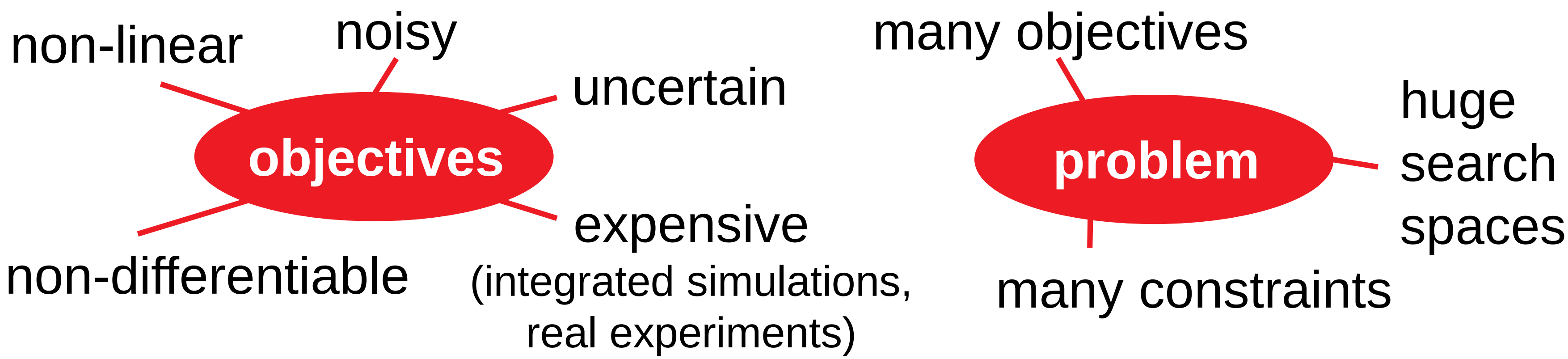
emo

- MCDM track at EMO conference since 2009
- special sessions on EMO at the MCDM conference since 2008
- joint Dagstuhl seminars since 2004

Blackbox optimization



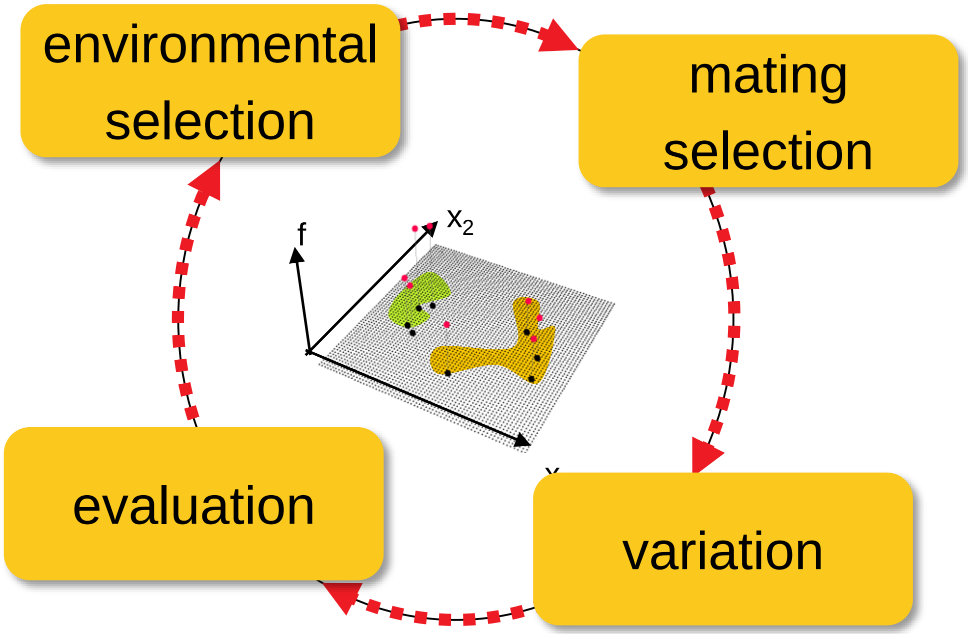
→ EMO therefore well-suited for real-world engineering problems

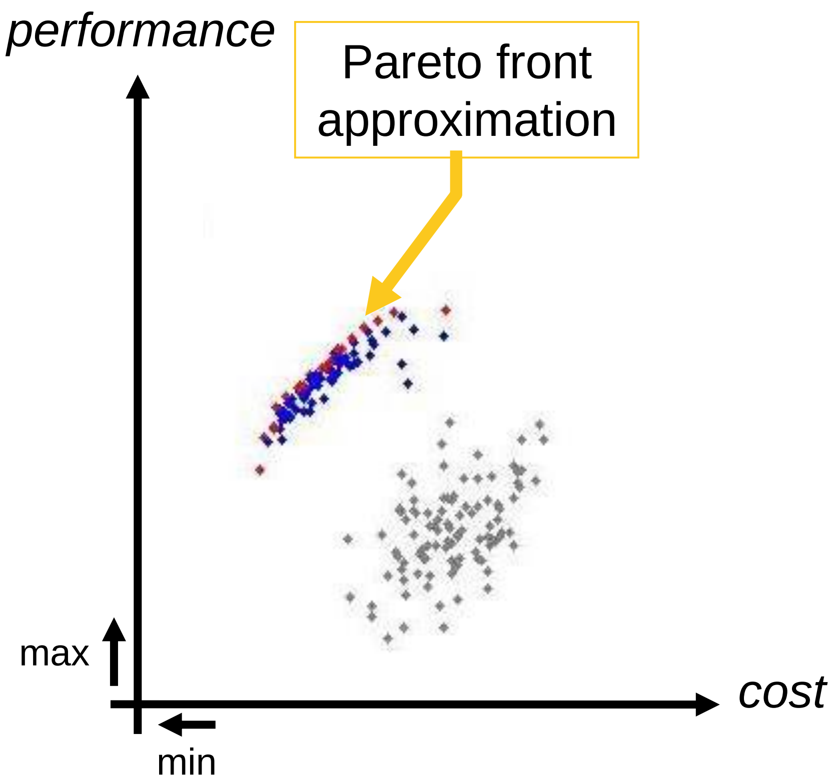


The Other Main Difference

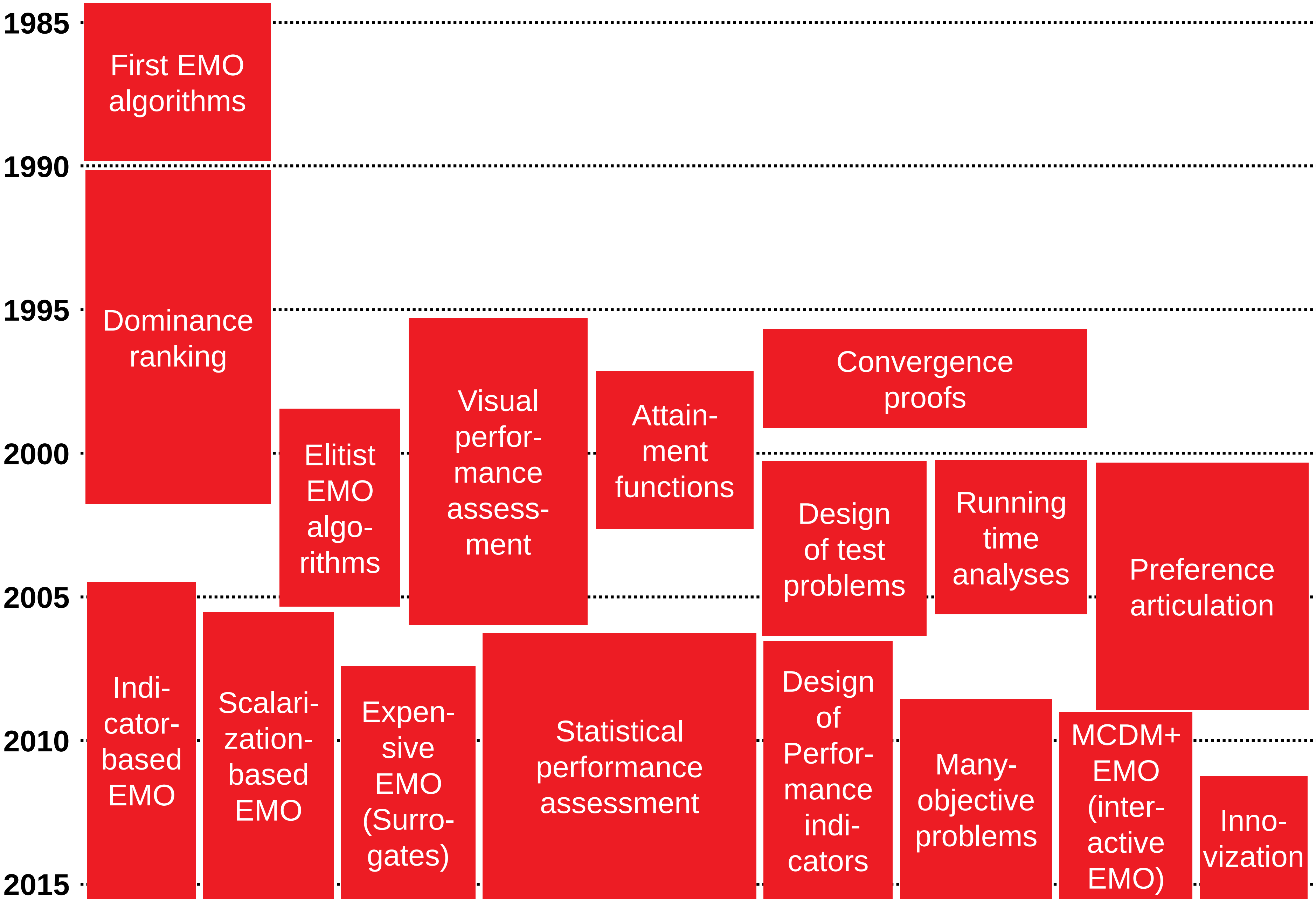
Evolutionary Multiobjective Optimization

- set-based algorithms
- therefore possible to approximate the Pareto front in one run





The History of EMO At A Glance



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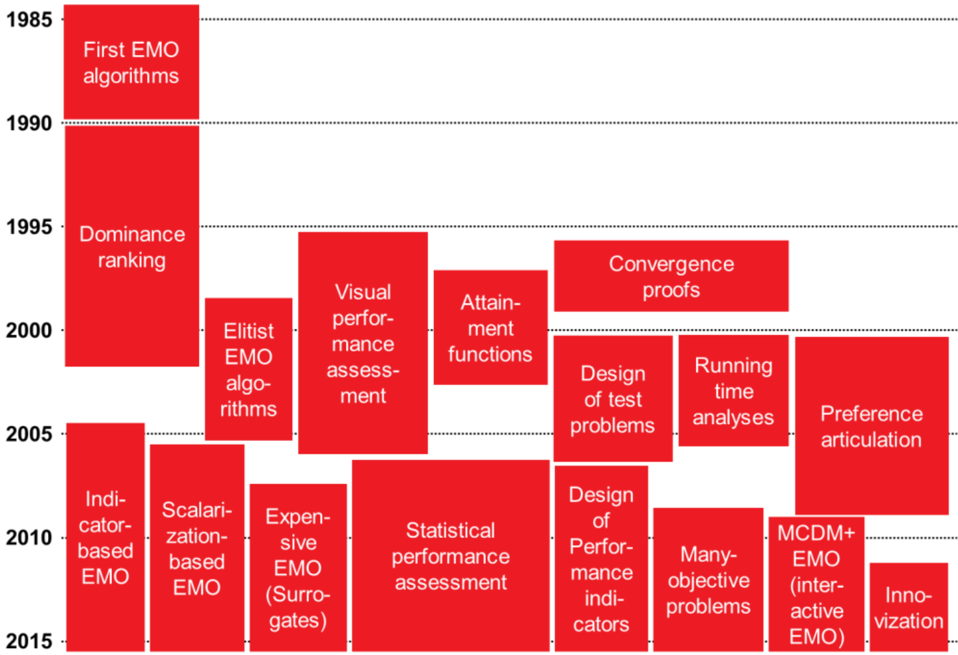
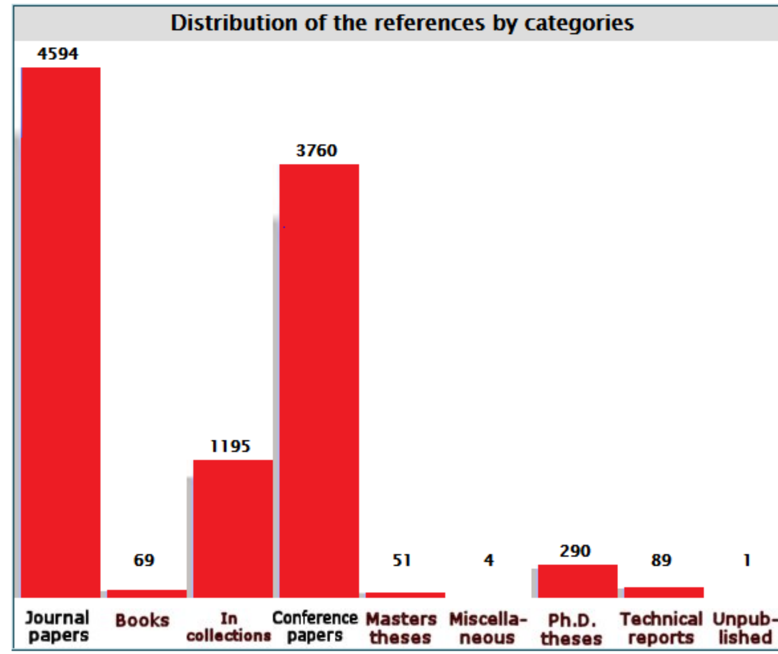
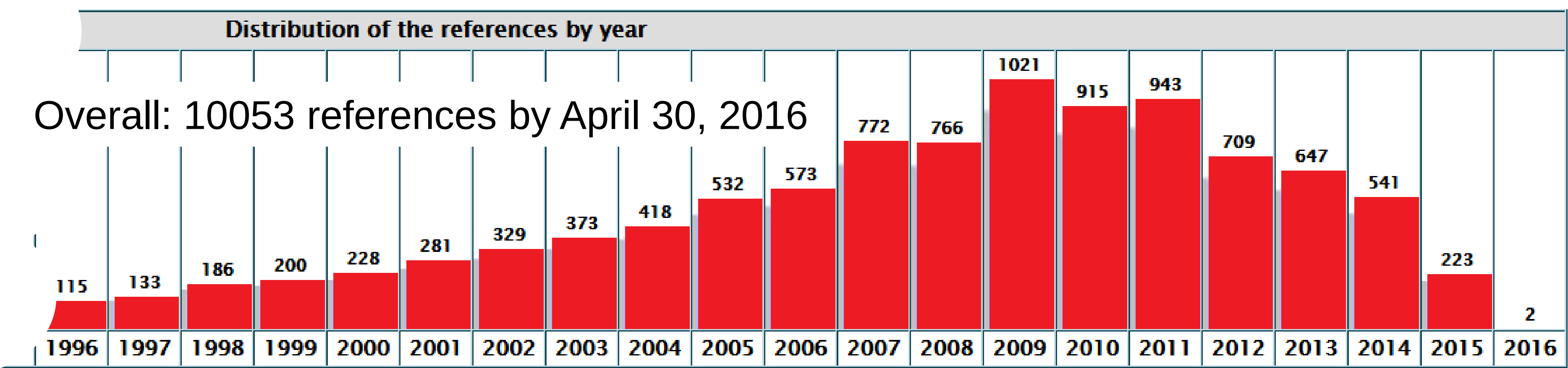




http://delta.cs.cinvestav.mx/~ccoello/EMOO



Overall: 10053 references by April 30, 2016 *from Google maps*

The EMO conference series:

Many further activities:

special sessions, special journal issues, workshops, tutorials, ...

