GECCO 2021 Tutorial on Benchmarking Multiobjective Optimizers 2.0

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The final slides are available at
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Isn’t Benchmarking Trivial?

Isn’t Benchmarking Trivial?

- Choose some algorithms
- Choose some test functions
- Discuss the results

Isn’t Benchmarking Trivial?

In principle: yes

But many details and pitfalls

Isn't Benchmarking Trivial?

In principle: yes

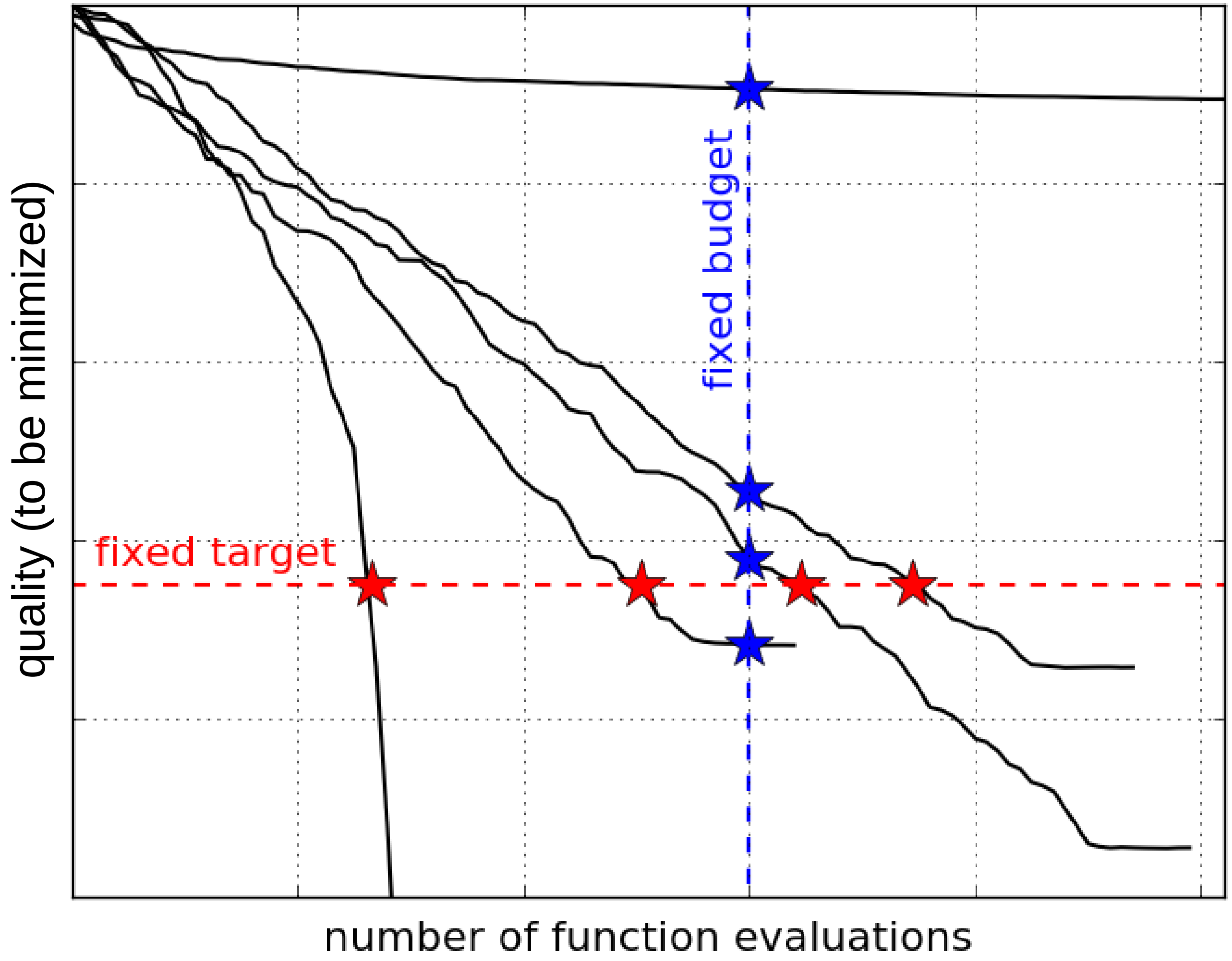
But many details and pitfalls

and in addition

- we compare *sets* of solutions
- which might be random (in size, position, dimension, …)
- …

We Typically Start with Convergence Graphs

(…in single-objective optimization)



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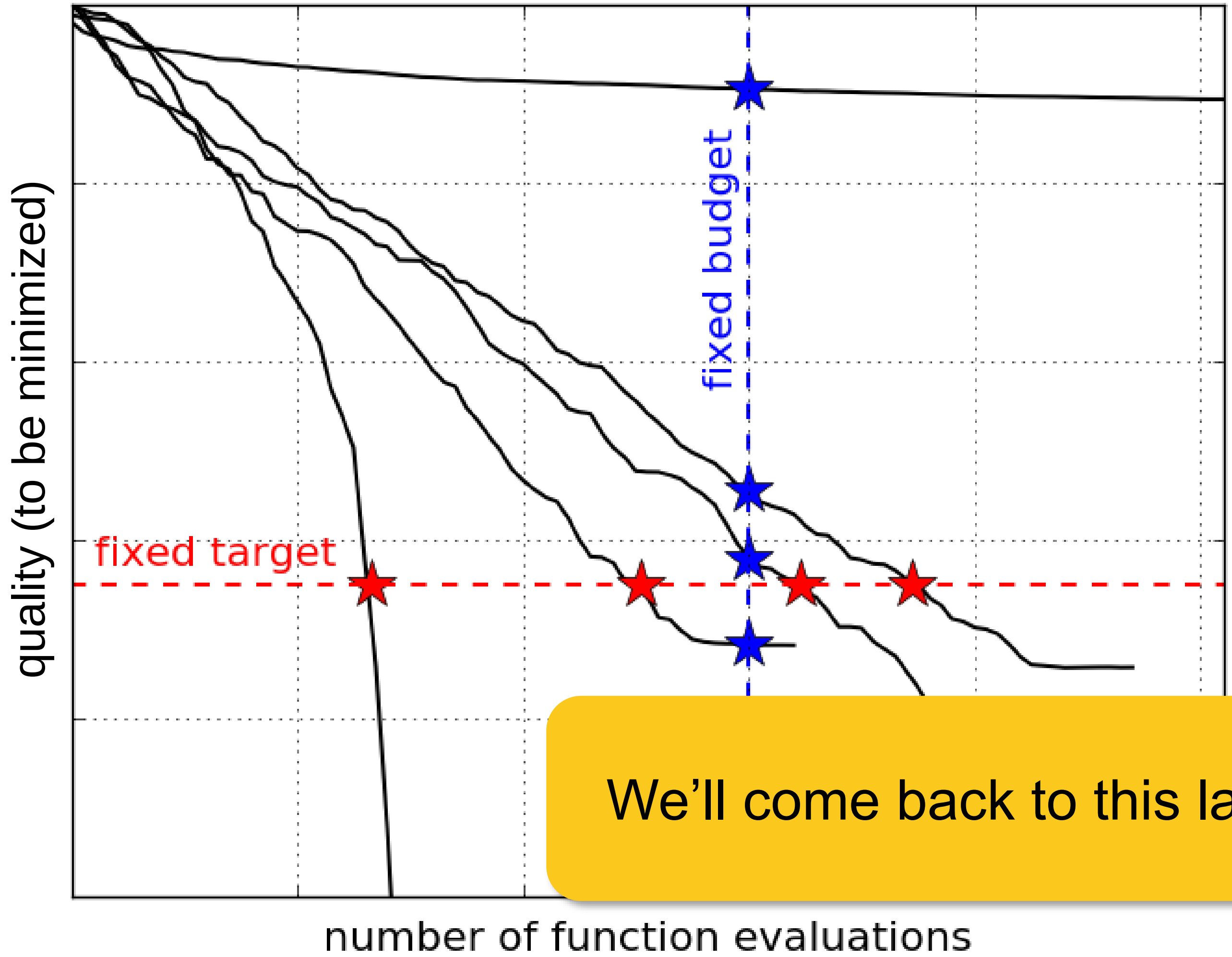

We'll come back to this later again

Overview

Our plan

Discuss history, present and future of multiobjective benchmarking

With respect to different topics

- performance assessment / methodology
- test functions

Finally, recommendations on good algorithms

Disclaimer

This is not an introductory tutorial to multiobjective optimization!

We assume you know basic definitions like

- Objective function
- Pareto dominance/Pareto front/Pareto set
- Ideal/Nadir points

Disclaimer II

We only consider continuous search spaces

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We only consider unconstrained problems

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What we present is highly subjective & selective

- how important do we find each milestone?
- use version numbering and branches
- what have we learned from the past?

Overview

❶ Performance Assessment

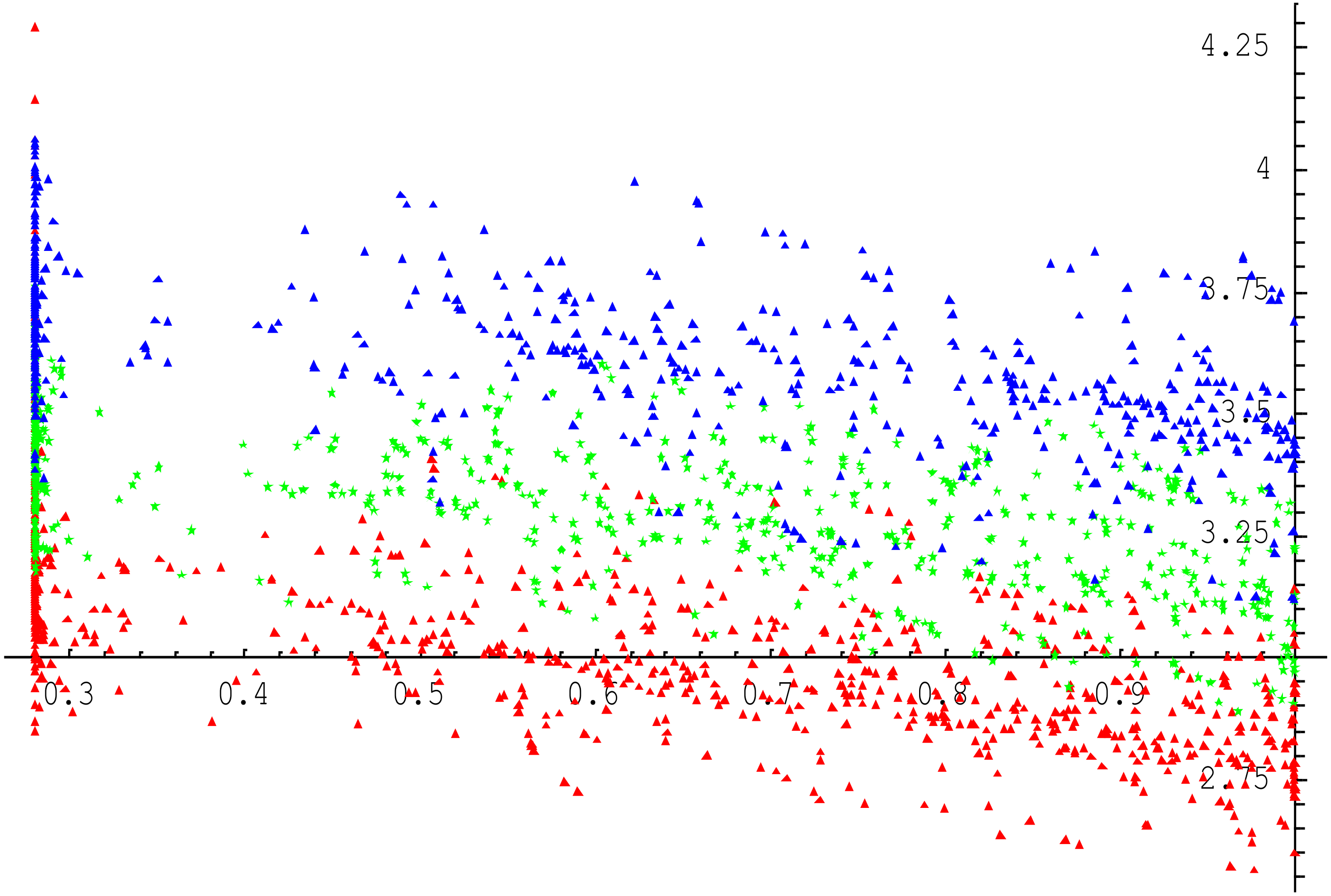
❷ Test Problems and Their Visualizations

❸ Recommendations from Numerical Results

v0.0.1alpha

In The Early Beginnings…

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

v0.1beta

Tables

Problem	MOCSA				NSGA2			
	$\langle d \rangle$	S	GD	ER	$\langle d \rangle$	S	GD	ER
ZDT1	0.0404	0.0055	0.0000	0	0.0270	0.0156	0.0011	0.04
ZDT2	0.0404	0.0082	0.0000	0	0.0292	0.0146	0.0212	0.02
ZDT3	0.0438	0.0148	0.0001	0	0.0329	0.0201	0.0020	0.02
ZDT4	0.0404	0.0097	0.0000	0	0.0328	0.0159	0.0006	0.02
ZDT6	0.0327	0.0150	0.0000	0	0.0216	0.0119	0.0000	0
DTLZ1	0.1114	0.0068	0.0000	0	0.0615	0.0319	0.0000	0
DTLZ2	0.2319	0.0646	0.0021	0.02	0.1361	0.0683	0.0020	0.04
DTLZ3	0.2770	0.0225	0.0000	0	0.1139	0.0739	0.0000	0
DTLZ4	0.2478	0.0424	0.0009	0	0.1630	0.0898	0.0019	0.02
DTLZ5	0.0487	0.0059	0.0000	0	0.0309	0.0176	0.0610	0.06
DTLZ6	0.0484	0.0156	0.0000	0	0.0306	0.0135	0.0000	0
DTLZ7	0.2897	0.0510	0.0011	0.04	0.1880	0.1322	0.0071	0.22

arXiv, 2012

Tables

Table 4: Influence of different κ values on the performance of the coneϵ-MOEA on test problems Deb52, ZDT1, and DTLZ2 with three and four objective functions. Median (M) and standard deviation (SD) over 30 independent runs are shown. Intermediate values for κ seem to yield reasonably good performance values for all metrics. A more appropriate study is required in order to formally characterize the effect of this parameter.

Metric		κ ; Deb52										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0006	0.0006	0.0005	0.0006	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.0001	0.0001
Δ	M	0.6766	0.6813	0.5244	0.2991	0.2552	0.2432	0.2648	0.2892	0.3147	0.3194	0.3199
	SD	0.0004	0.0021	0.0025	0.0027	0.0034	0.0039	0.0017	0.0019	0.0016	0.0042	0.0066
HV	M	0.2735	0.2779	0.2794	0.2802	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
$\|\mathcal{H}\|$	M	19.00	51.00	74.00	93.00	101.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	$< 10^{-4}$	$< 10^{-4}$	0.2537	0.3457	0.4842	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.1826	0.1826	0.1826

Metric		κ ; ZDT1										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0103	0.0069	0.0055	0.0059	0.0074	0.0040	0.0042	0.0051	0.0053	0.0050	0.0038
	SD	0.0072	0.0038	0.0057	0.0047	0.0049	0.0042	0.0060	0.0058	0.0040	0.0050	0.0034
Δ	M	0.3046	0.5543	0.3678	0.2084	0.1818	0.1812	0.1898	0.1937	0.1934	0.1956	0.1891
	SD	0.0122	0.0607	0.0480	0.0408	0.0235	0.0220	0.0234	0.0251	0.0240	0.0232	0.0155
HV	M	0.8435	0.8561	0.8602	0.8607	0.8598	0.8652	0.8650	0.8636	0.8633	0.8638	0.8657
	SD	0.0115	0.0066	0.0094	0.0079	0.0082	0.0069	0.0099	0.0096	0.0066	0.0083	0.0057
$\|\mathcal{H}\|$	M	37.00	63.00	84.50	98.00	100.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	0.6397	5.7211	2.8730	5.0901	3.8201	0.5467	0.8584	0.9371	0.9377	1.3515	0.7112

Metric		κ ; DTLZ2 (m = 3)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0062	0.0069	0.0072	0.0070	0.0074	0.0079	0.0074	0.0076	0.0074	0.0078	0.0072
	SD	0.0002	0.0013	0.0015	0.0013	0.0012	0.0014	0.0010	0.0019	0.0007	0.0014	0.0009
Δ	M	0.0503	0.6066	0.3029	0.2411	0.2386	0.2308	0.2274	0.2175	0.2079	0.2173	0.1982
	SD	0.0041	0.0422	0.0357	0.0302	0.0264	0.0219	0.0316	0.0275	0.0306	0.0295	0.0239
HV	M	0.6731	0.7149	0.7383	0.7435	0.7458	0.7469	0.7467	0.7469	0.7470	0.7470	0.7471
	SD	0.0066	0.0042	0.0023	0.0012	0.0007	0.0006	0.0005	0.0005	0.0005	0.0005	0.0003
$\|\mathcal{H}\|$	M	21.00	69.00	88.00	93.00	94.50	95.00	95.00	95.00	95.00	95.00	94.00
	SD	1.3047	3.1639	2.8367	2.0424	1.7750	1.9464	2.2894	2.0197	1.5643	2.2614	1.7100

Metric		κ ; DTLZ2 (m = 4)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0001	0.0311	0.0385	0.0312	0.0449	0.0404	0.0445	0.0488	0.0590	0.0489	0.0534
	SD	0.0001	0.0198	0.0218	0.0239	0.0283	0.0240	0.0369	0.0281	0.0284	0.0241	0.0304
Δ	M	0.1390	0.4700	0.3602	0.3296	0.3299	0.3429	0.3377	0.3258	0.3253	0.3304	0.3319
	SD	0.1173	0.0304	0.0307	0.0226	0.0255	0.0263	0.0187	0.0262	0.0210	0.0254	0.0259
$\|\mathcal{H}\|$	M	14.00	79.50	90.00	92.00	95.00	96.00	95.50	97.00	98.00	95.50	97.00
	SD	1.9815	4.9642	4.8476	4.2372	4.6307	4.2129	5.8530	5.0496	4.5945	4.6233	4.3423

arXiv, 2020

Table 7: Problemwise comparison of the algorithms on the four performance metrics used, for problems Deb52, Pol and the ZDT family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ-MOEA	coneϵ-MOEA	C-NSGA-II	SPEA2	NSGA-II*
Deb52	γ	0.58±7e-3	0.56±3e-3	0.56±2e-3	0.58±1e-2	0.55±7e-3	0.55±6e-3
	Δ	0.53±8e-3	0.99±6e-4	0.32±3e-4	0.33±6e-3	0.20±3e-3	0.41±8e-3
	HV	0.99±7e-5	0.99±6e-5	0.99±0	0.99±1e-4	0.99±3e-5	0.99±7e-5
	CS	0.02±7e-4	0.03±10e-4	0.03±8e-4	0.02±8e-4	0.03±9e-4	0.02±8e-4
Pol	γ	0.20±2e-2	0.13±6e-4	0.19±2e-2	0.19±9e-3	0.15±2e-3	0.16±2e-3
	Δ	0.58±1e-2	0.98±9e-4	0.29±6e-3	0.38±7e-3	0.24±3e-3	0.36±8e-3
	HV	1.00±1e-5	1.00±5e-6	1.00±4e-6	1.00±3e-5	1.00±4e-6	1.00±8e-6
	CS	0.04±1e-3	0.04±1e-3	0.04±2e-3	0.04±10e-4	0.06±1e-3	0.05±1e-3
Zdt1	γ	0.16±2e-2	0.30±3e-2	0.23±3e-2	0.18±2e-2	0.30±2e-2	0.19±2e-2
	Δ	0.79±1e-2	0.70±4e-3	0.37±6e-3	0.50±8e-3	0.29±7e-3	0.56±1e-2
	HV	0.99±10e-4	0.98±2e-3	0.98±2e-3	0.98±9e-4	0.98±1e-3	0.98±1e-3
	CS	0.33±2e-2	0.20±3e-2	0.27±3e-2	0.30±2e-2	0.15±2e-2	0.27±2e-2
Zdt2	γ	0.43±9e-3	0.65±8e-3	0.30±4e-3	0.80±1e-2	0.41±8e-3	0.41±8e-3
	Δ	0.76±1e-2	0.56±3e-3	0.38±4e-3	0.50±7e-3	0.28±4e-3	0.58±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.99±4e-5	0.98±1e-4	0.99±8e-5	0.99±9e-5
	CS	0.07±3e-3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Zdt3	γ	0.16±2e-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Δ	0.67±1e-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	HV	0.98±2e-3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	CS	0.41±3e-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Zdt4	γ	0.27±2e-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Δ	0.61±9e-3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	HV	0.84±9e-3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	CS	0.73±3e-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Zdt6	γ	0.06±3e-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Δ	0.52±1e-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	HV	0.96±1e-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	CS	0.18±2e-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 8: Problemwise comparison of the algorithms on the four performance metrics used, for the DTLZ family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ-MOEA	coneϵ-MOEA	C-NSGA-II	SPEA2	NSGA-II*
Dtlz1	γ	0.23±7e-3	0.13±1e-3	0.17±2e-2	0.39±1e-2	0.18±2e-3	0.17±2e-3
	Δ	0.34±3e-3	0.12±2e-3	0.05±1e-2	0.20±2e-2	0.08±1e-3	0.34±4e-3
	HV	0.95±6e-4	0.92±4e-4	0.95±2e-4	0.96±5e-4	0.97±1e-4	0.96±5e-4
	CS	0.02±8e-4	0.01±9e-4	0.03±1e-3	0.00±5e-4	0.02±1e-3	0.02±10e-4
Dtlz2	γ	0.62±10e-3	0.70±7e-3	0.48±8e-3	0.75±1e-2	0.55±8e-3	0.48±6e-3
	Δ	0.81±10e-3	0.42±4e-3	0.42±6e-3	0.29±5e-3	0.16±3e-3	0.83±9e-3
	HV	0.89±9e-4	0.92±4e-4	0.94±1e-4	0.90±7e-4	0.93±4e-4	0.89±9e-4
	CS	0.03±1e-3	0.02±8e-4	0.06±2e-3	0.01±6e-4	0.03±1e-3	0.04±1e-3
Dtlz3	γ	0.35±2e-2	0.25±1e-2	0.42±3e-2	0.50±3e-2	0.32±2e-2	0.26±1e-2
	Δ	0.33±9e-3	0.19±1e-2	0.29±2e-2	0.22±3e-2	0.15±2e-2	0.34±8e-3
	HV	0.90±10e-4	0.91±2e-2	0.91±1e-2	0.91±1e-3	0.93±3e-4	0.90±9e-4
	CS	0.02±1e-3	0.03±2e-3	0.04±2e-3	0.00±5e-4	0.02±2e-3	0.02±2e-3
Dtlz4	γ	0.32±8e-3	0.41±2e-2	0.53±3e-2	0.34±1e-2	0.33±1e-2	0.30±3e-3
	Δ	0.67±2e-2	0.37±3e-2	0.43±2e-2	0.36±4e-2	0.22±3e-2	0.66±8e-3
	HV	0.88±1e-2	0.86±2e-2	0.86±2e-2	0.84±2e-2	0.87±2e-2	0.90±7e-4
	CS	0.04±2e-3	0.02±1e-3	0.03±2e-3	0.02±2e-3	0.03±2e-3	0.03±1e-3
Dtlz5	γ	0.14±2e-3	0.26±3e-3	0.56±3e-2	0.22±5e-3	0.15±2e-3	0.13±1e-3
	Δ	0.74±2e-2	0.78±5e-3	0.83±9e-3	0.43±6e-3	0.26±4e-3	0.61±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.98±3e-5	0.98±1e-4	0.99±4e-5	0.99±1e-4
	CS	0.06±2e-3	0.02±1e-3	0.04±1e-3	0.03±1e-3	0.05±2e-3	0.07±2e-3
[illegible]	γ	0.84±8e-3	0.94±3e-3	0.83±3e-3	0.83±5e-3	0.84±7e-3	0.84±8e-3
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.32±5e-3	0.62±1e-2
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.99±3e-5	0.99±1e-4
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.03±8e-4	0.01±7e-4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.67±8e-3	0.74±10e-3
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.52±8e-3	0.78±8e-3
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.94±6e-4	0.92±9e-4
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.09±3e-3	0.07±2e-3
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.83±9e-3	0.58±1e-2
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.36±8e-3	0.80±1e-2
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.10±3e-3	0.09±3e-3
Dtlz9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.77±7e-3	0.90±5e-3
	Δ	0.62±5e-3	0.71±1e-2	0.52±6e-3	0.49±6e-3	0.45±4e-3	0.61±6e-3
	CS	0.11±8e-3	0.51±1e-2	0.34±7e-3	0.23±8e-3	0.24±8e-3	0.13±7e-3