Overview

The Big Picture

Algorithm Design Principles and Concepts

Performance Assessment

Fitness Assignment: Principal Approaches

aggregation-based

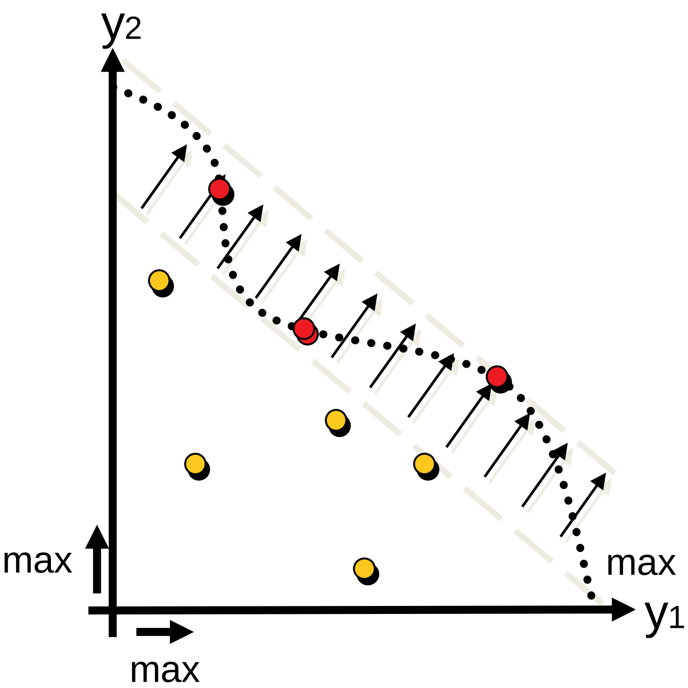
problem decomposition (multiple single-objective optimization problems)

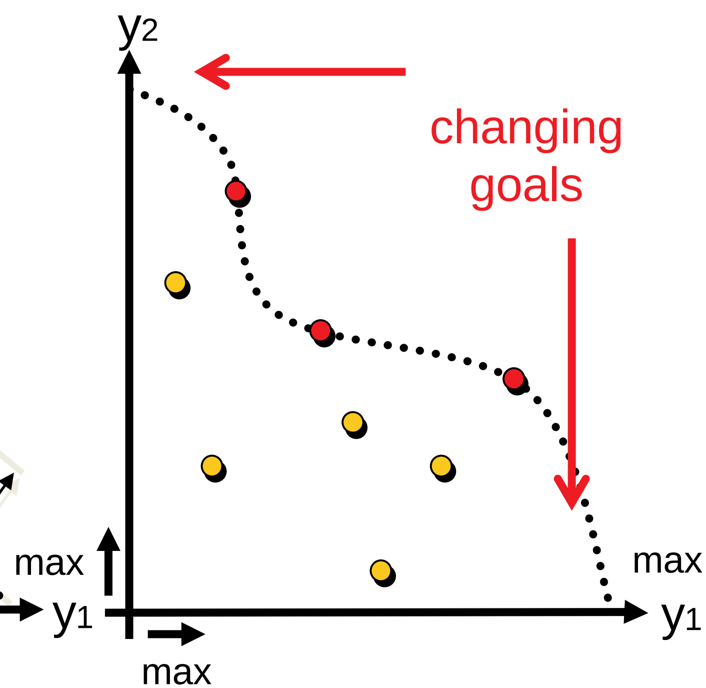
criterion-based

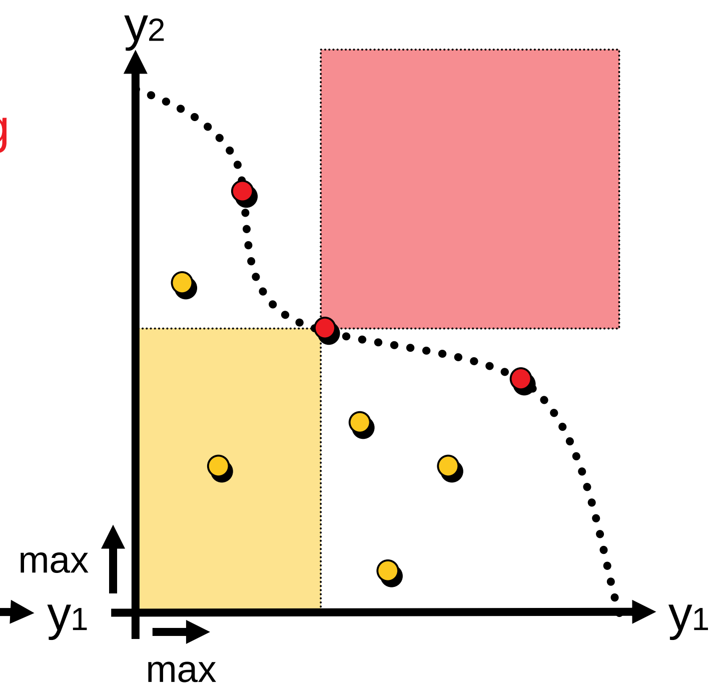
VEGA

dominance-based

SPEA2, NSGA-II
"modern" EMOA



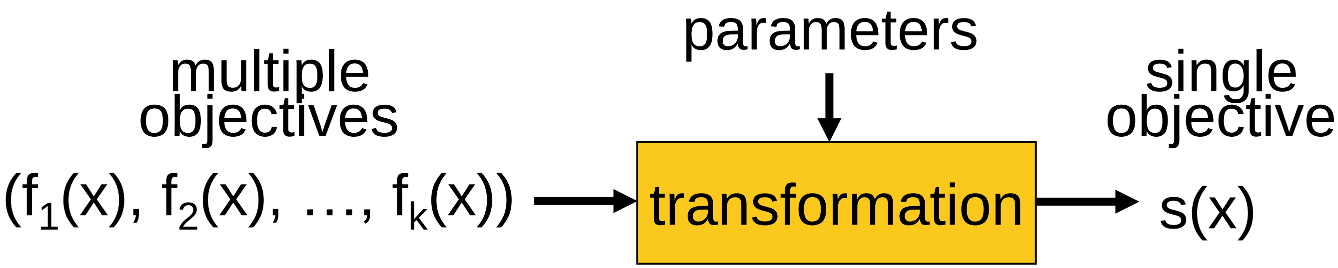




solution-oriented
scaling-dependent

set-oriented
scaling-independent

Solution-Oriented Problem Transformations



A scalarizing function s is a function $s: Z \to \mathbb{R}$ that maps each objective vector $u = (u_1, \ldots, u_n) \in Z$ to a real value $s(u) \in \mathbb{R}$

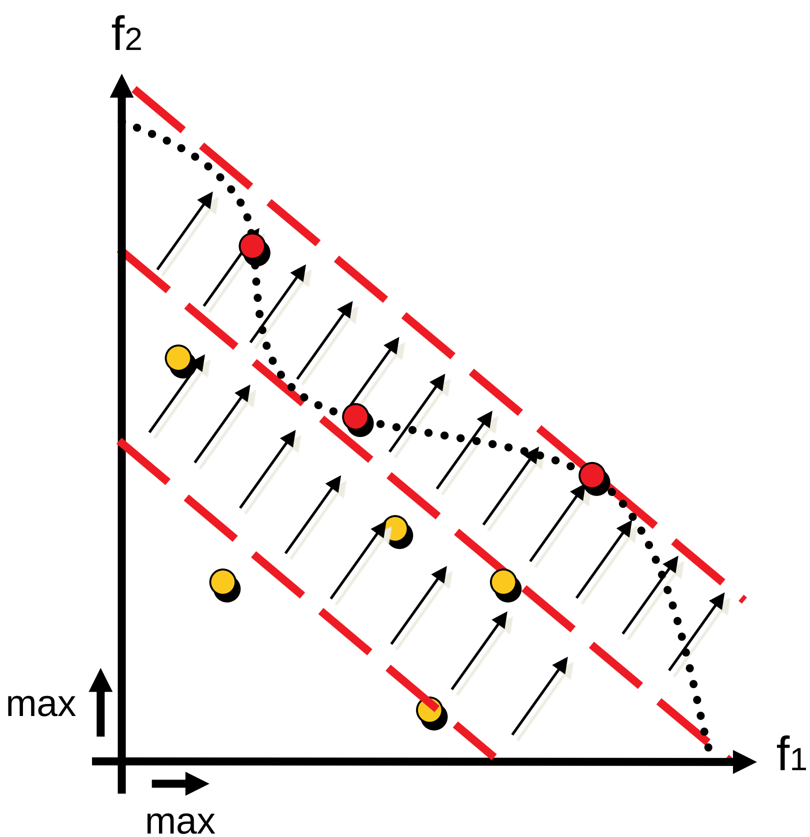
Solution-Oriented Problem Transformations

parameters

multiple objectives

$(f_1(x), f_2(x), \ldots, f_k(x))$ → transformation → $s(x)$

single objective



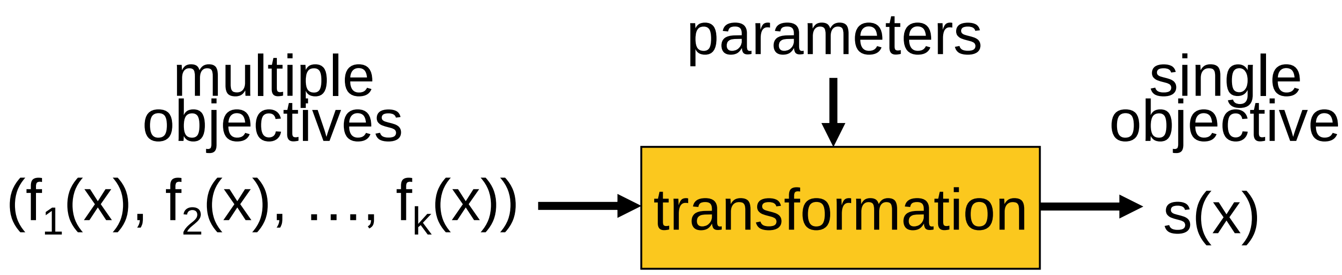
Example 1: weighted sum approach

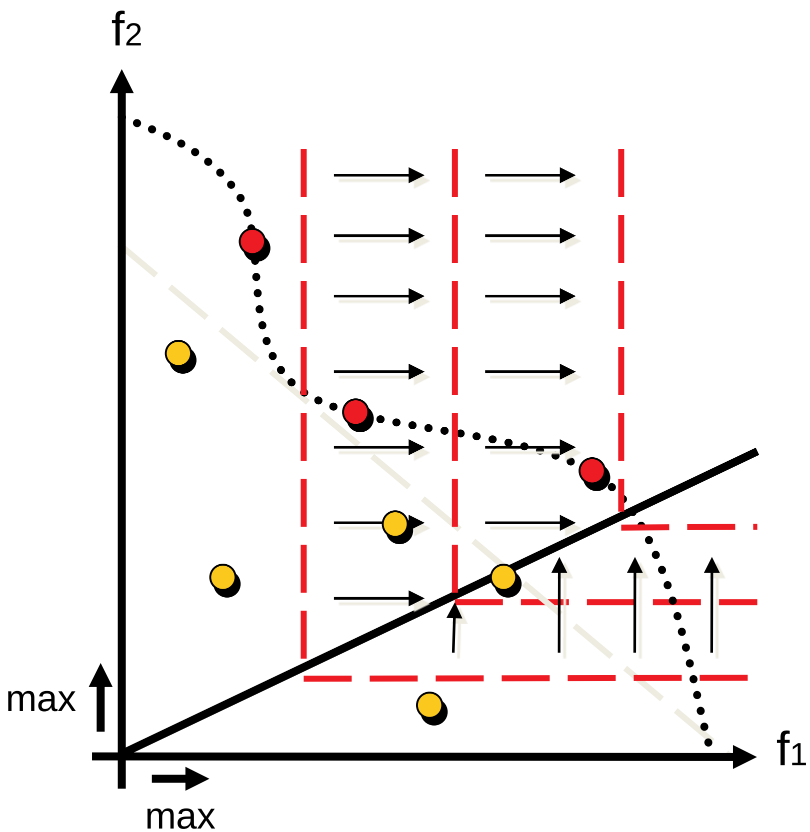
$(w_1, w_2, \ldots, w_k)$

$y = w_1 y_1 + \ldots + w_k y_k$

Disadvantage: not all Pareto-optimal solutions can be found if the front is not convex