Numbers have their value.

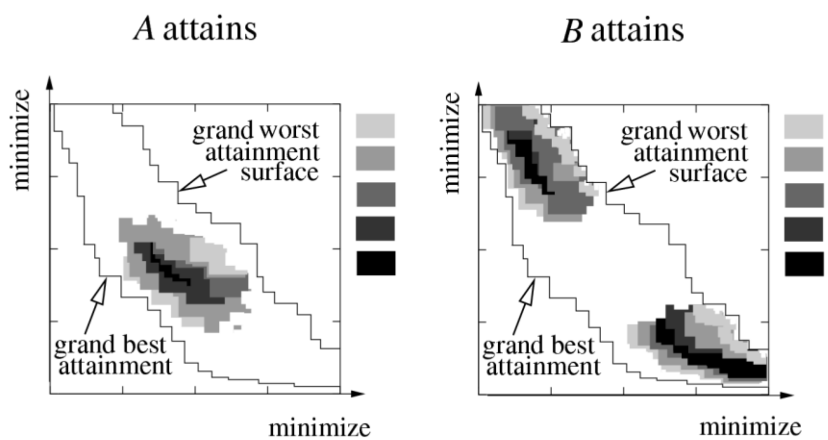
But not *only* tables, please!

v1.0

v1.0: Two Approaches for Empirical Studies

Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur



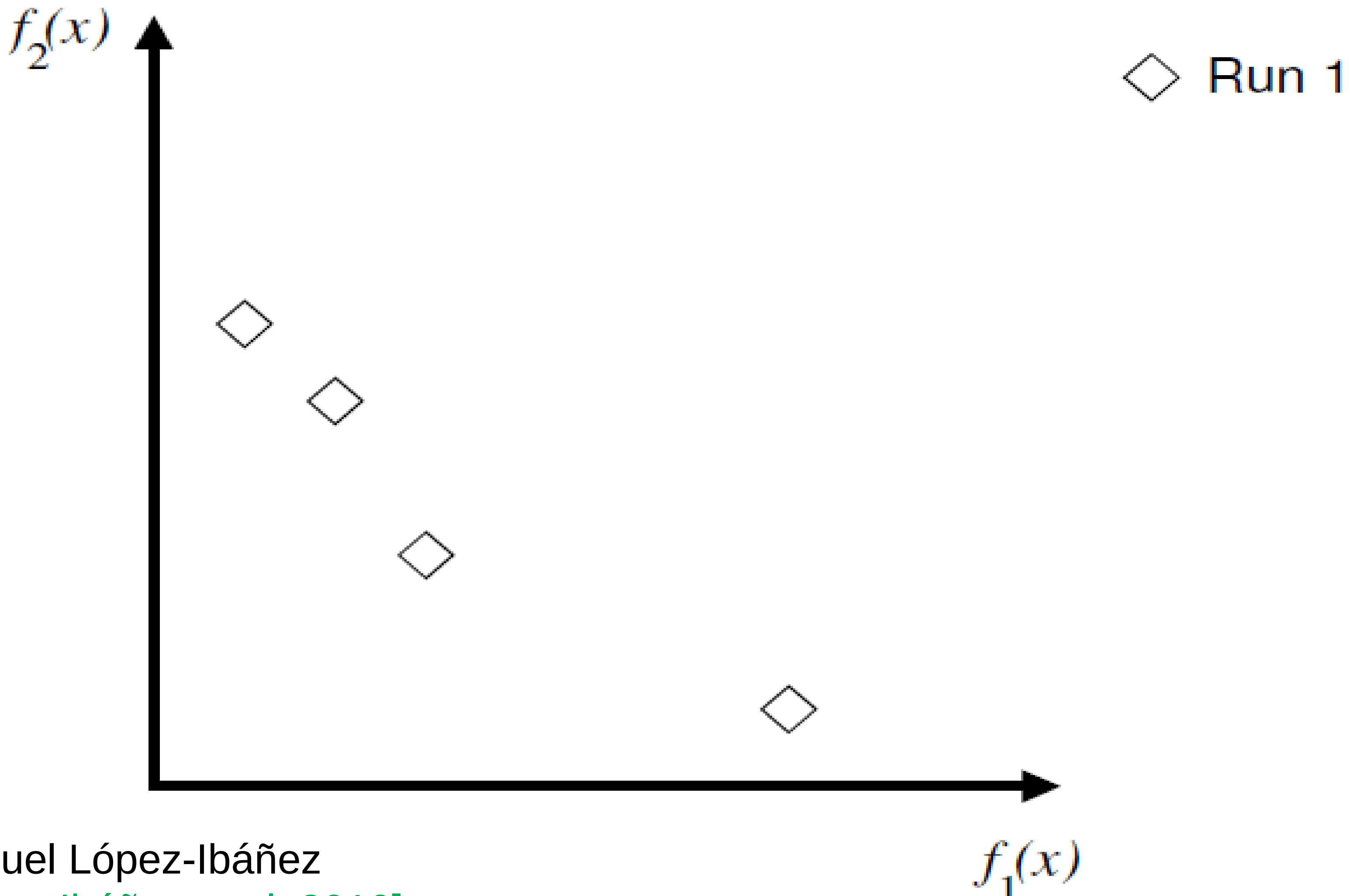
Quality indicator approach

- reduces each approximation set to a single quality value
- applies statistical tests to the quality values

Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

Empirical Attainment Functions



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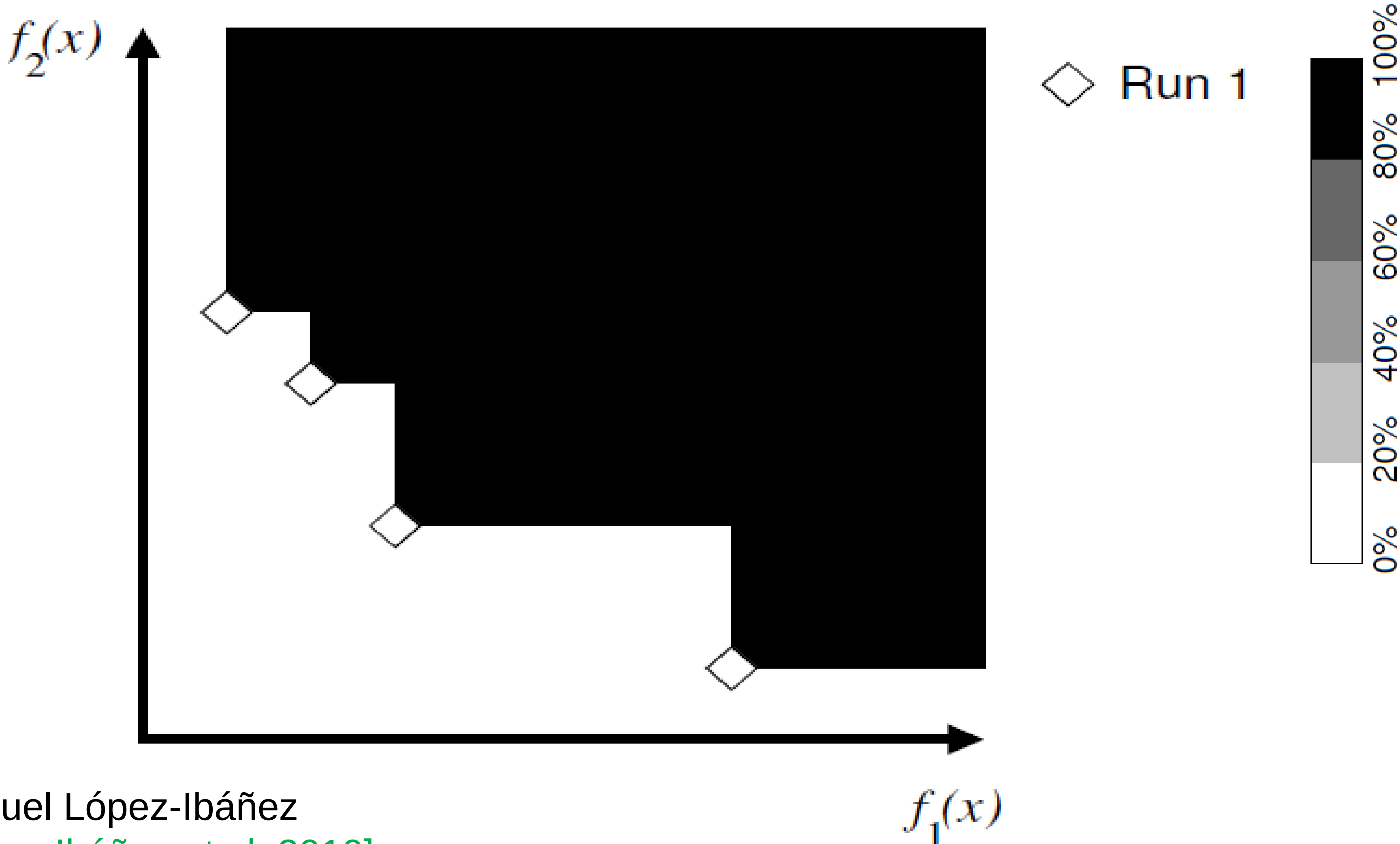
[López-Ibáñez et al. 2010]