Solution-Oriented Problem Transformations





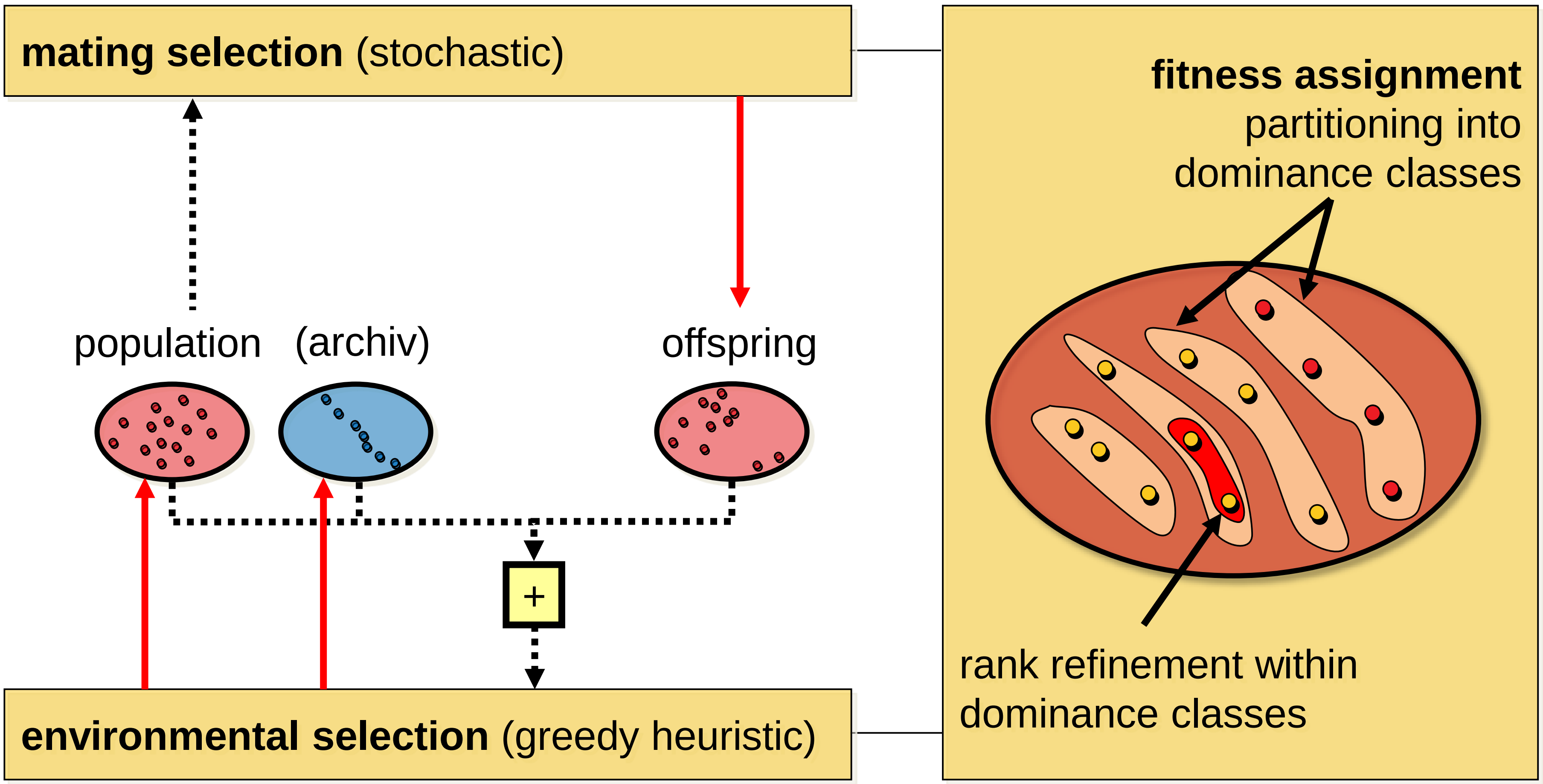
Example 2: weighted Tchebycheff

$(\lambda_1, \lambda_2, \ldots, \lambda_k)$

$$y = \max_i |\lambda_i(u_i - z_i)|$$

Several other scalarizing functions are known, see e.g. [Miettinen 1999]

General Scheme of Most Set-Oriented EMO



Ranking of the Population Using Dominance

... goes back to a proposal by David Goldberg in 1989.

... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  MOGA, NPGA
- **dominance count:** how many individuals does an individual dominate?
  SPEA, SPEA2
- **dominance depth:** at which front is an individual located?
  NSGA, NSGA-II, most of the recently proposed algorithms

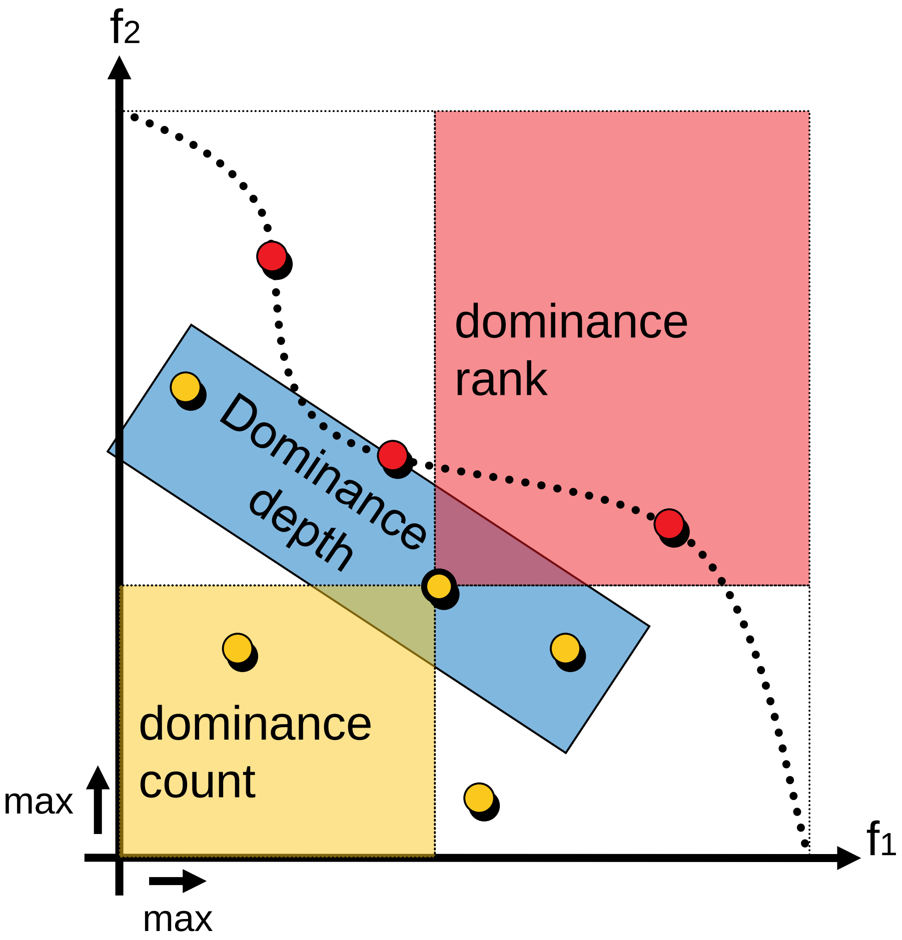
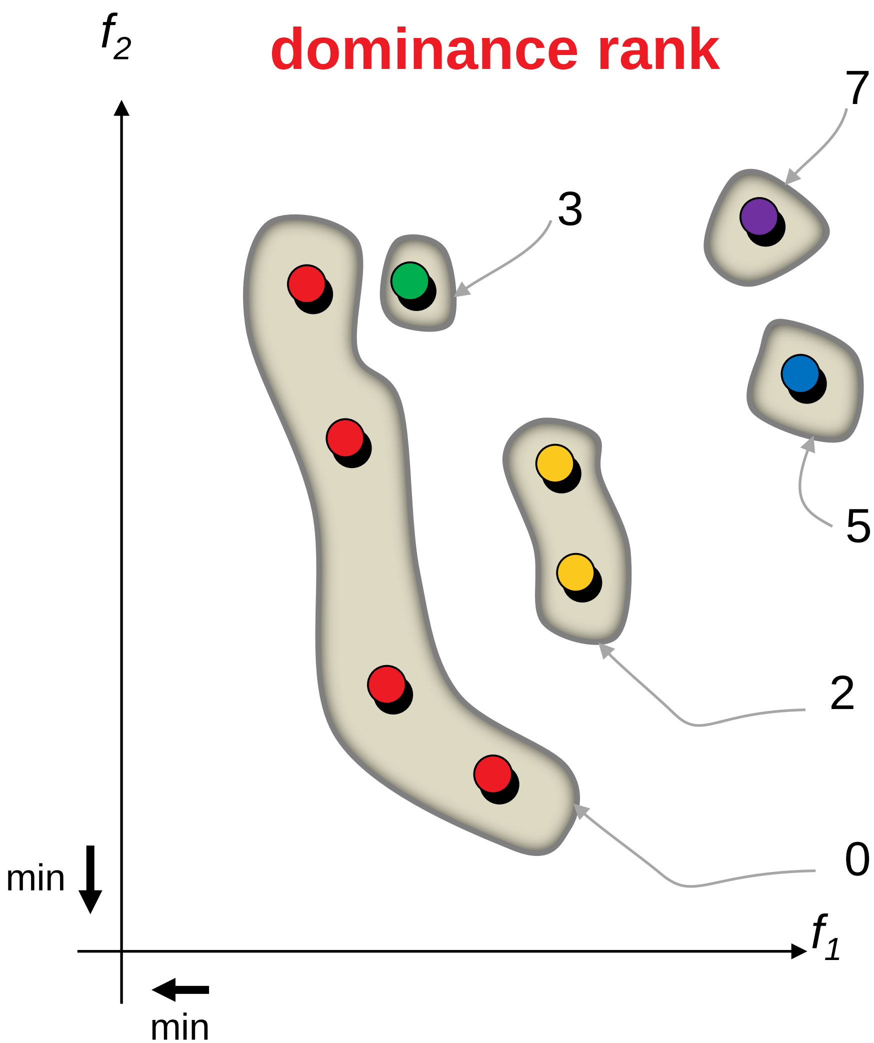
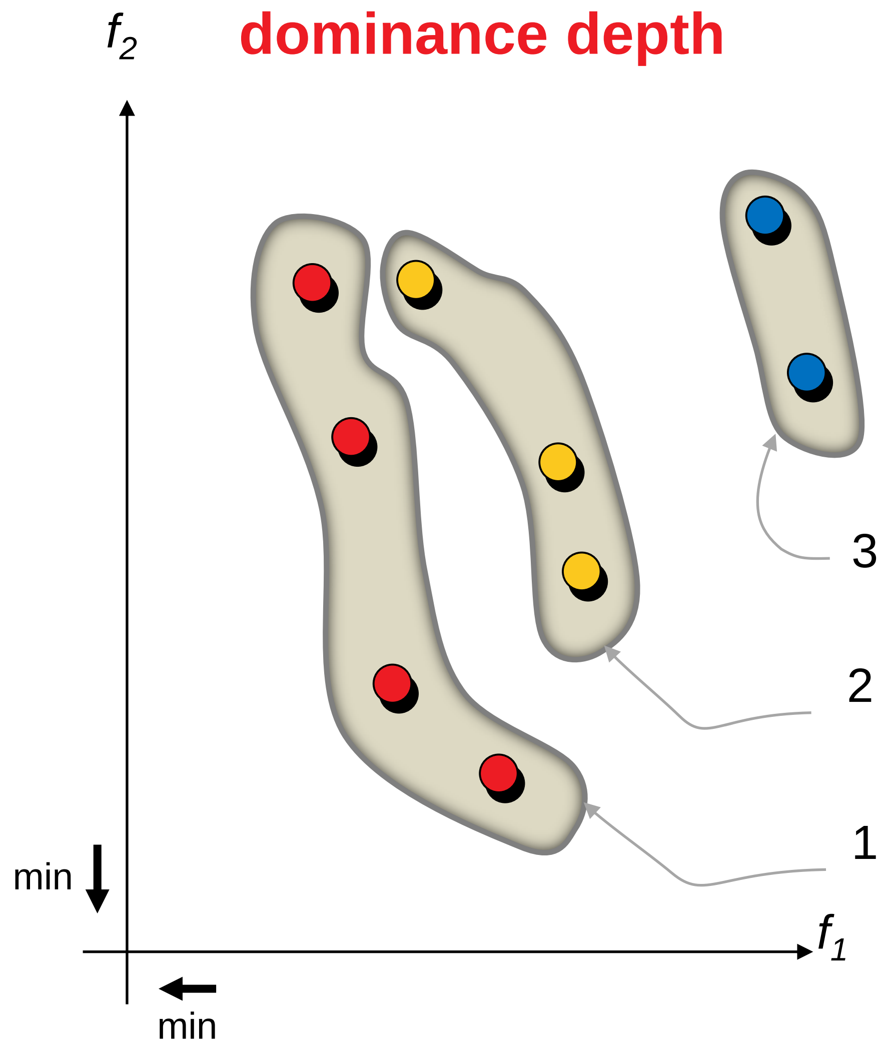