Empirical Attainment Functions



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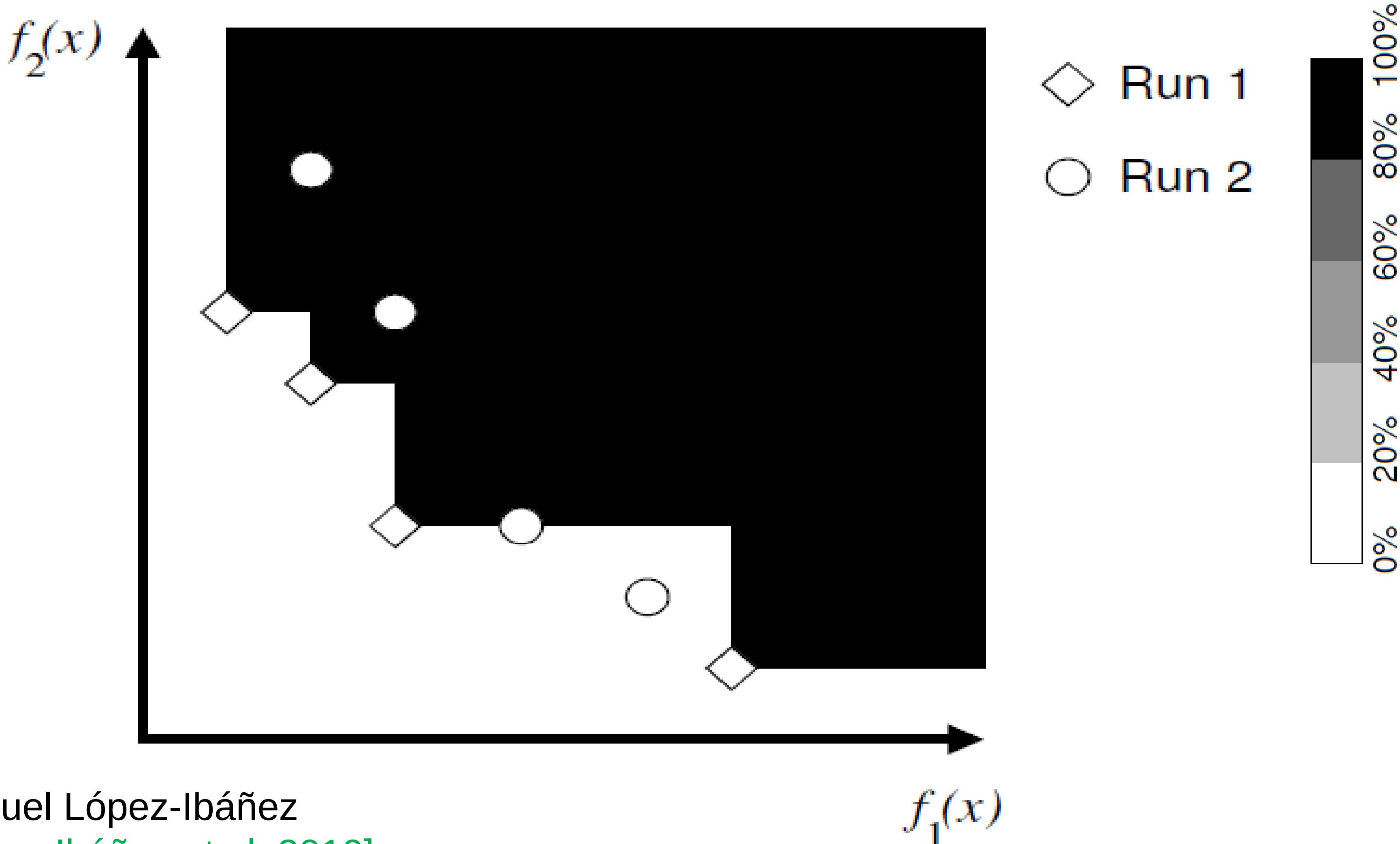

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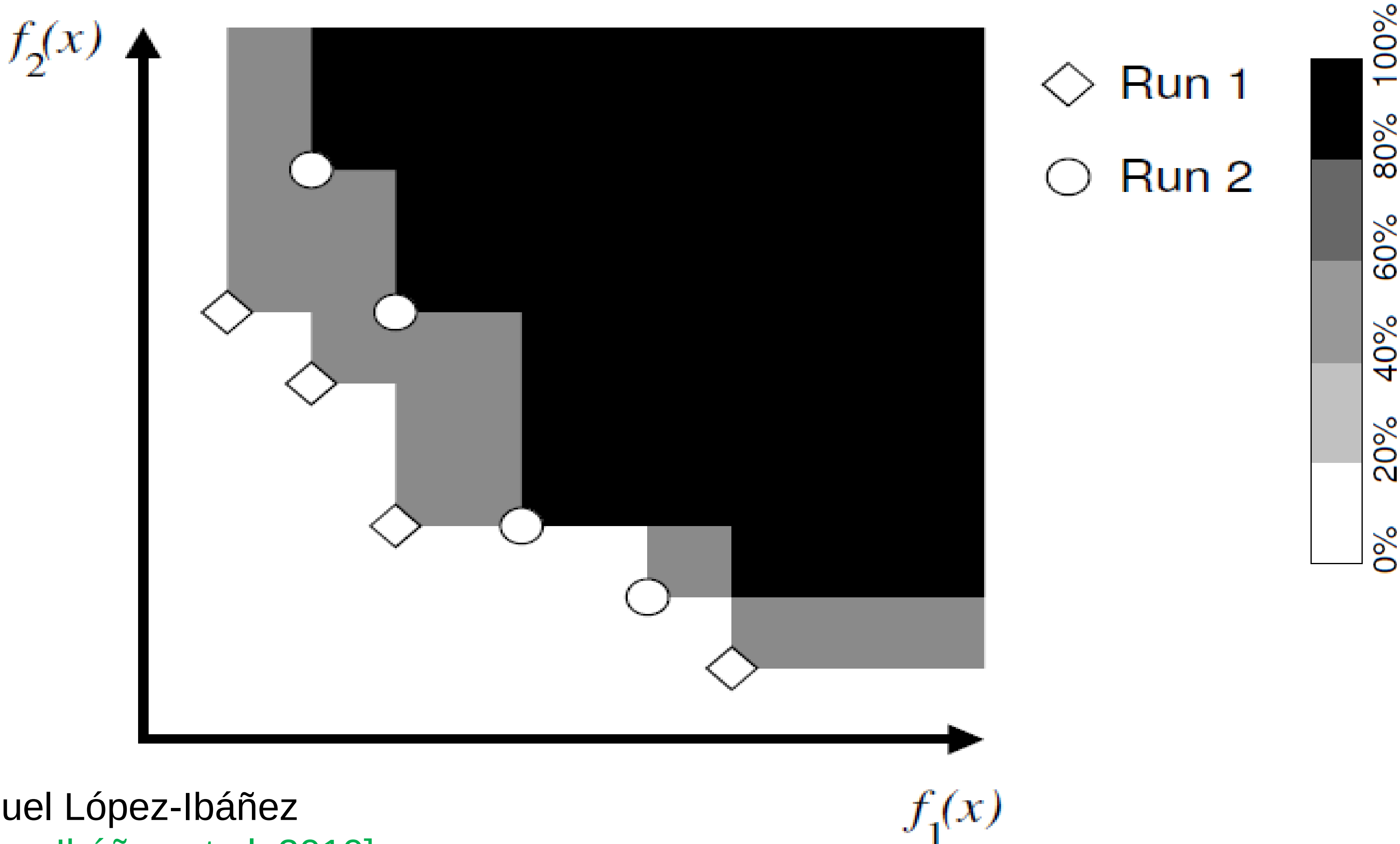
Empirical Attainment Functions



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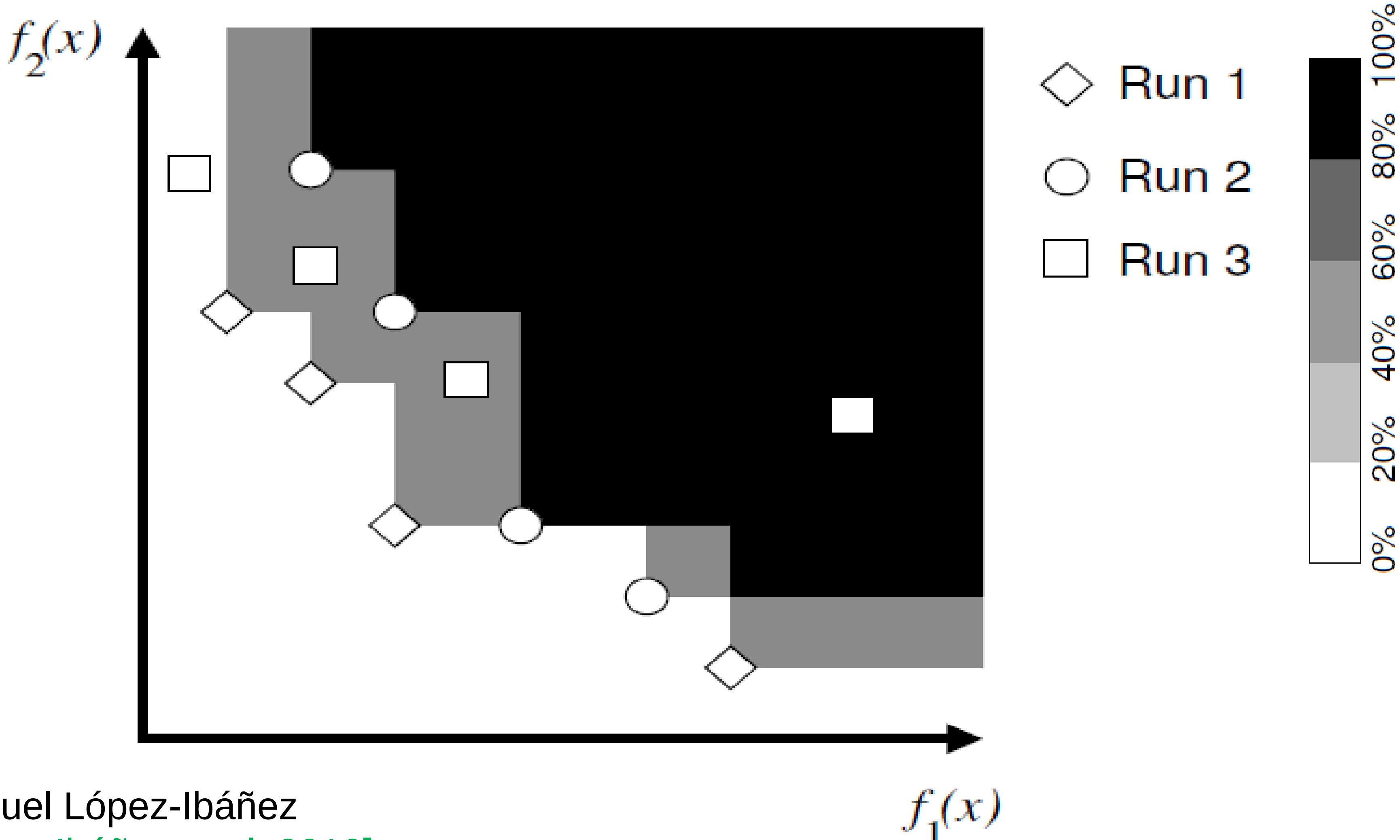
[López-Ibáñez et al. 2010]