Illustration of Dominance-Based Partitioning





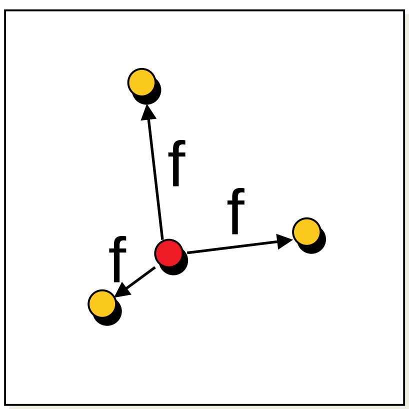
Refinement of Dominance Rankings

Goal: rank incomparable solutions within a dominance class

❶ Diversity information

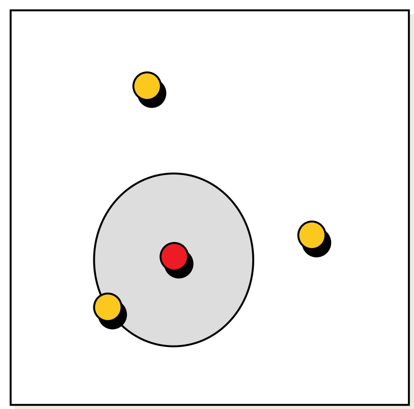
Kernel method

diversity =
function of the
distances



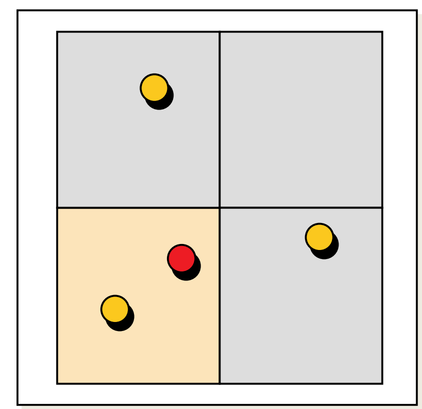
k-th nearest neighbor

diversity =
function of distance
to k-th nearest neighbor



Histogram method

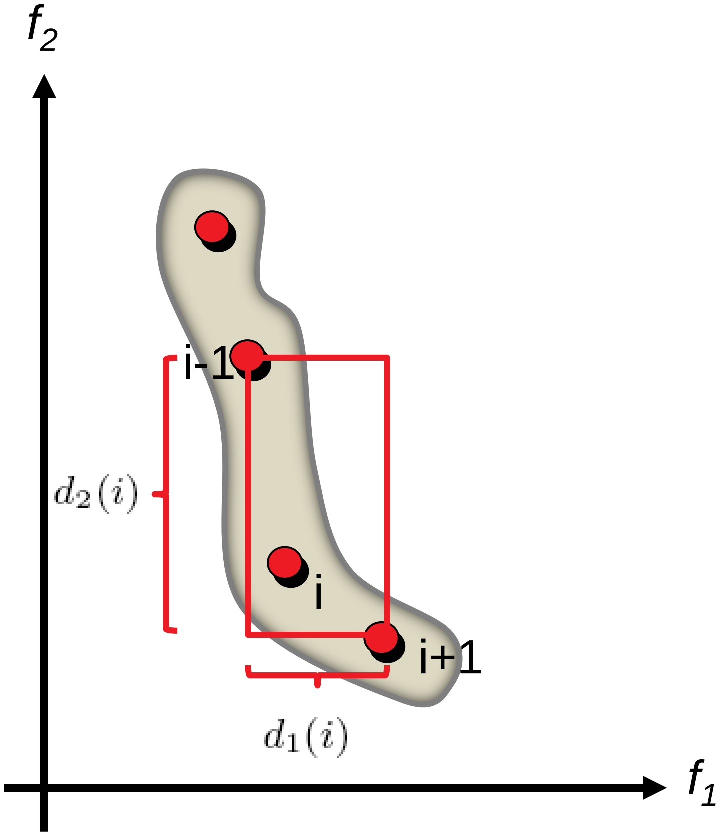
diversity =
number of elements
within box(es)



❷ (Contribution to a) quality indicator

Crowding Distance (CD)

- sort solutions with regard to each objective
- assign CD maximum value to extremal objective vectors
- compute CD based on the distance to the neighbors in each objective

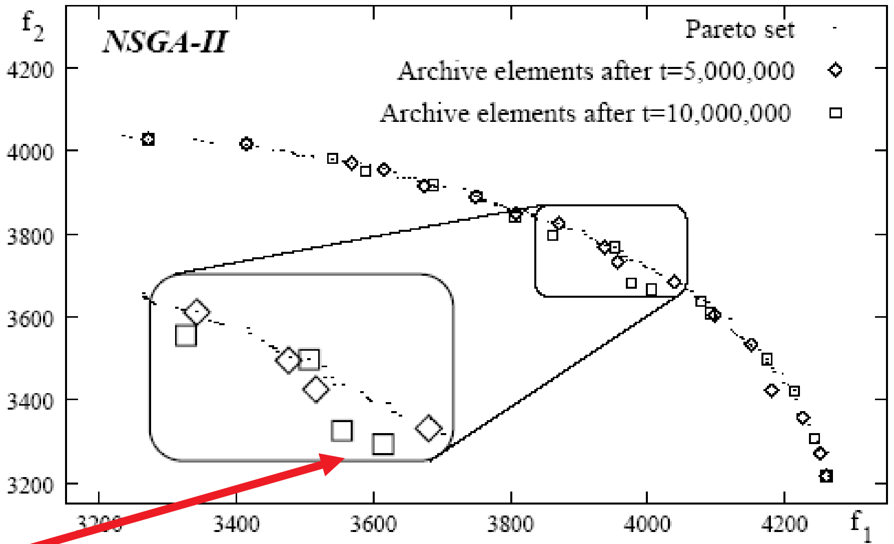


$$\mathrm{CD}(i) = \frac{d_1(i)}{f_{1,\max} - f_{1,\min}} + \cdots + \frac{d_m(i)}{f_{m,\max} - f_{m,\min}}$$

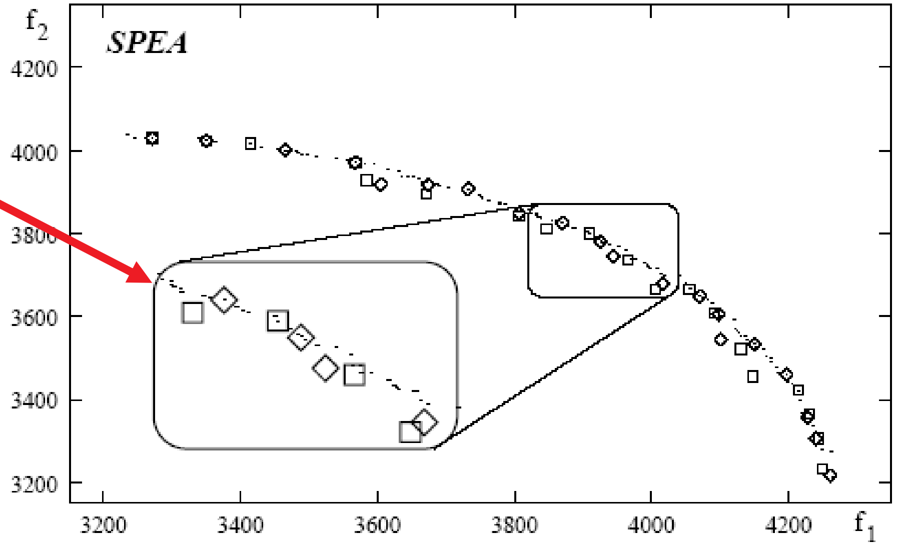
SPEA2 and NSGA-II: Deteriorative Cycles

Selection in SPEA2 and NSGA-II can result in

deteriorative cycles

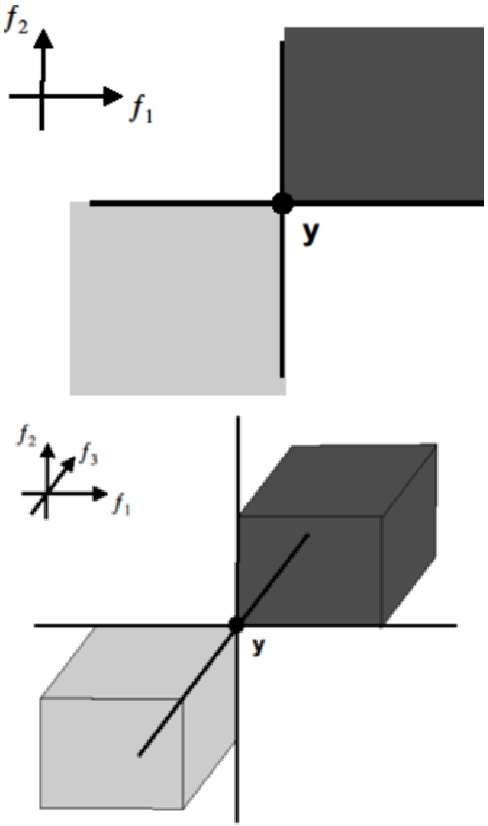


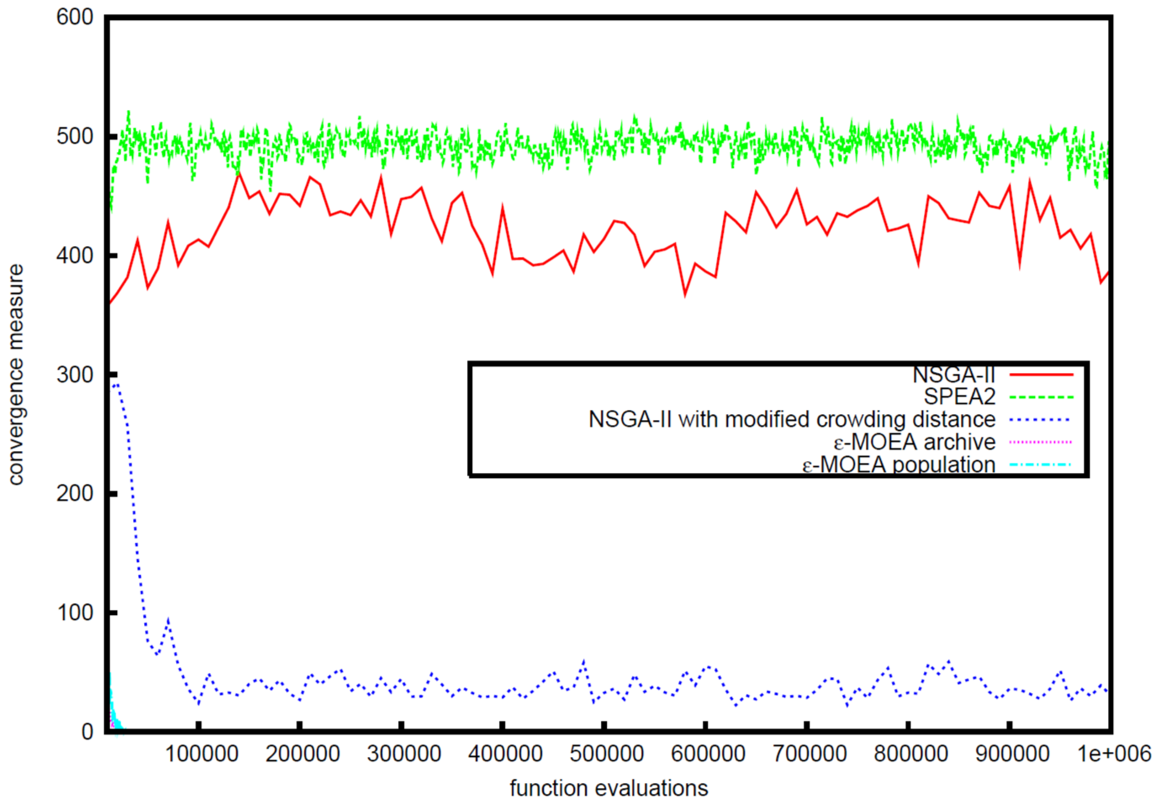
non-dominated solutions already found can be lost



Remark: Many-Objective Optimization

- high number of objectives
  → percentage of non-dominated solutions within a random sample quickly approaches 100 %
  → optimization is mainly guided by diversity criterion
  → apply secondary criterion compliant with dominance relation





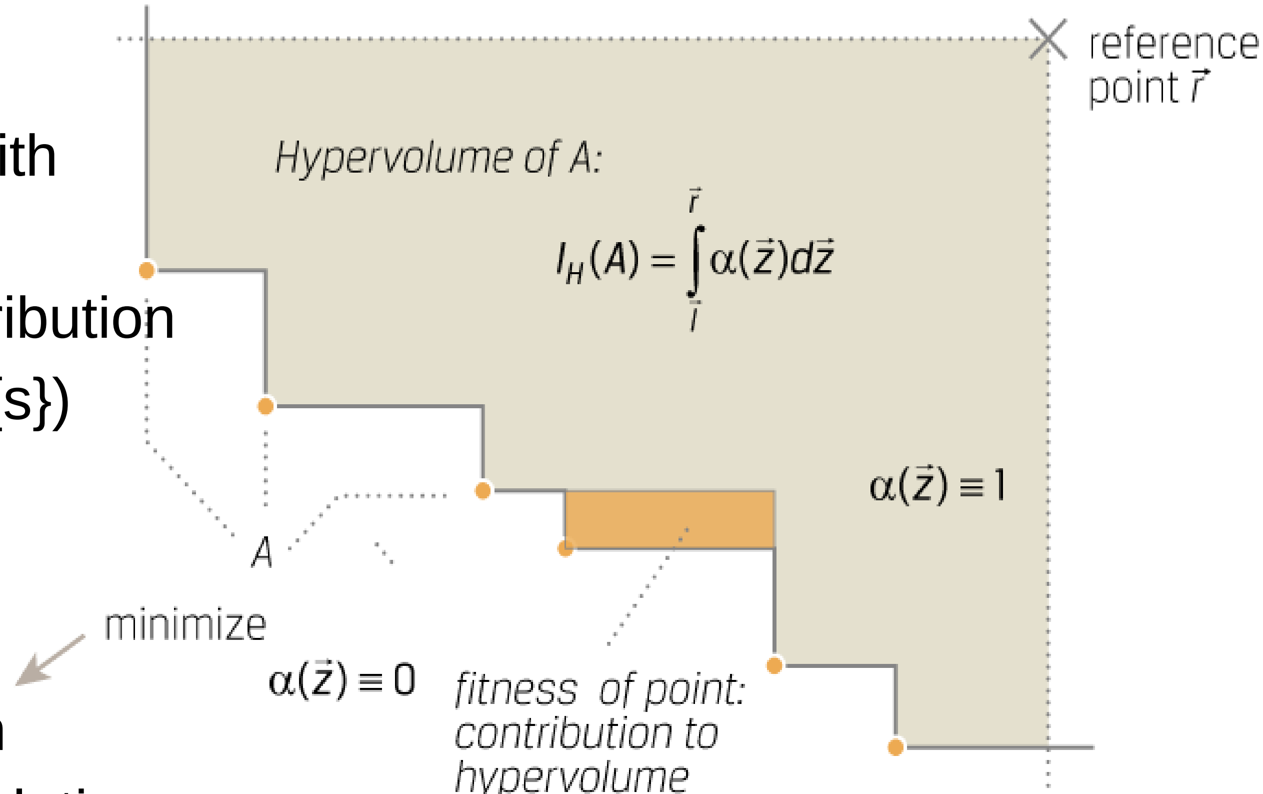
Hypervolume-Based Selection

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …)

use hypervolume indicator to guide the search: refines dominance

Main idea

Delete solutions with the smallest hypervolume contribution $d(s) = I_H(P) - I_H(P / \{s\})$ iteratively



But: can also result in cycles on single solutions [Judt et al. 2011], [López-Ibáñez et al. 2011]

and is expensive to compute exactly for many objectives

[Bringmann and Friedrich 2009]

Indicator-Based Selection

- Concept can be generalized to any quality indicator

A (unary) quality indicator I is a function $I : \Psi = 2^X \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.