Empirical Attainment Functions



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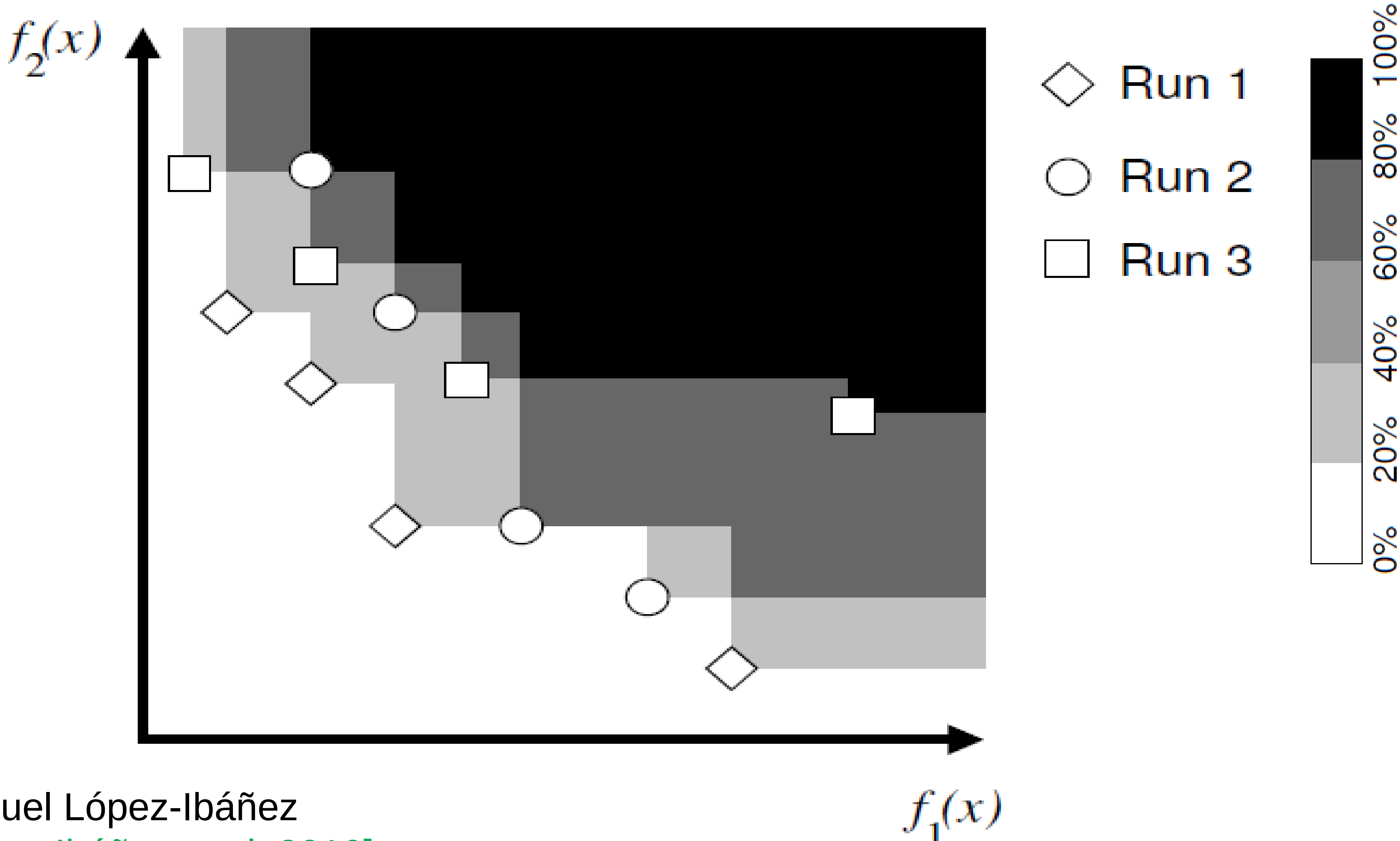
[López-Ibáñez et al. 2010]

Empirical Attainment Functions



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[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Definition

The Empirical Attainment Function $\alpha(z)$ "counts" how many solution sets $\mathcal{X}_i$ attain or dominate a vector z at time T:

$$\alpha_T(z) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\{\mathcal{X}_i \trianglelefteq_T z\}}$$

with $\trianglelefteq_T$ being the weak dominance relation between a solution set and an objective vector at time T.

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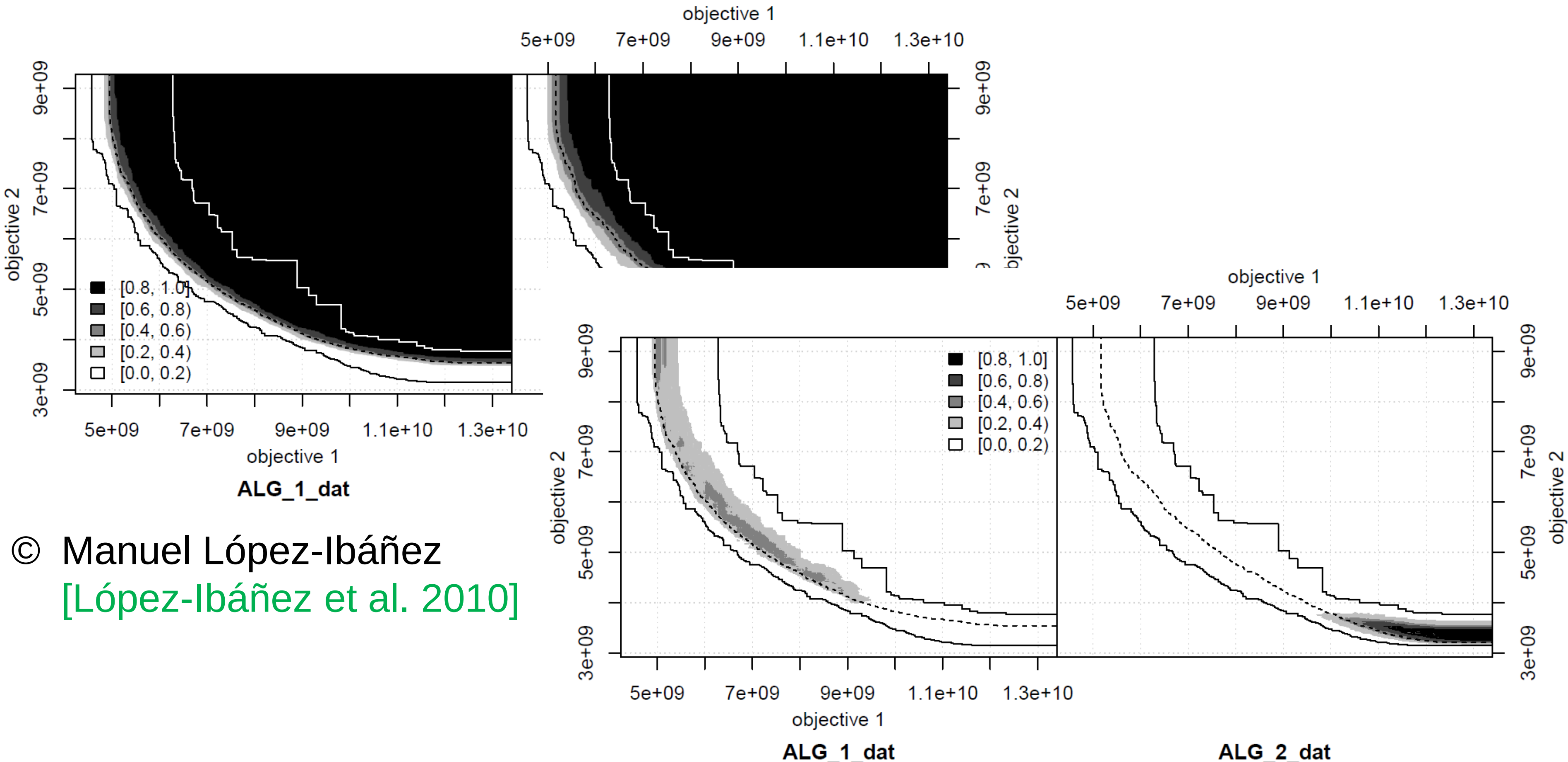
Note that $\alpha_T(z)$ is the empirical cumulative distribution function of the achieved objective function distribution at time T in the single-objective case ("fixed budget scenario").

Empirical Attainment Functions in Practice



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[López-Ibáñez et al. 2010]

latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
R package: **`http://lopez-ibanez.eu/eaftools`**
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function ("unary quality indicator") on sets
- use the resulting total (pre-)order (on the quality values)

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- transfer multiobjective problem into a set problem
- define an objective function ("unary quality indicator") on sets
- use the resulting total (pre-)order (on the quality values)

Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation should be reflected!

$$A \preccurlyeq B :\Leftrightarrow \forall_{b \in B} \exists_{a \in A}\, a \preccurlyeq b$$

Monotonicity and Strict Monotonicity

- Monotonicity when quality indicator does not contradict relation

$$A \preccurlyeq B \Rightarrow I(A) \geq I(B)$$

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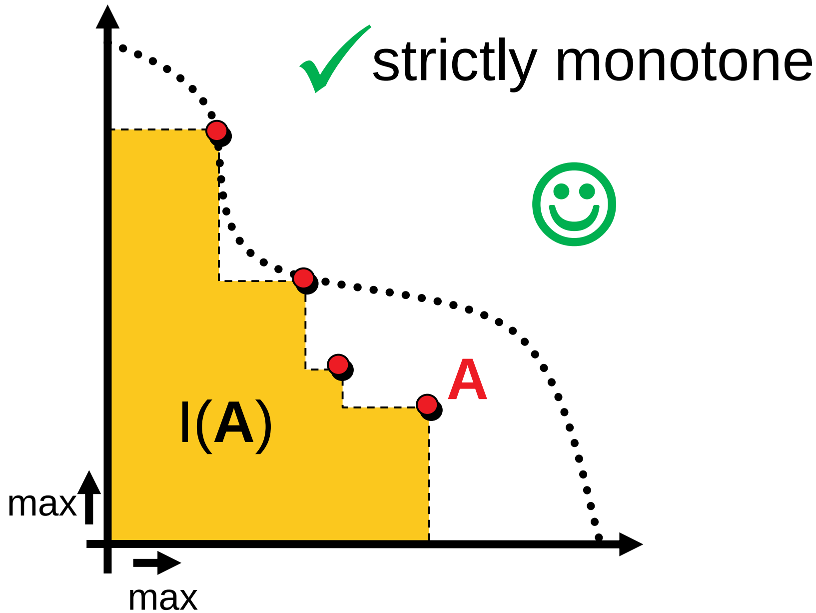
- Strict monotonicity: better = higher indicator

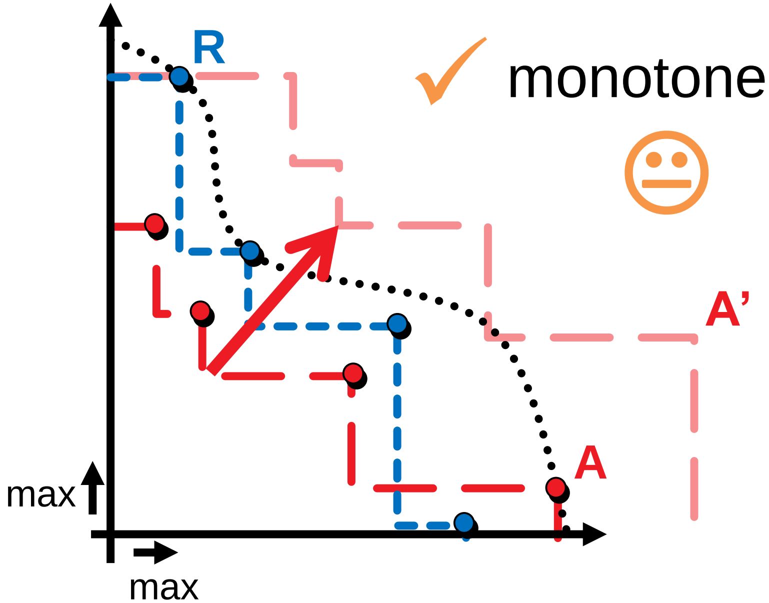
$$A \preccurlyeq B \text{ and } A \neq B \Rightarrow I(A) > I(B)$$