Multiobjective Problem $\xrightarrow{\text{Indicator}}$ Single-objective Problem

- for example: R2-indicator [Brockhoff et al. 2012], [Trautmann et al. 2013], [Díaz-Manríquez et al. 2013]

- Generalizable also to contribution to larger sets

  HypE [Bader and Zitzler 2011]: Hypervolume sampling + contribution if more than 1 (random) solution deleted

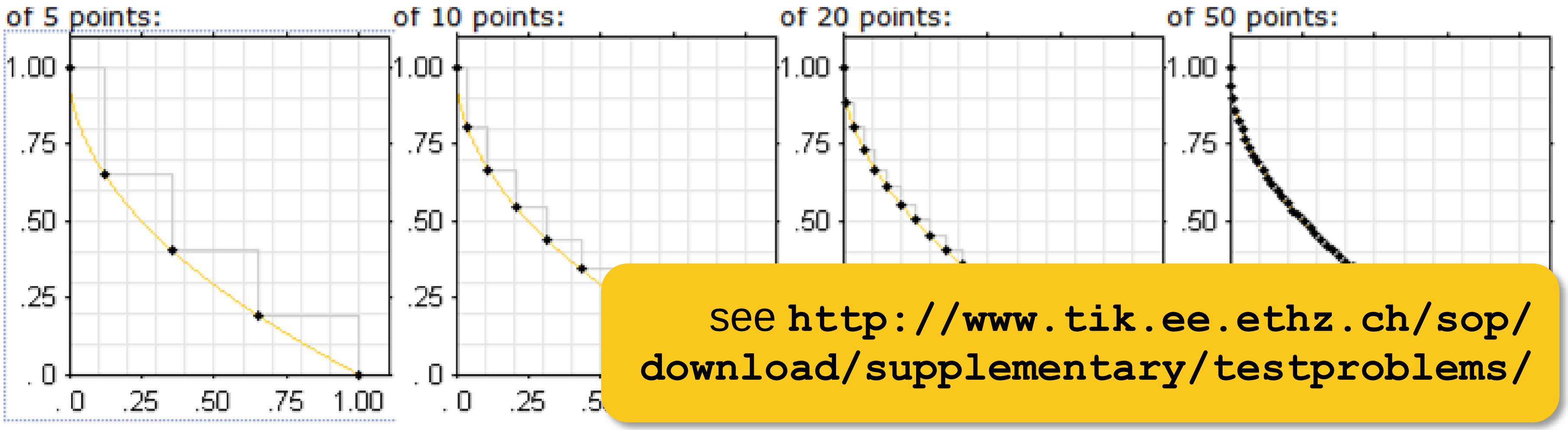
The Optimization Goal in Indicator-Based EMO

When the goal is to maximize a unary indicator…

- we have a single-objective problem on sets
- but what is the optimum?
- important: population size μ plays a role!

Optimal μ-Distribution:

A set of μ solutions that maximizes a certain unary indicator I among all sets of μ solutions is called optimal μ-distribution for I. [Auger et al. 2009a]



see **http://www.tik.ee.ethz.ch/sop/download/supplementary/testproblems/**

Optimal µ-Distributions for the Hypervolume

Hypervolume indicator refines dominance relation

$\Longrightarrow$ most results on optimal µ-distributions for hypervolume

Optimal µ-Distributions (example results)

[Auger et al. 2009a]:

- contain equally spaced points iff front is linear
- density of points $\propto \sqrt{-f'(x)}$ with f' the slope of the front

[Friedrich et al. 2011]:

optimal µ-distributions for the hypervolume correspond to ε-approximations of the front

OPT	$1 + \frac{\log(\min\{A/a, B/b\})}{n}$
HYP	$1 + \frac{\sqrt{A/a} + \sqrt{B/b}}{n-4}$
logHYP	$1 + \frac{\sqrt{\log(A/a)\log(B/b)}}{n-2}$

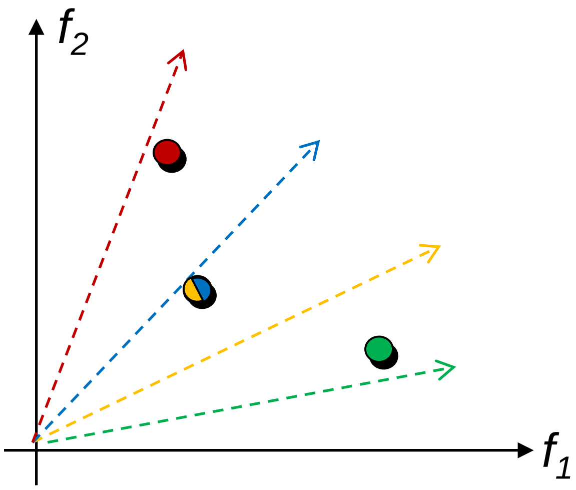
! (probably) does not hold for > 2 objectives

Decomposition-Based Selection: MOEA/D

MOEA/D: Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

Ideas:

- optimize N scalarizing functions in parallel
- use best solutions of neighbor subproblems for mating
- keep the best solution for each scalarizing function
- update neighbors
- use external archive for non-dominated solutions
- several variants and enhancements



Remark: Variation in EMO

- at first sight not different from single-objective optimization
- most research on selection mechanisms (until now)
- but: convergence to a set ≠ convergence to a point

Open Question:

- how to achieve fast convergence to a *set*?

Related work:

- set-based gradient of the HV [Emmerich et al. 2007]
- multiobjective CMA-ES [Igel et al. 2007] [Voß et al. 2010]
- RM-MEDA [Zhang et al. 2008]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]
- offline and online configuration based on libraries of variation operators [Bezerra et al. 2015] [Hadka and Reed 2013]

Overview

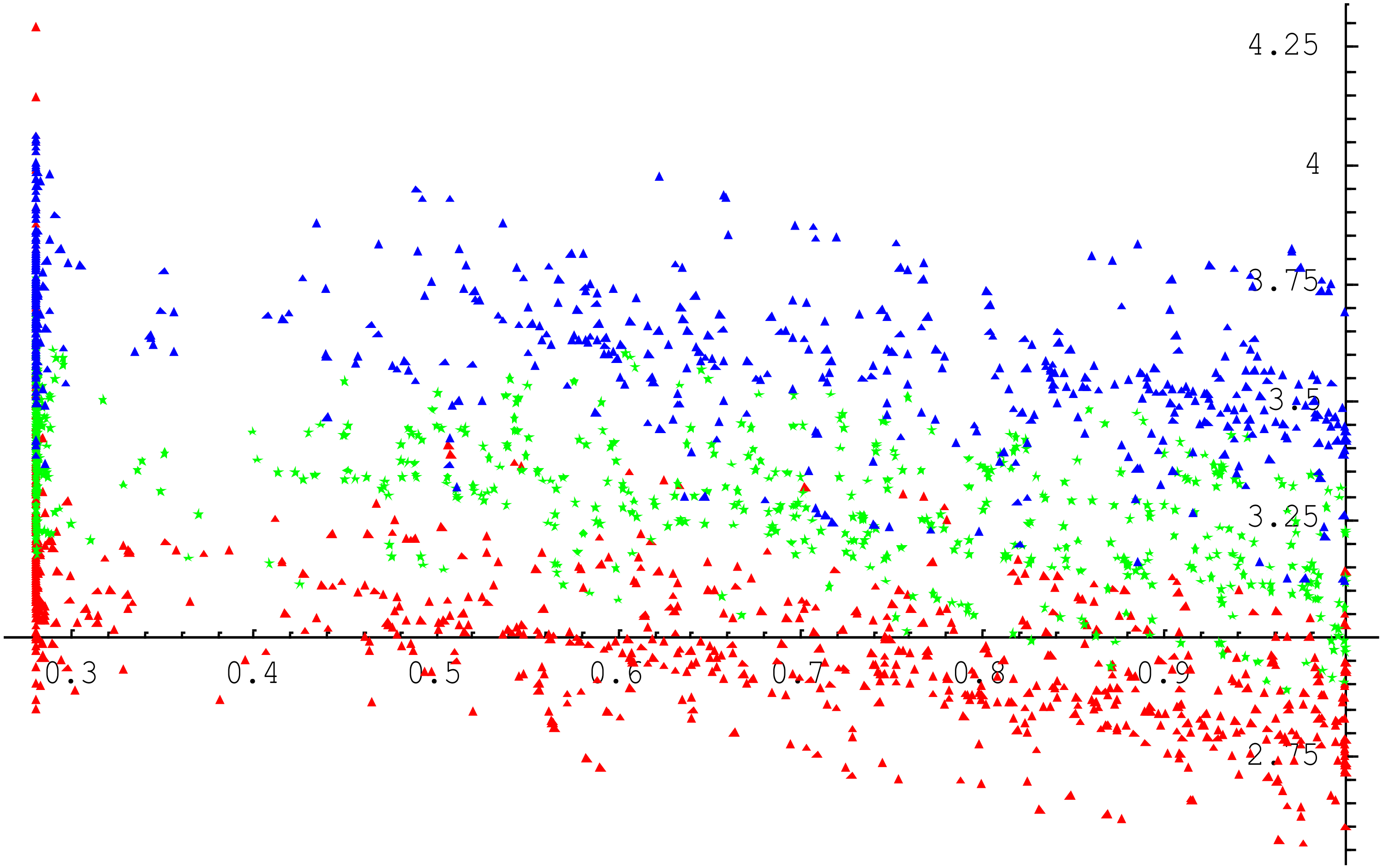
The Big Picture

Algorithm Design Principles and Concepts

Performance Assessment

Once Upon a Time...

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

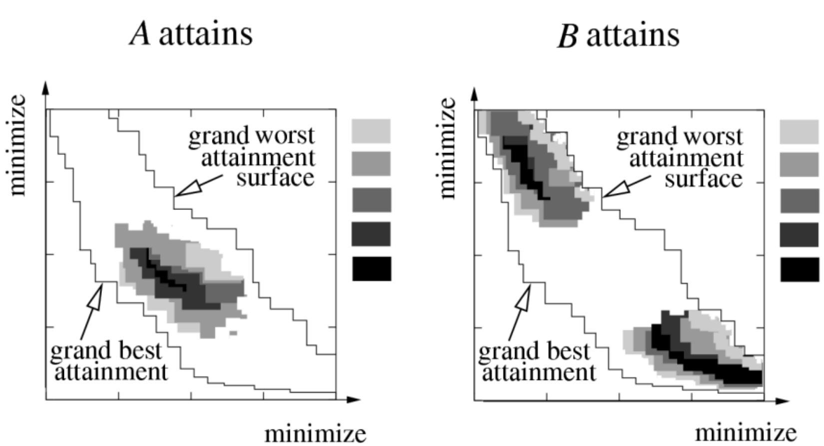
Two Approaches for Empirical Studies

Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur

Quality indicator approach

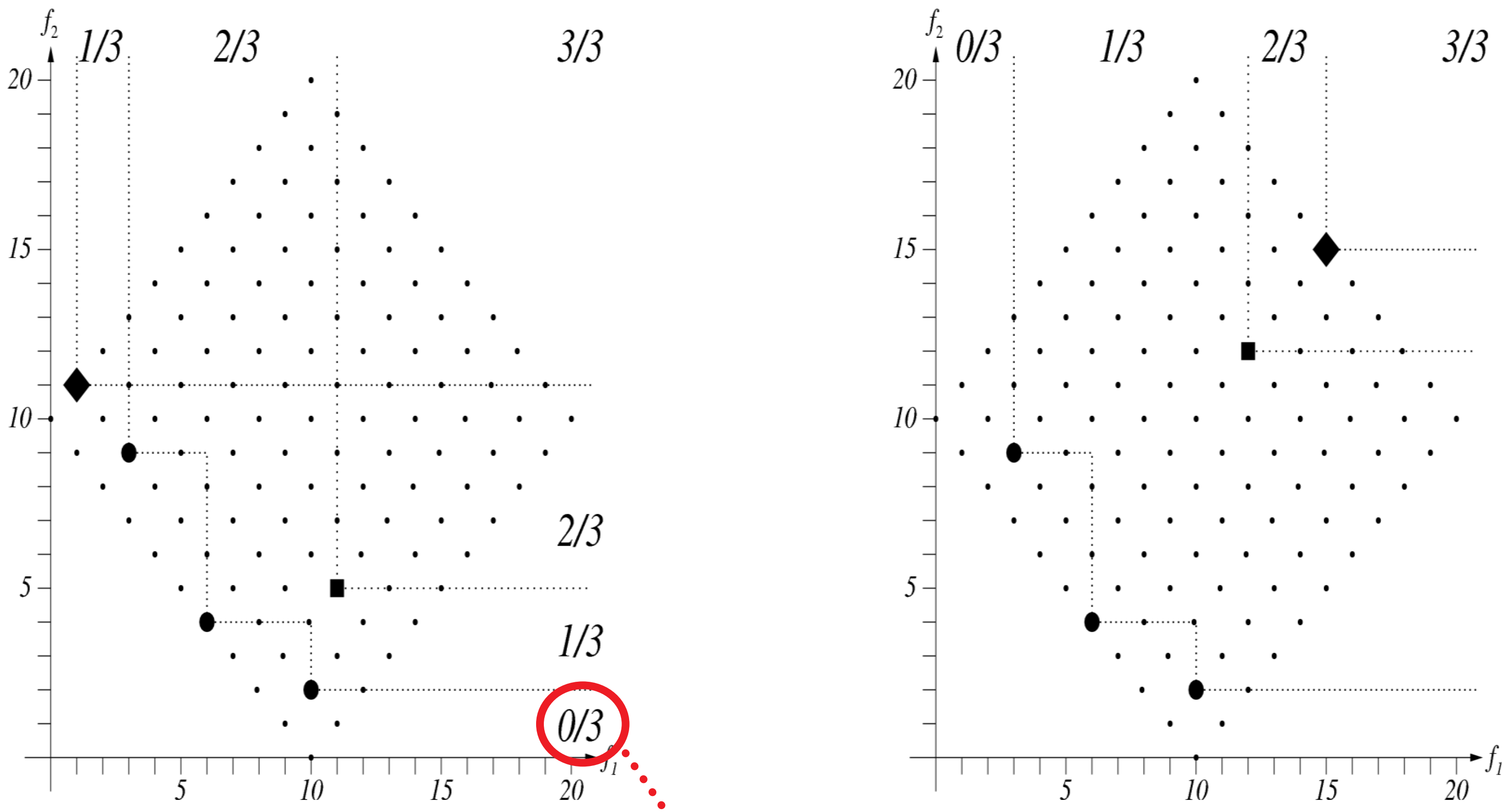
- reduces each approximation set to a single quality value
- applies statistical tests to the quality values



Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

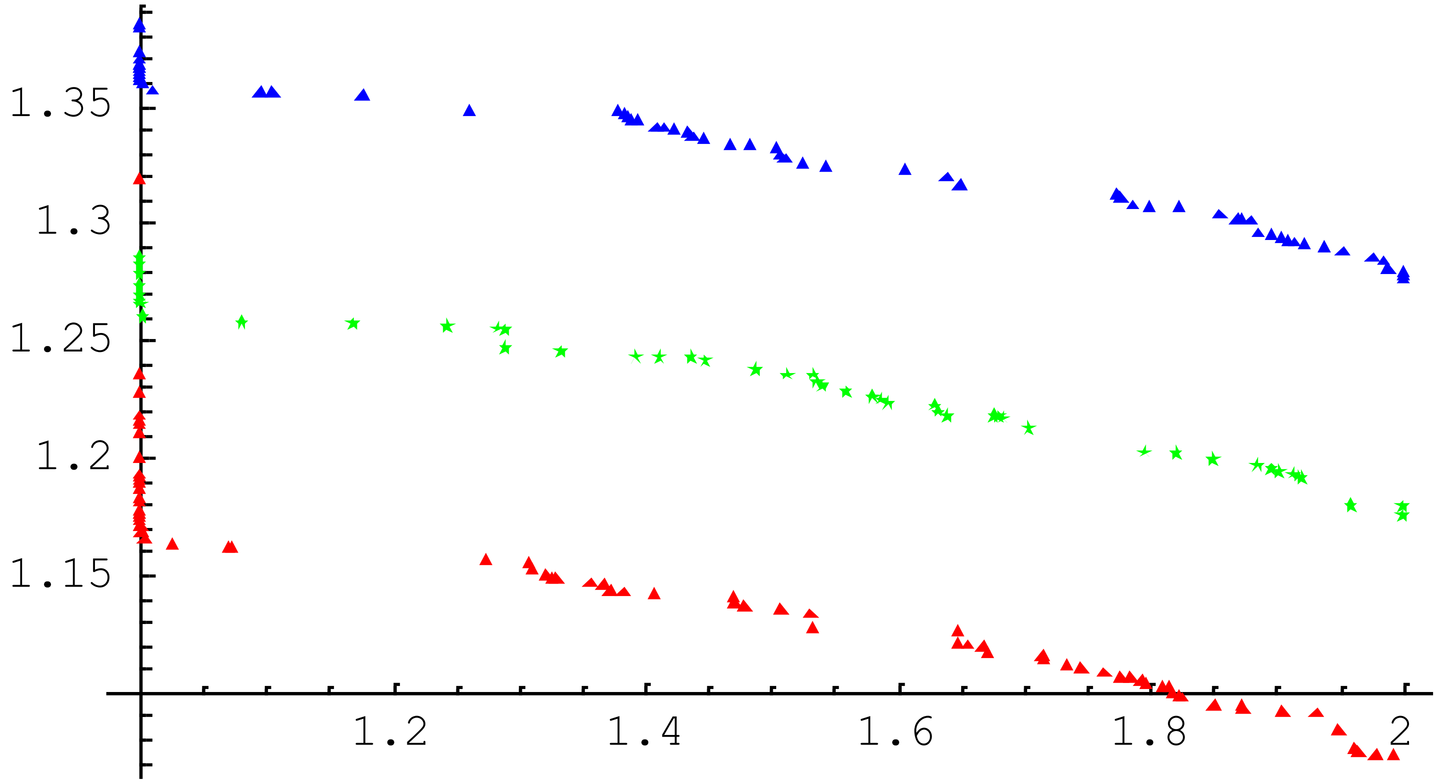
three runs of two multiobjective optimizers



frequency of attaining regions

Attainment Plots

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)



latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
R package: **`http://lopez-ibanez.eu/eaftools`**
see also [Fonseca et al. 2011]

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function (“quality indicator”) on sets
- use the resulting total (pre-)order (on the quality values)

Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation (on sets) should be reflected!

$$A \preceq B :\Leftrightarrow \forall_{y \in B} \exists_{x \in A} x \leq_{par} y$$

Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$ fulfills requirement

❷ $\overset{\text{ref}}{\preccurlyeq}$ **weakly refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

$\Rightarrow$ does not fulfill requirement, but $\overset{\text{ref}}{\preccurlyeq}$ does not contradict $\preccurlyeq$

! sought are total refinements… [Zitzler et al. 2010]

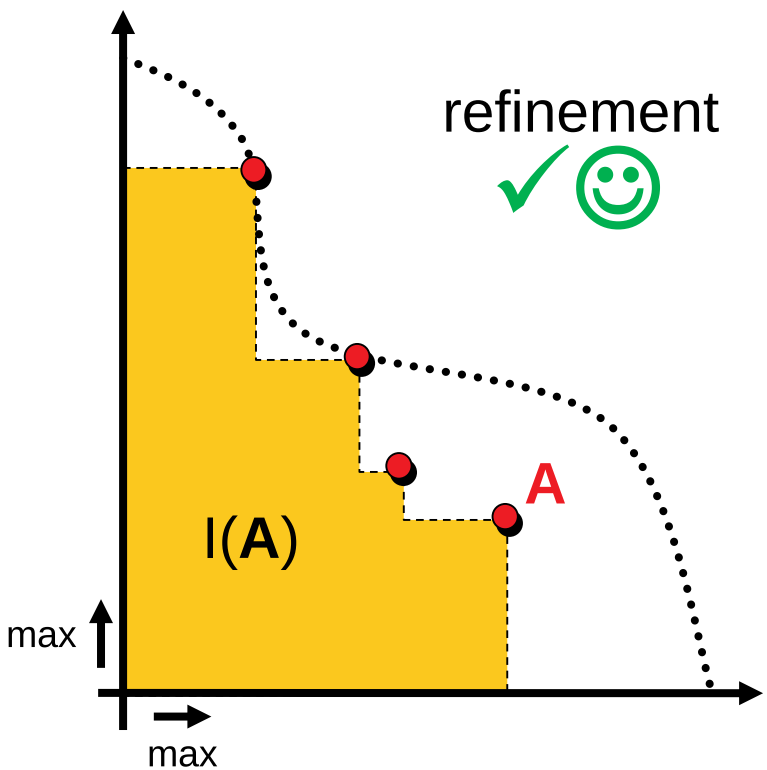
Example: Refinements Using Indicators

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \geq I(B)$$

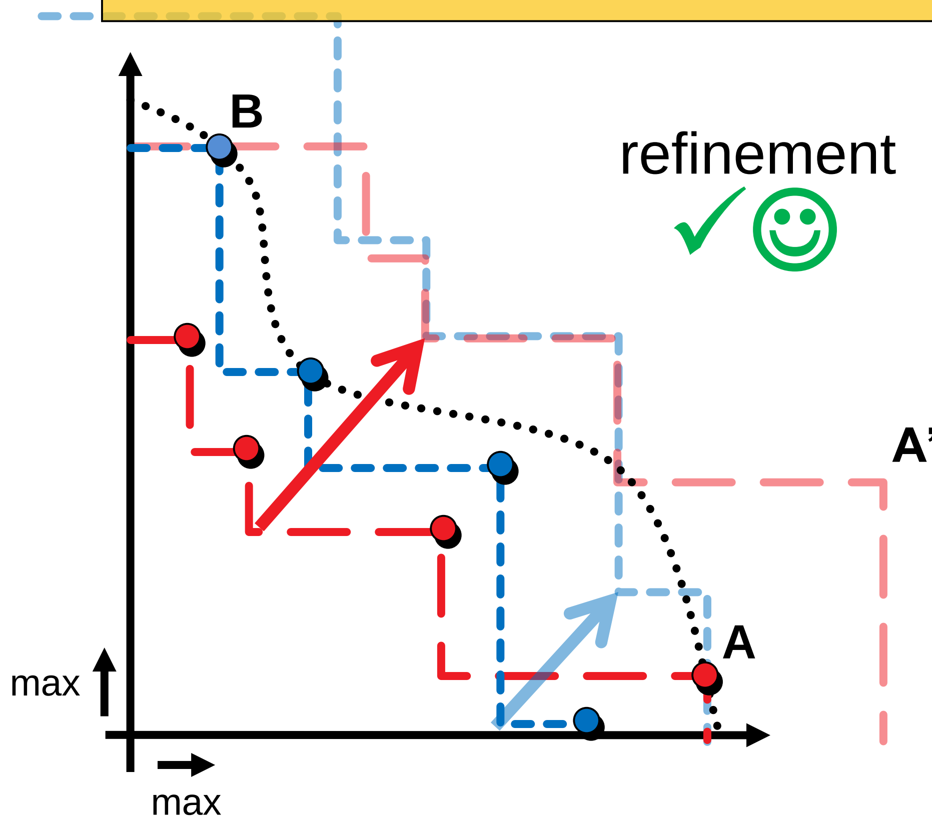
$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,B) \leq I(B,A)$$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B



unary hypervolume indicator



binary epsilon indicator

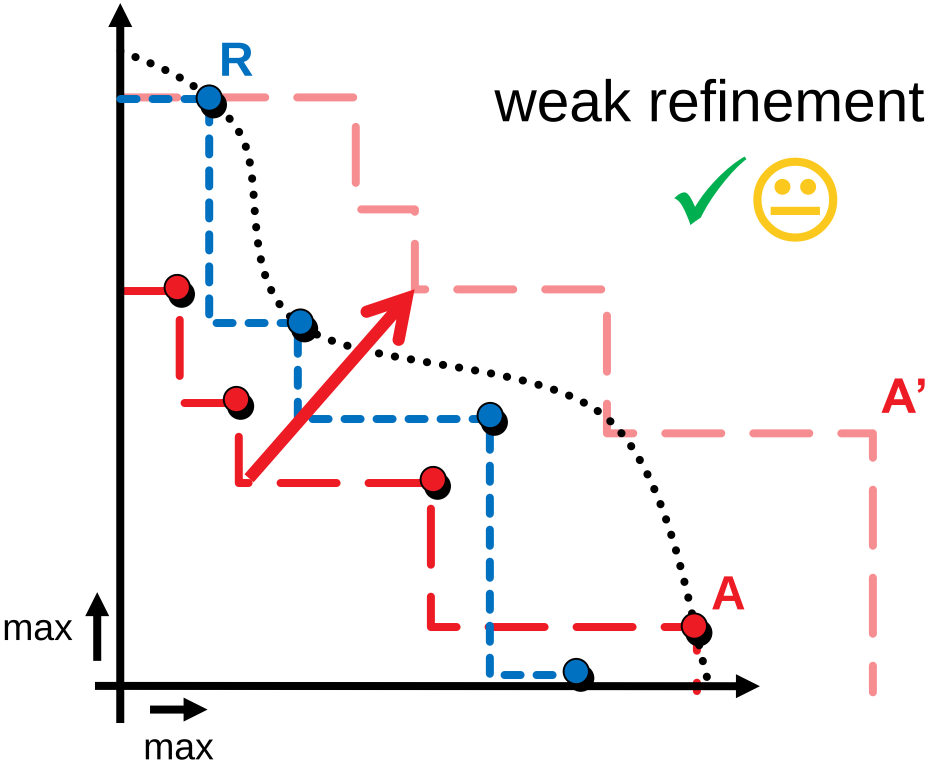
Example: Weak Refinement / No Refinement

$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A,R) \leq I(B,R)$$

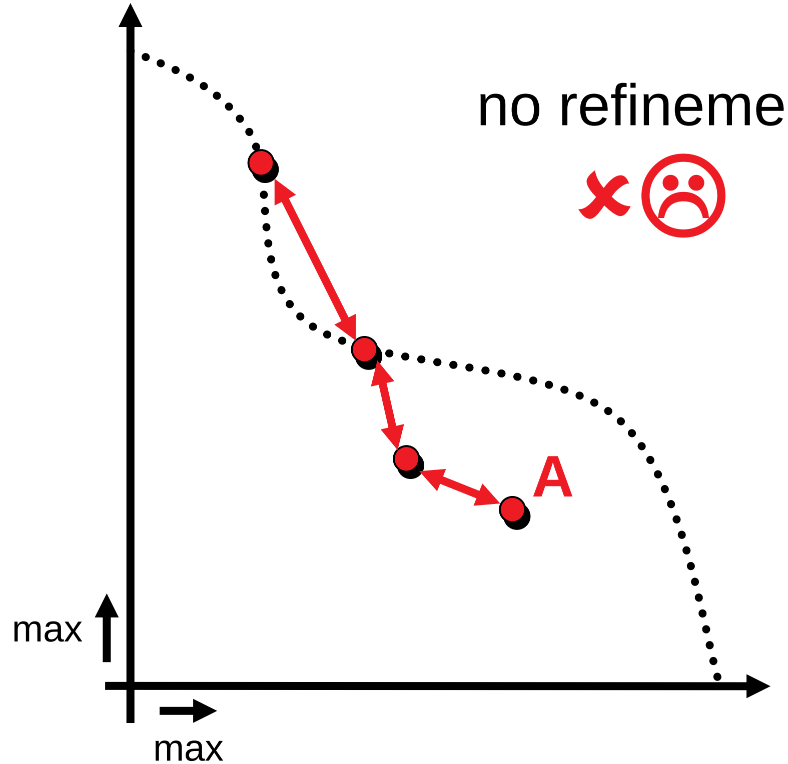
$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A) \leq I(B)$$