Example: Refinements Using Indicators

I(A) = volume of the weakly dominated area in objective space

I(A,R) = how much needs A to be moved to weakly dominate R





unary hypervolume indicator

unary epsilon indicator

v1.0.1 – v1.0.100 and counting

Many Indicators Available

Performance Assessment of Multiobjective Optimizers: An Analysis and Review

Eckart Zitzler[1], Lothar Thiele[1], Marco Laumanns[1],
Carlos M. Fonseca[2], and Viviane Grunert da Fonseca[2]

[1] Computer Engineering and Networks Laboratory (TIK)
Department of Information Technology and Electrical Engineering
Swiss Federal Institute of Technology (ETH) Zurich, Switzerland
Email: {zitzler, thiele, laumanns}@tik.ee.ethz.ch

[2]ADEEC and ISR (Coimbra)
Faculty of Sciences and Technology
University of Algarve, Portugal
Email: cmfonsec@ualg.pt, vgrunert@csi.fct.ualg.pt

[Zitzler et al. 2003]

22 indicators

Even More Indicators…

Performance indicators in multiobjective optimization

Charles Audet[a], Jean Bigeon[b], Dominique Cartier[c], Sébastien Le Digabel[a], Ludovic Salomon[a,1]

[a]*GERAD and Département de mathématiques et génie industriel, École Polytechnique de Montréal, C.P. 6079, Succ. Centre-ville, Montréal, Québec, H3C 3A7, Canada.*
[b]*Univ. Grenoble Alpes, CNRS, Grenoble INP, G-SCOP, 38000 Grenoble, France.*
[c]*Collège de Maisonneuve, 3800 Rue Sherbrooke E, Montréal, Québec, H1X 2A2, Canada.*

[Audet et al 2021]

63 indicators

Quality Evaluation of Solution Sets in Multiobjective Optimisation: A Survey

Miqing Li, and Xin Yao[1]

[1]CERCIA, School of Computer Science, University of Birmingham, Birmingham B15 2TT, U. K.
*Email: limitsing@gmail.com, x.yao@cs.bham.ac.uk

[Li and Yao 2019]

100 indicators

Many Indicators: What Do We Do?

Focus on indicators which are (strictly) monotone

- all hypervolume-based indicators
- unary epsilon indicator
- R2
- IGD+

Many Indicators: What Do We Do?

Focus on indicators which are (strictly) monotone

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- unary epsilon indicator
- R2
- IGD+

Why is monotonicity important?

- Pareto dominance is the lowest form of preference
- If dominance relation does not hold, we have not defined a true multiobjective problem.

v2.0

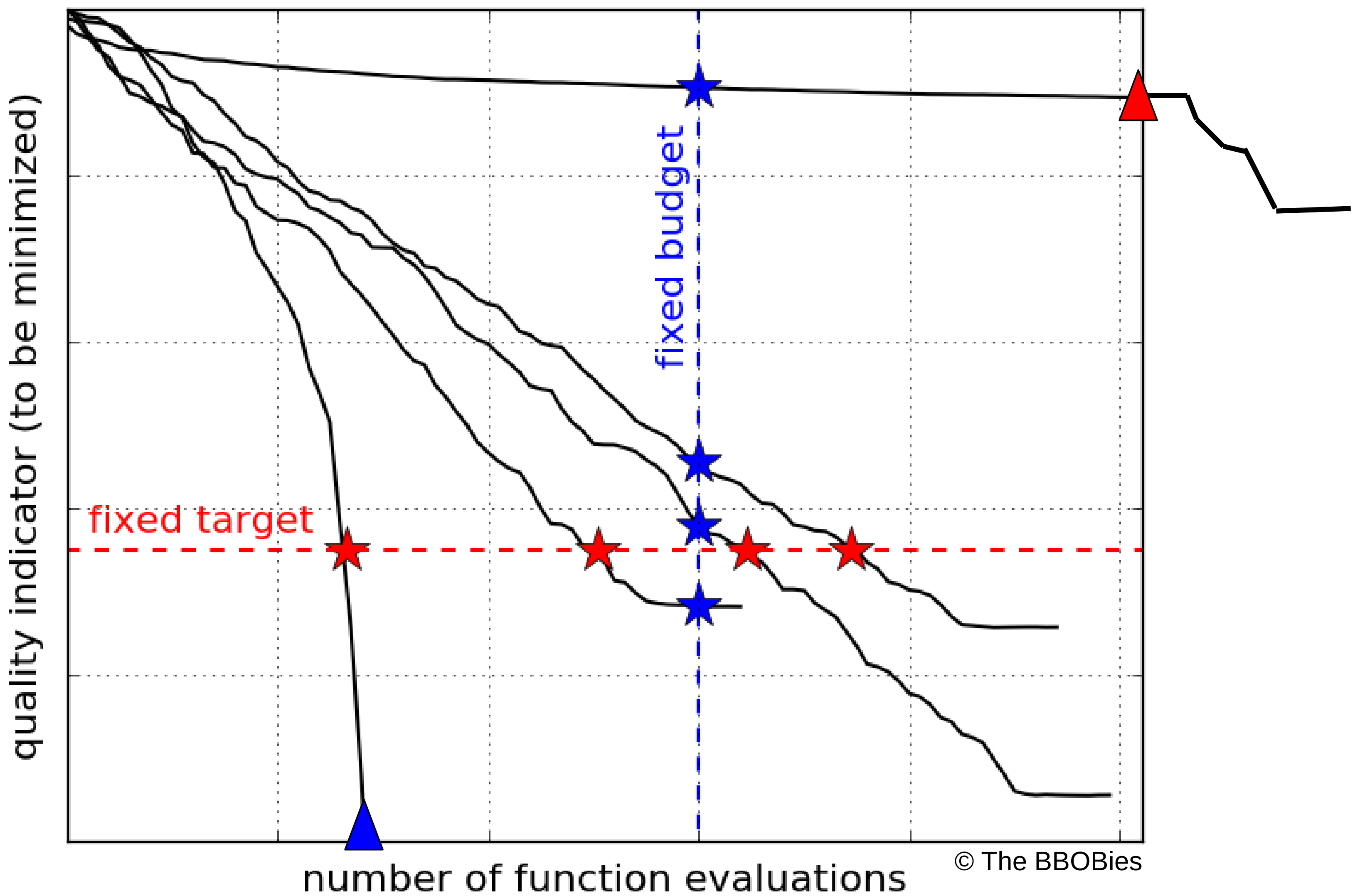
Benchmarking Multiobjective Optimizers 2.0

- With the right (strictly) monotone indicator, multiobjective optimization is not different from single-objective optimization (!)

We can use our normal tools from single-objective optimization, including

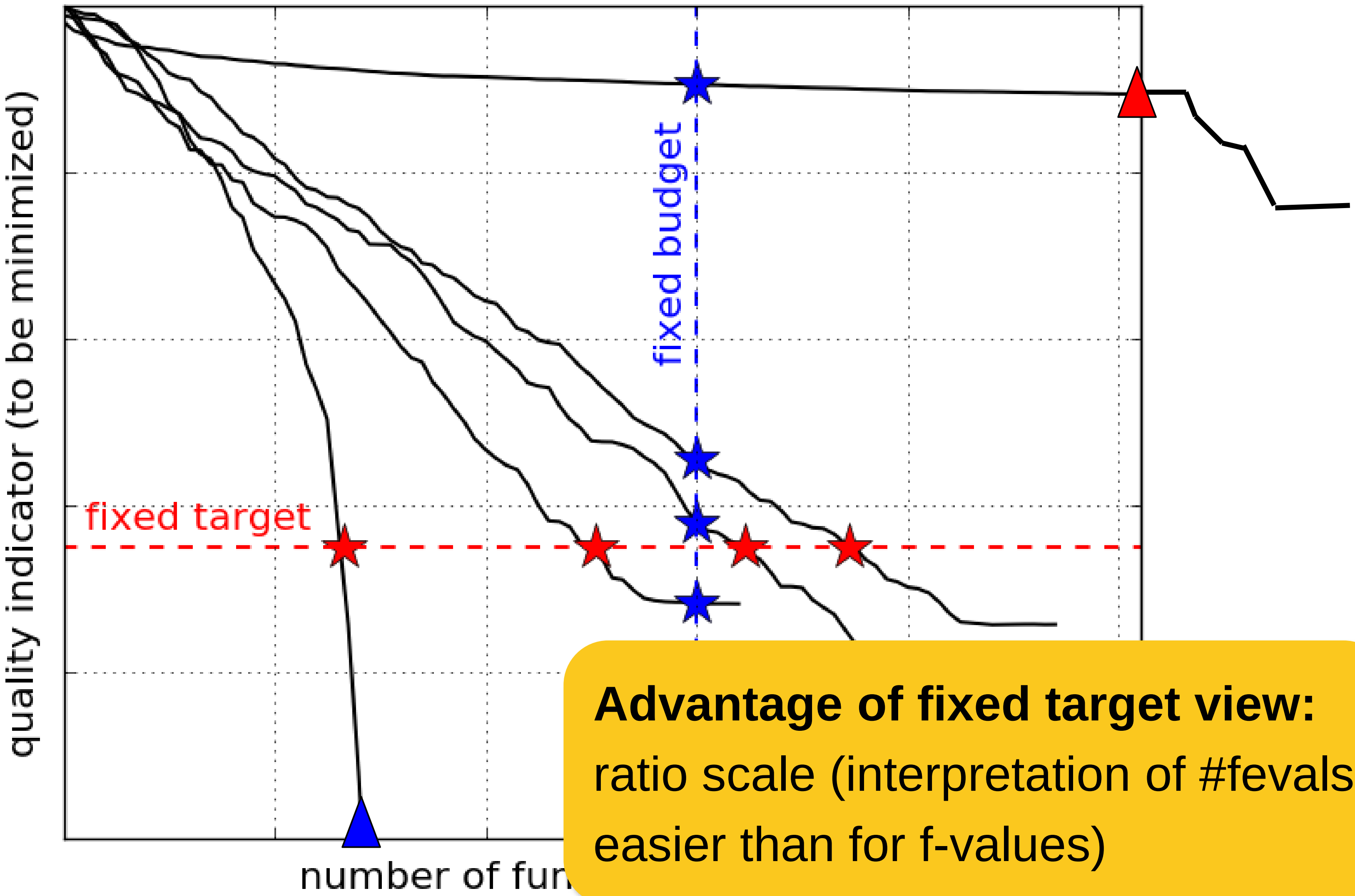
- reporting of target-based runtimes
- ECDFs of runtimes, performance profiles, data profiles
- statistical tests, box plots, …

convergence graphs is all we have to start with...



Measuring Performance Empirically

convergence graphs is all we have to start with...