I(A,R) = how much needs A to be moved to weakly dominate R

I(A) = variance of pairwise distances



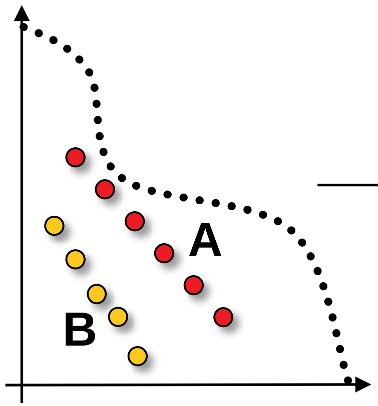
unary epsilon indicator



unary diversity indicator

Quality Indicator Approach

Goal: compare two Pareto set approximations A and B



	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

→ "**A** better"

Comparison method C = quality measure(s) + Boolean function

$$A, B \xrightarrow[\textbf{reduction}]{\text{quality measure}} \mathbb{R}^n \xrightarrow[\textbf{interpretation}]{\text{Boolean function}} \text{statement}$$

Summary: Quality Indicators

A quality indicator

- maps a solution set to a real number
- can be used with standard performance assessment
  - report median, variance, ...
  - boxplots
  - statistical tests
- should optimally refine the dominance relation on sets

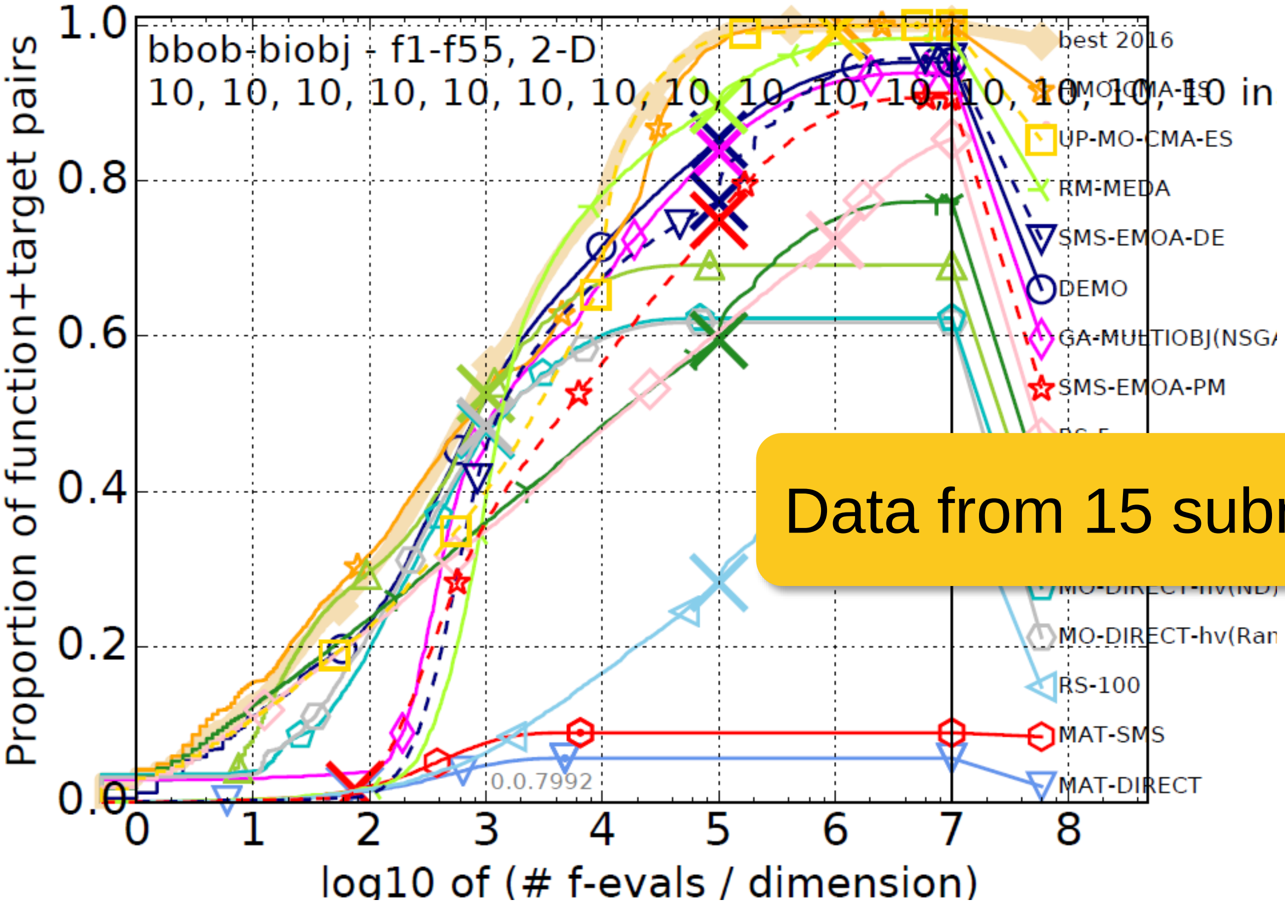
Recommendation:

- use hypervolume (refinement)
- or epsilon indicator or R2 indicator (are weak refinements)

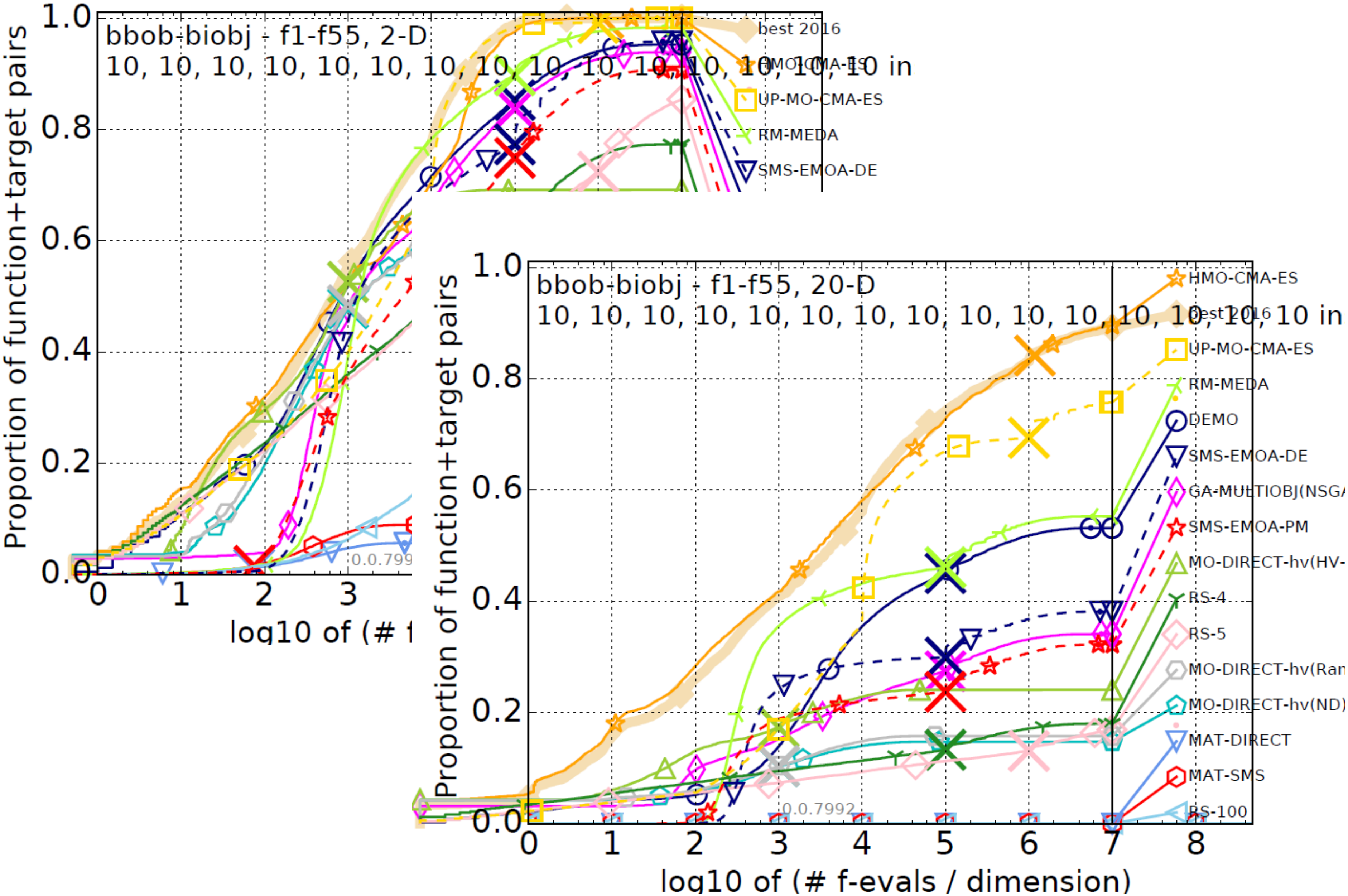
Automated Benchmarking

- State-of-the-art in single-objective optimization: Blackbox Optimization Benchmarking (BBOB) with COCO platform

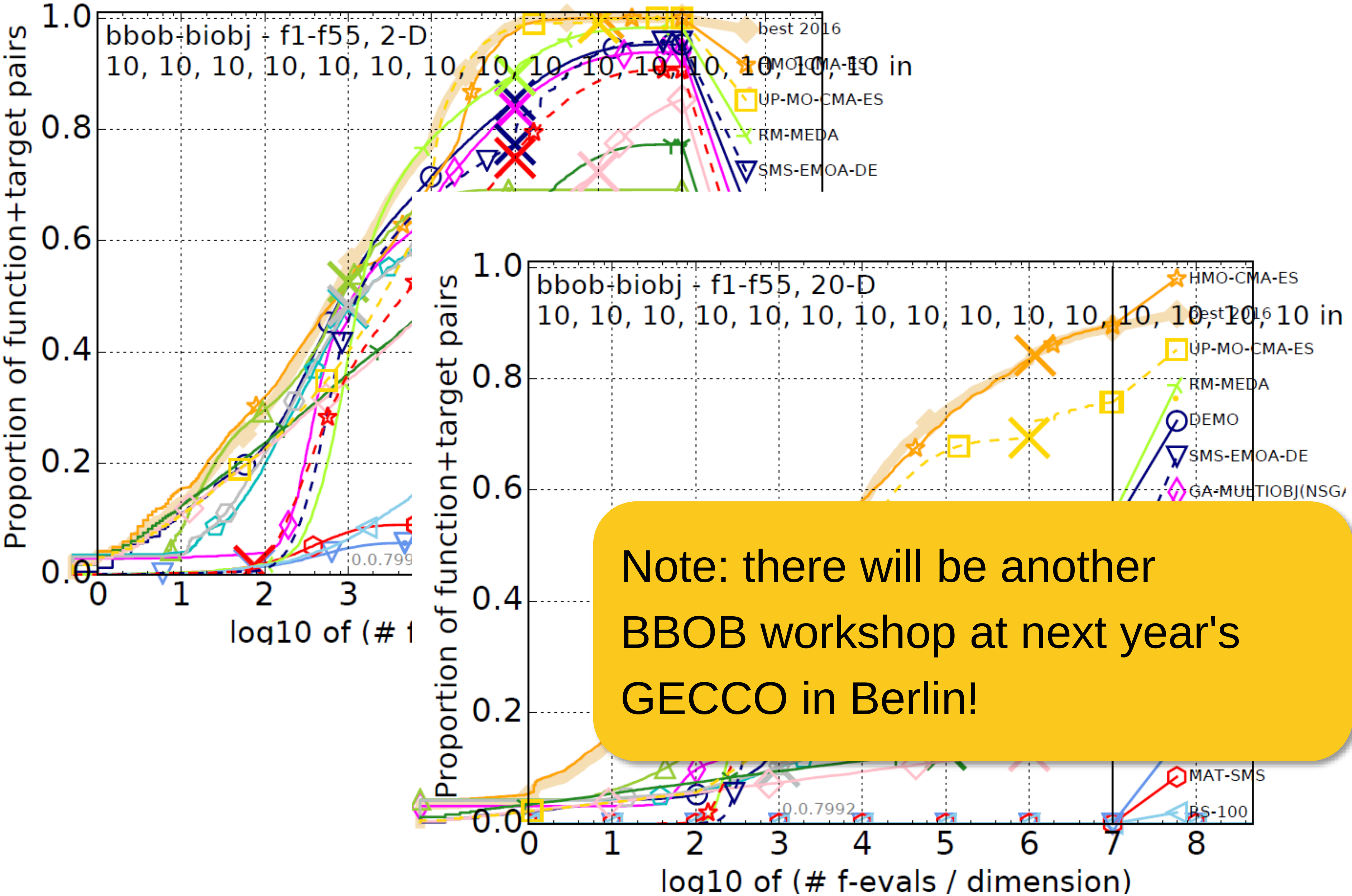
  `https://github.com/numbbo/coco`
- This year: first release of a bi-objective test suite and corresponding BBOB-2016 workshop @ GECCO
- Focus on target-based runlengths
  - gives (nearly) anytime, interpretable results
  - defines problem=(test function instance, single-objective goal e.g. min. indicator difference to reference set, target precision)
  - reports average runtimes (aRT) to reach target precision
- COCO provides data profiles, scaling plots, scatter plots, tables, statistical tests, etc. automatically



Data from 15 submitted algorithms

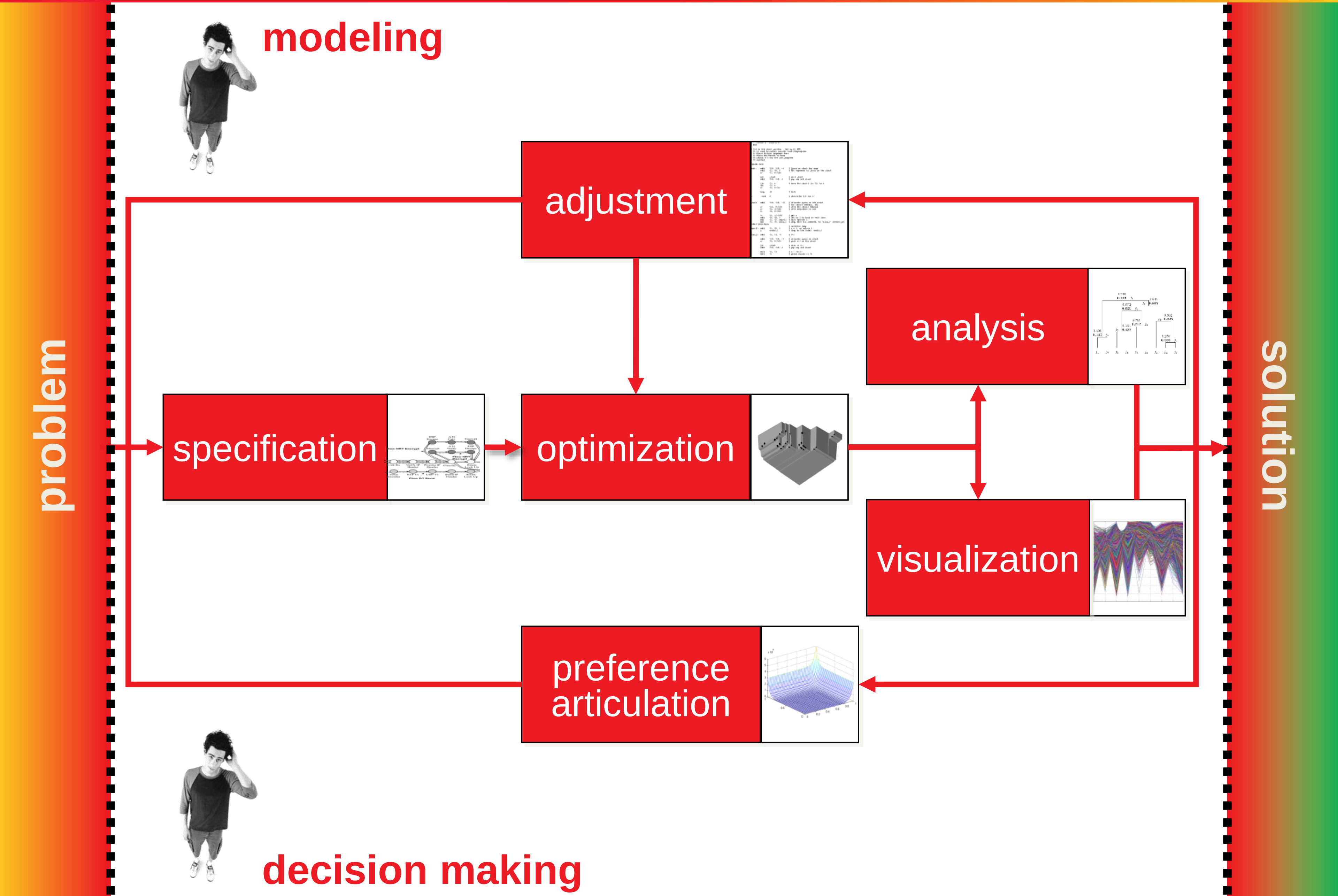

bbob-biobj - f1-f55, 2-D
10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10 in
Proportion of function+target pairs
best 2016
HMO-CMA-ES
UP-MO-CMA-ES
RM-MEDA
SMS-EMOA-DE
0.0.799
log10 of (# f
bbob-biobj - f1-f55, 20-D
10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10 in
Proportion of function+target pairs
HMO-CMA-ES
best 2016
UP-MO-CMA-ES
RM-MEDA
DEMO
SMS-EMOA-DE
GA-MULTIOBJ(NSGA
SMS-EMOA-PM
MO-DIRECT-hv(HV-
RS-4
RS-5
MO-DIRECT-hv(Ran
MO-DIRECT-hv(ND)
MAT-DIRECT
MAT-SMS
RS-100
0.0.7992
log10 of (# f-evals / dimension)

Exemplary BBOB-2016 Results



Note: there will be another BBOB workshop at next year's GECCO in Berlin!

Conclusions: EMO as Interactive Decision Support



The EMO Community

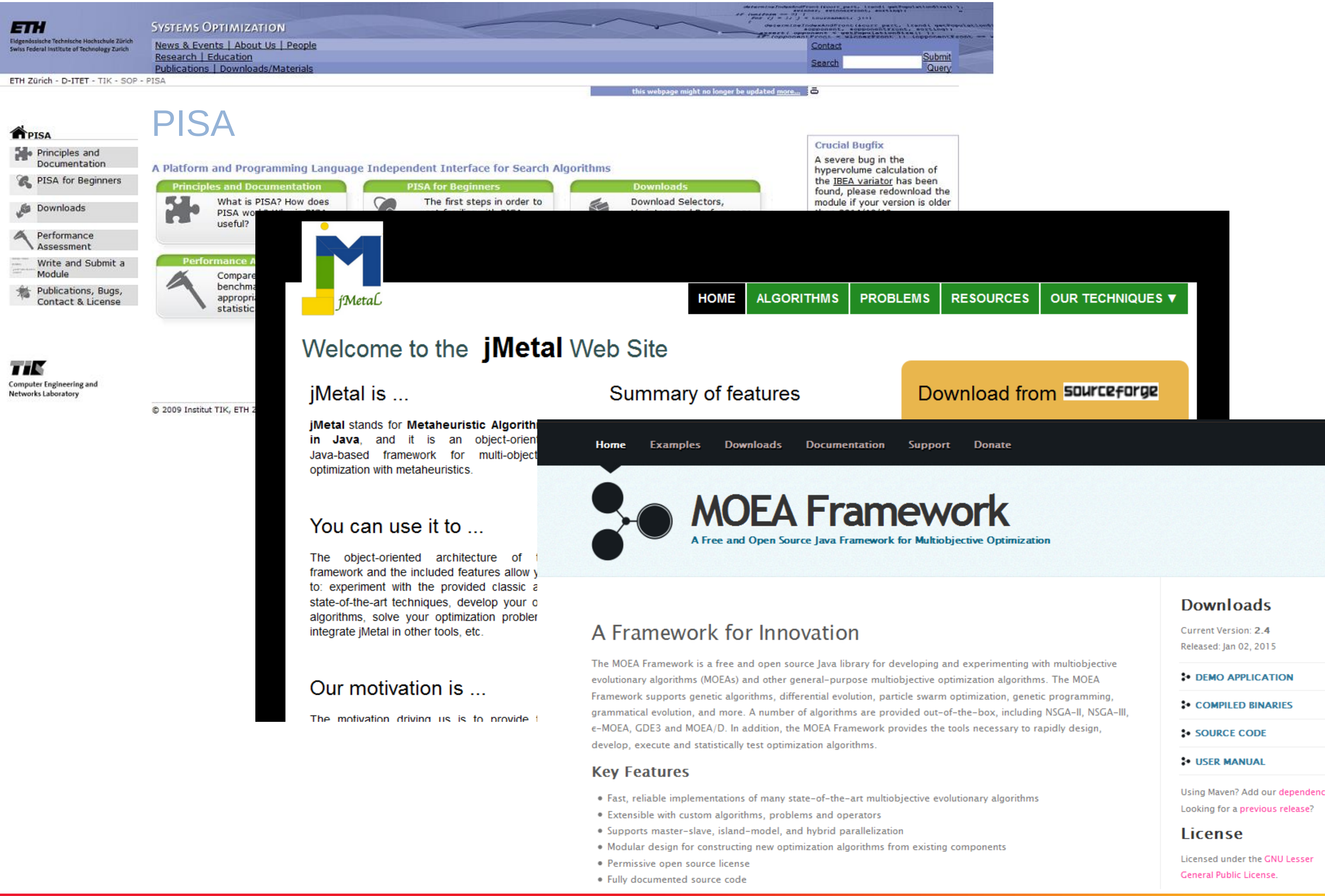
Links:

- EMO mailing list: *https://lists.dei.uc.pt/mailman/listinfo/emo-list*
- MCDM mailing list: *http://lists.jyu.fi/mailman/listinfo/mcdm-discussion*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.emo2017.org/*

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms***
  Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [(still) many open questions!]
- and more…

Software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to: experiment with the provided classic a
state-of-the-art techniques, develop your o
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integrate jMetal in other tools, etc.
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MOEA Framework
A Free and Open Source Java Framework for Multiobjective Optimization
A Framework for Innovation
The MOEA Framework is a free and open source Java library for developing and experimenting with multiobjective evolutionary algorithms (MOEAs) and other general-purpose multiobjective optimization algorithms. The MOEA Framework supports genetic algorithms, differential evolution, particle swarm optimization, genetic programming, grammatical evolution, and more. A number of algorithms are provided out-of-the-box, including NSGA-II, NSGA-III, ε-MOEA, GDE3 and MOEA/D. In addition, the MOEA Framework provides the tools necessary to rapidly design, develop, execute and statistically test optimization algorithms.
Key Features
Fast, reliable implementations of many state-of-the-art multiobjective evolutionary algorithms
Extensible with custom algorithms, problems and operators
Supports master-slave, island-model, and hybrid parallelization
Modular design for constructing new optimization algorithms from existing components
Permissive open source license
Fully documented source code
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Current Version: 2.4
Released: Jan 02, 2015
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USER MANUAL
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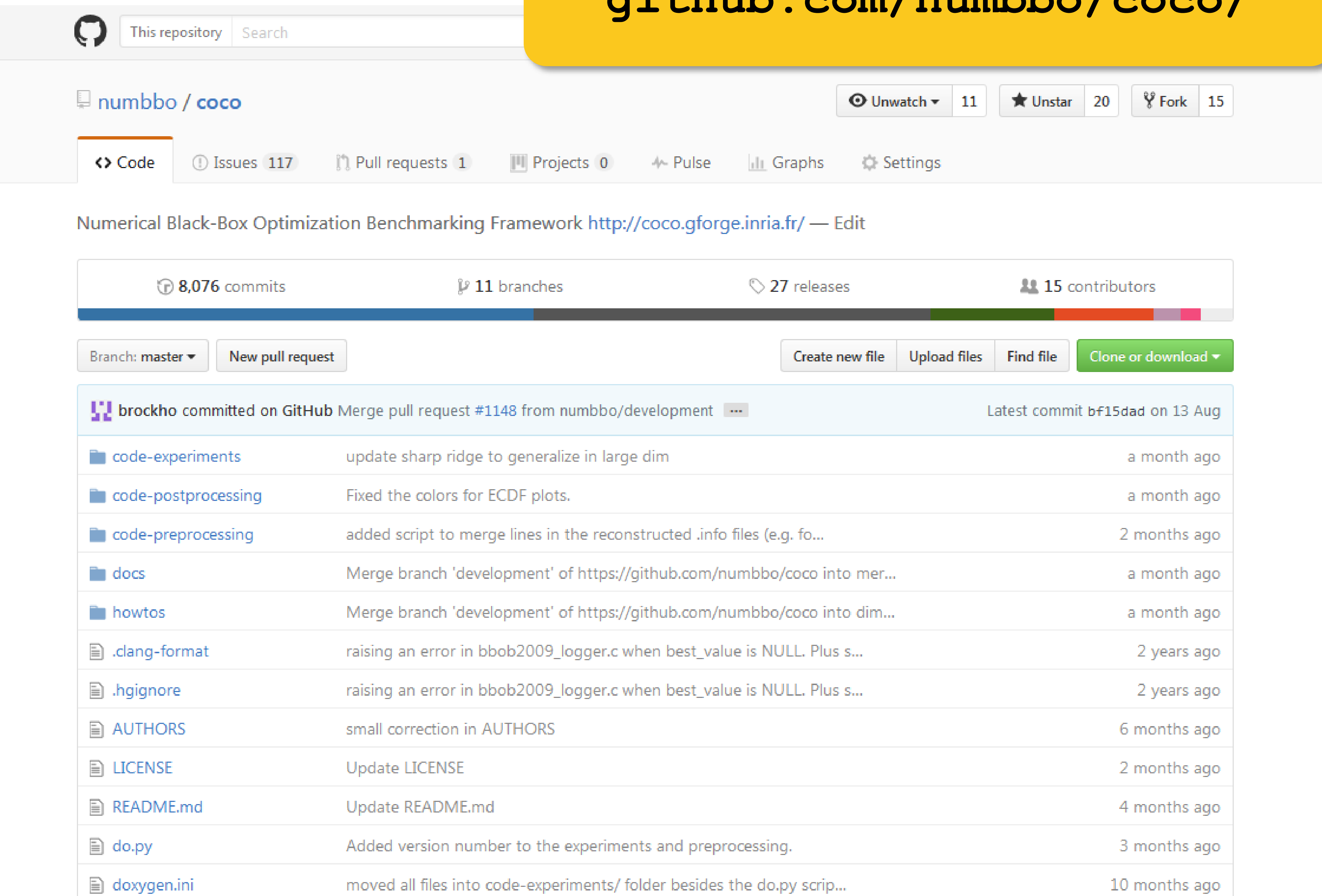
Software

github.com/numbbo/coco/



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Perspectives

Challenging Open (Research) Directions

- from algorithms to toolkits
  - libraries of modules for each task (selection, variation, etc.)
  - problem-specific algorithm configuration/ parameter tuning
- benchmarking
  - comparison with classical approaches
  - design/selection of practically relevant problems
  - Algorithm/toolkit recommendations for practice
- integration of EMO and MCDM into one field
- interactive preference articulation and learning
- interactive problem design
- integration of problem-specific knowledge

Questions?

Additional Slides

Instructor Biography: Dimo Brockhoff

Dimo Brockhoff

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DOLPHIN team
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After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. Between June 2009 and October 2011 he held postdoctoral research positions---first at INRIA Saclay Ile-de-France in Orsay and then at Ecole Polytechnique in Palaiseau, both in France. Since November 2011 he has been a junior researcher (now CR1) at INRIA Lille - Nord Europe in Villeneuve d'Ascq, France. His most recent research interests are focused on evolutionary multiobjective optimization (EMO) and other (single-objective) blackbox optimization techniques, in particular with respect to benchmarking, theoretical aspects, and expensive optimization.

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