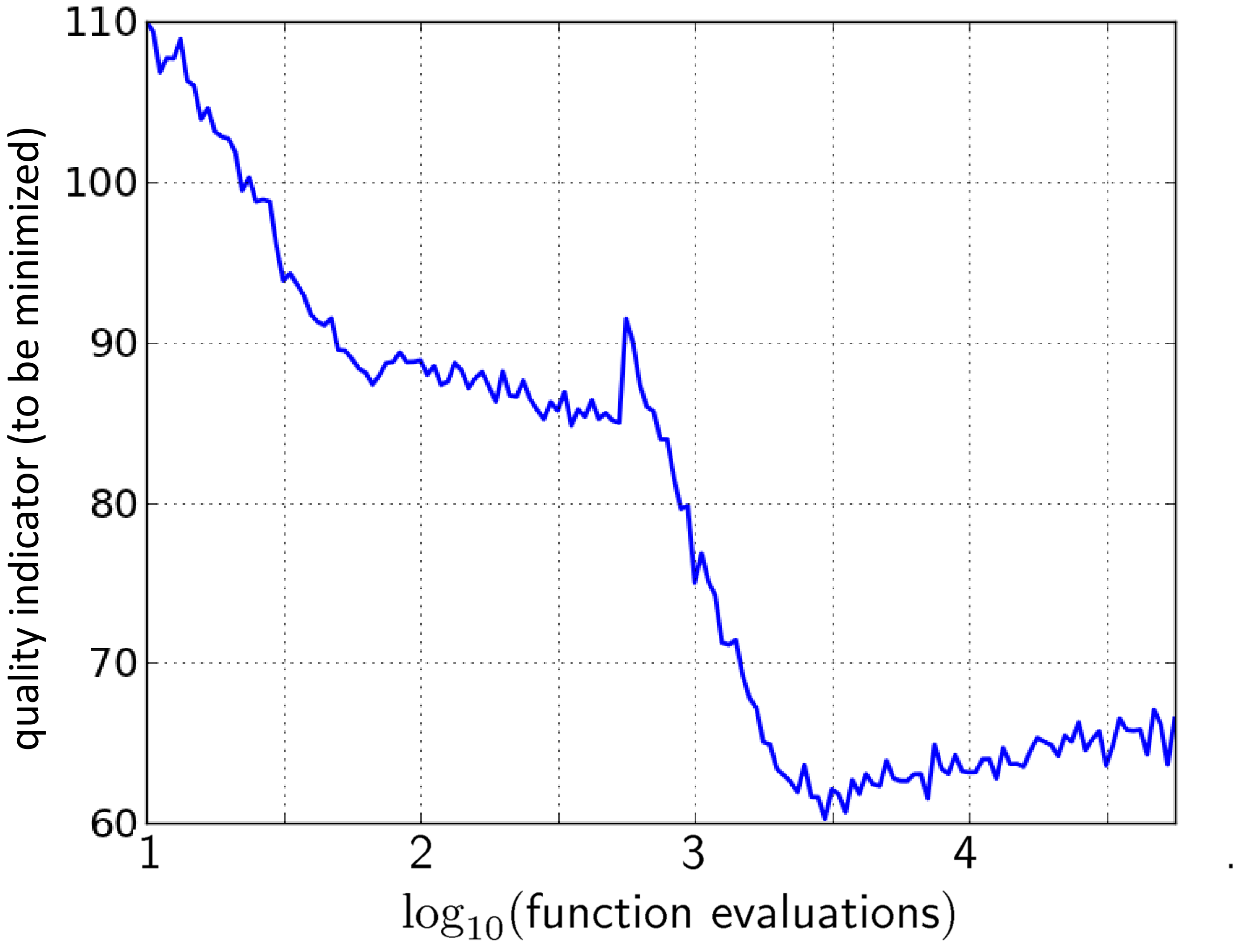


Advantage of fixed target view:
ratio scale (interpretation of #fevals easier than for f-values)

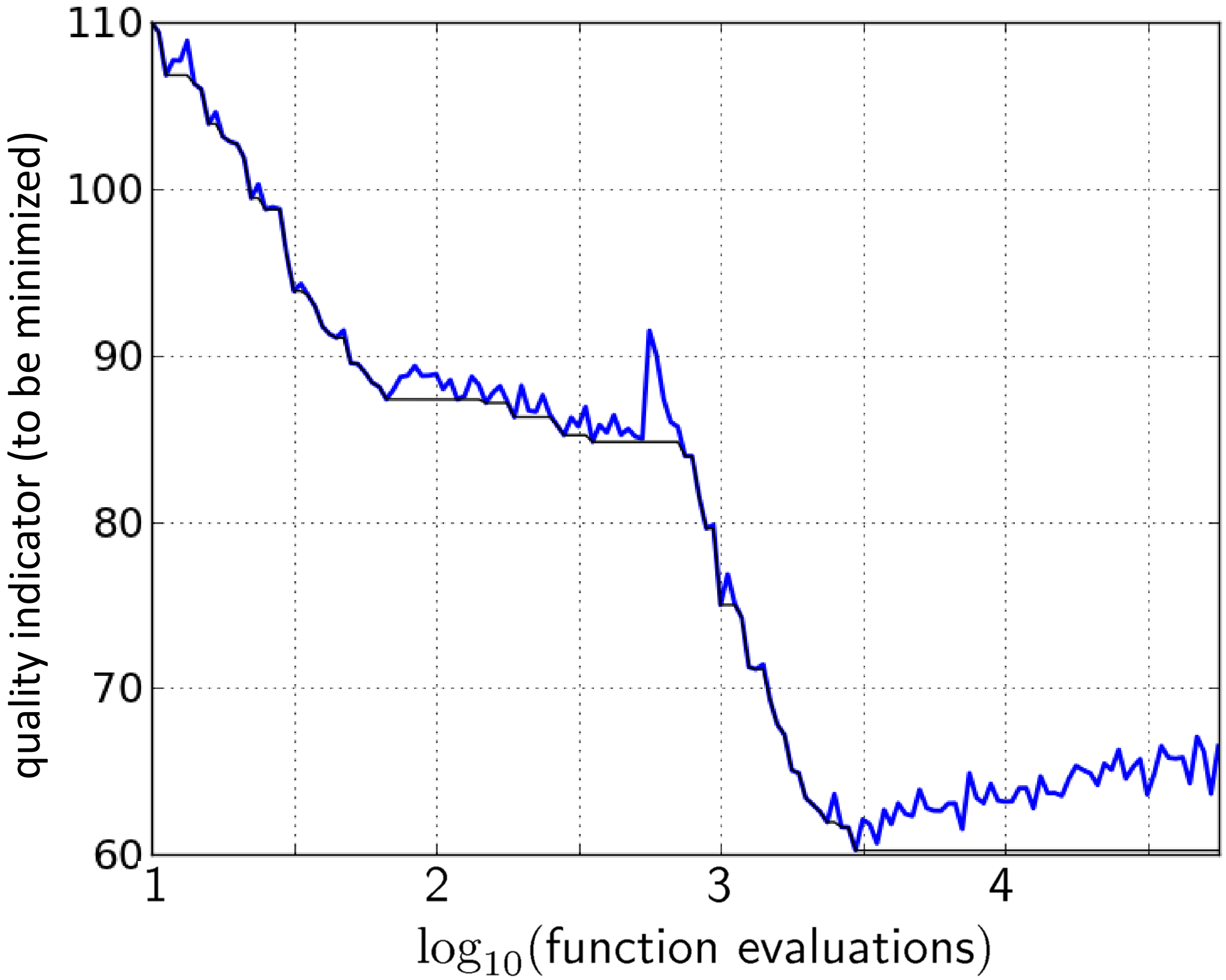
ECDF:

Empirical Cumulative Distribution Function of the Runtime

A Convergence Graph



First Hitting Time is Monotonous



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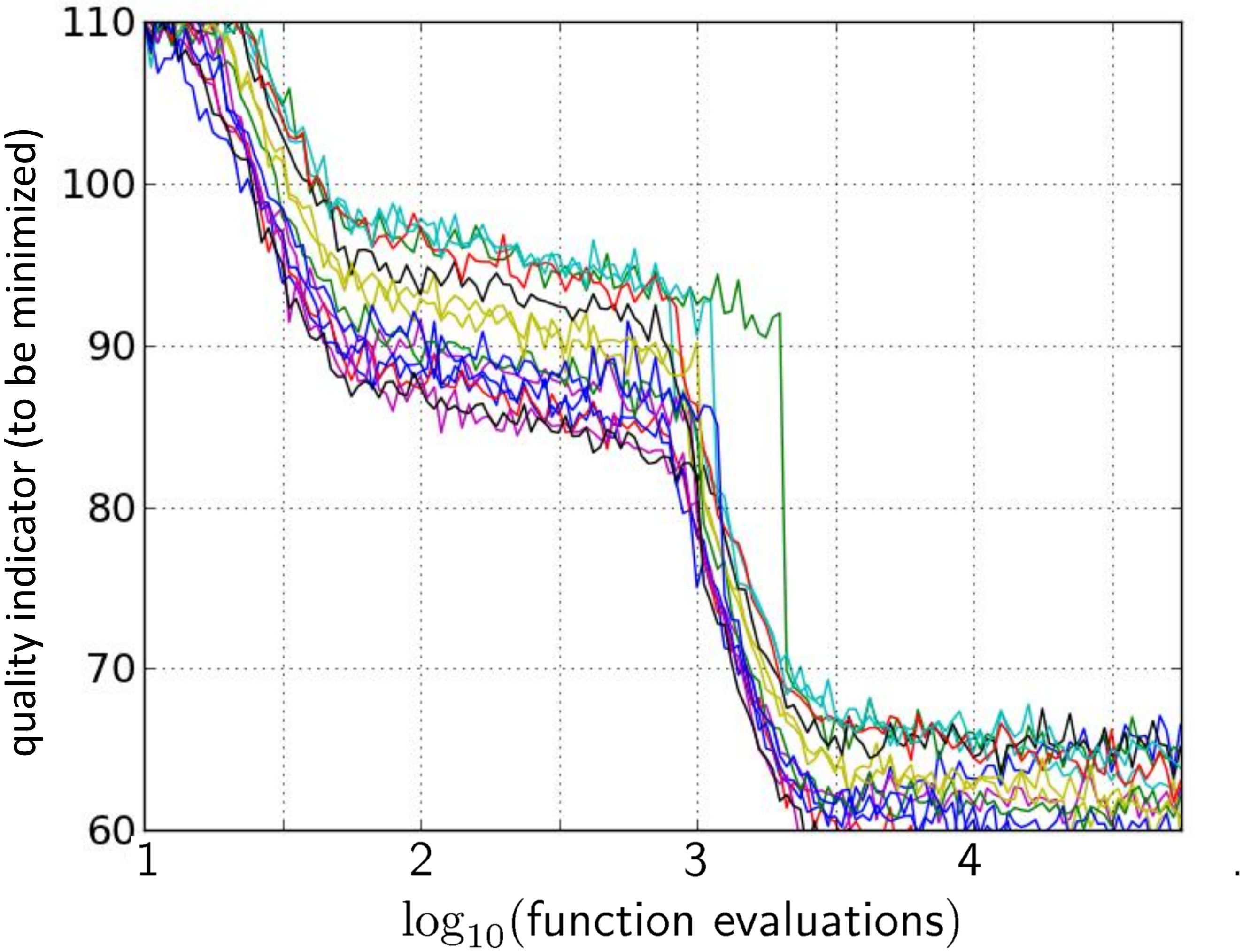
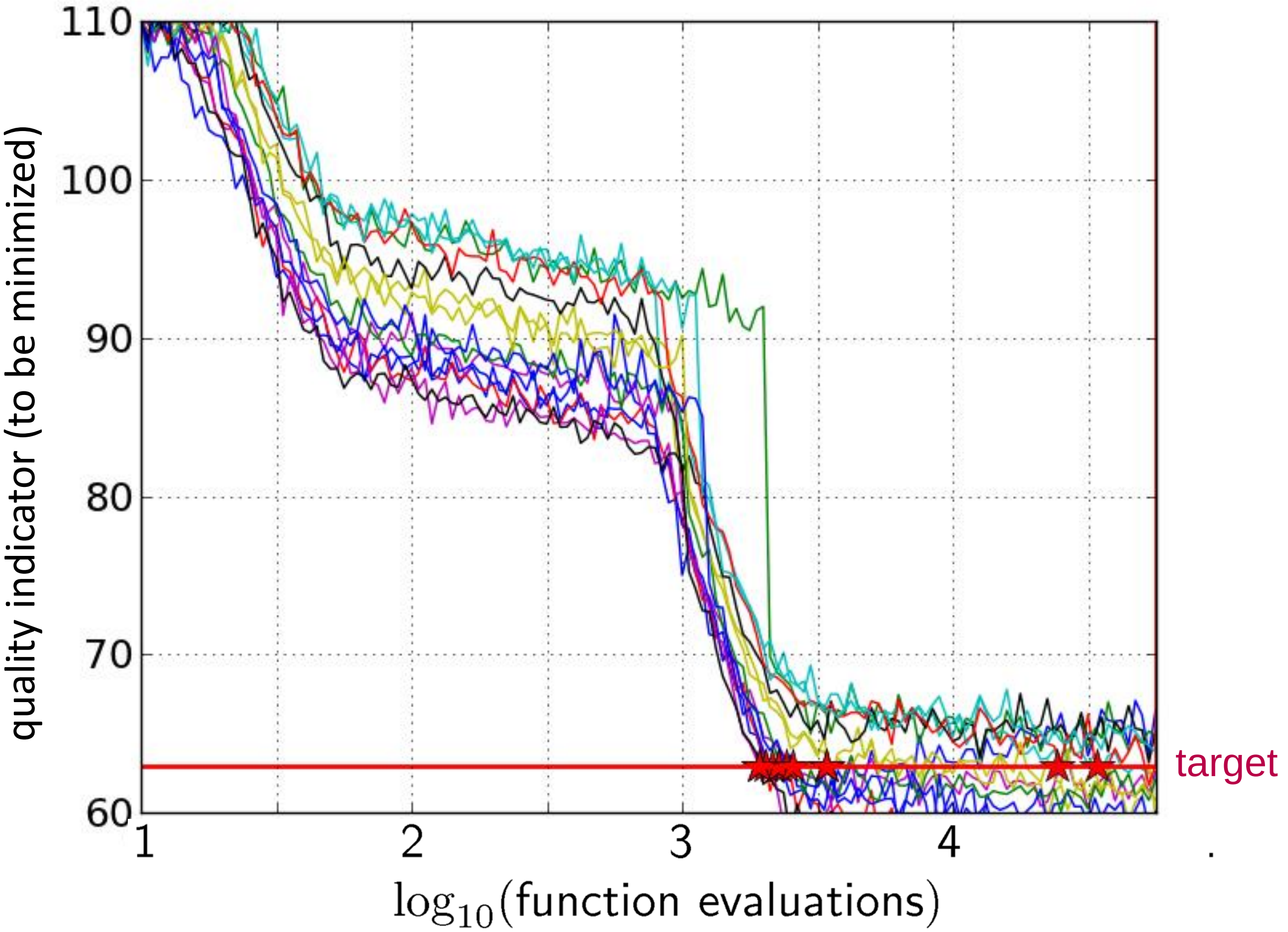
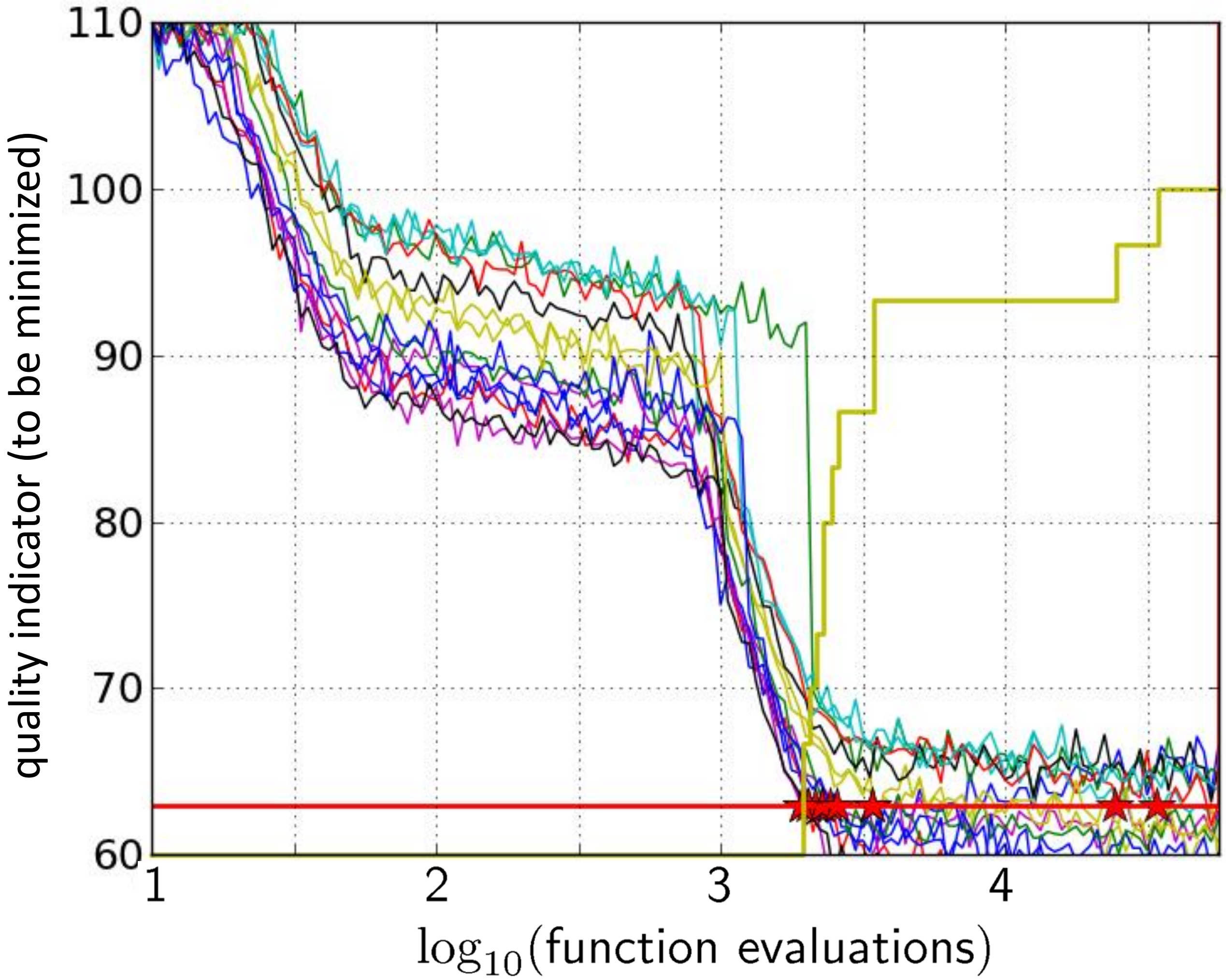
15 Runs



110
100
90
80
70
60
1
2
3
4
quality indicator (to be minimized)
log10(function evaluations)

15 Runs ≤ 15 Runtime Data Points

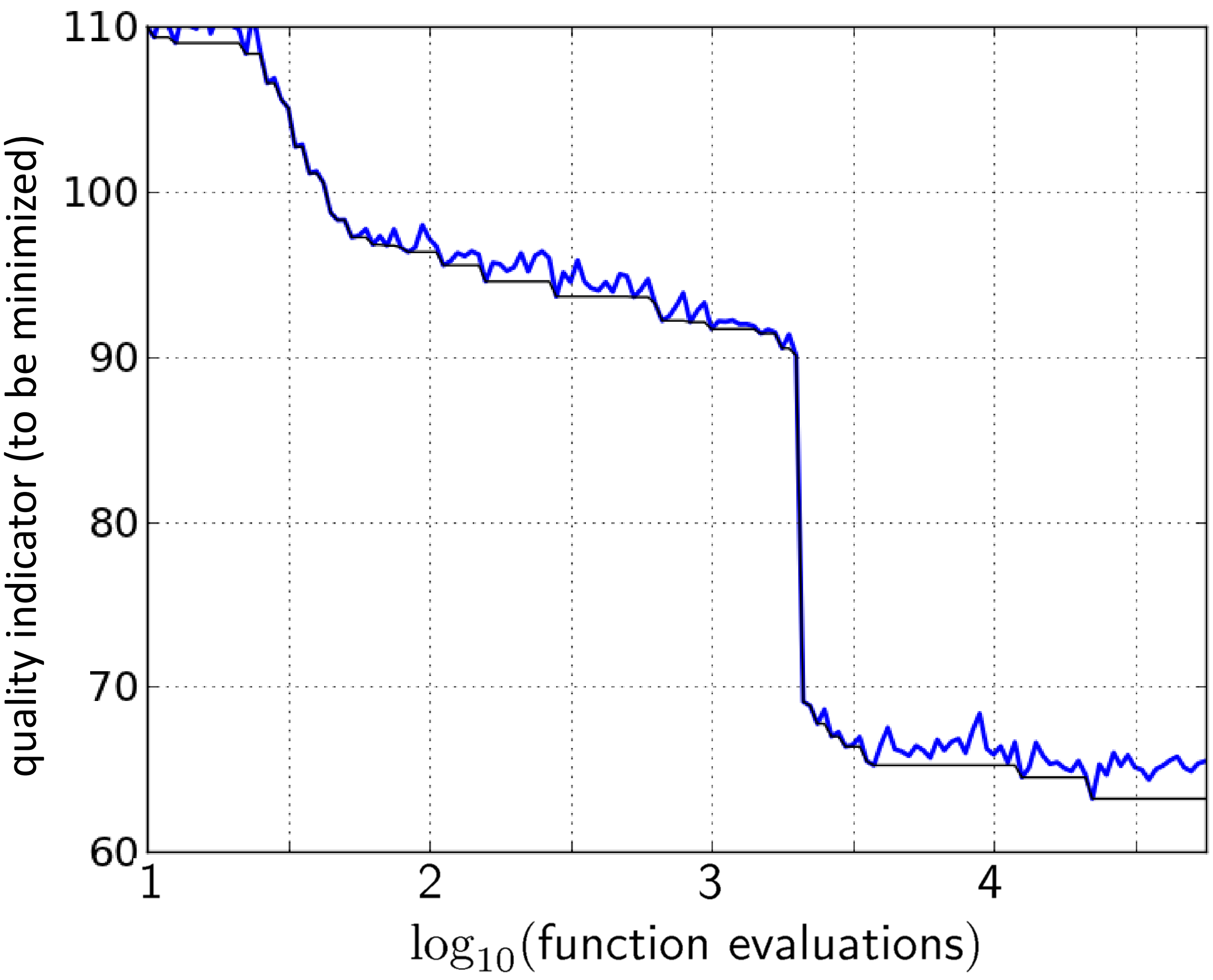




the ECDF of run lengths to reach the target

- has for each data point a vertical step of constant size
- displays for each x-value (budget) the count of observations to the left (first hitting times)

Reconstructing A Single Run



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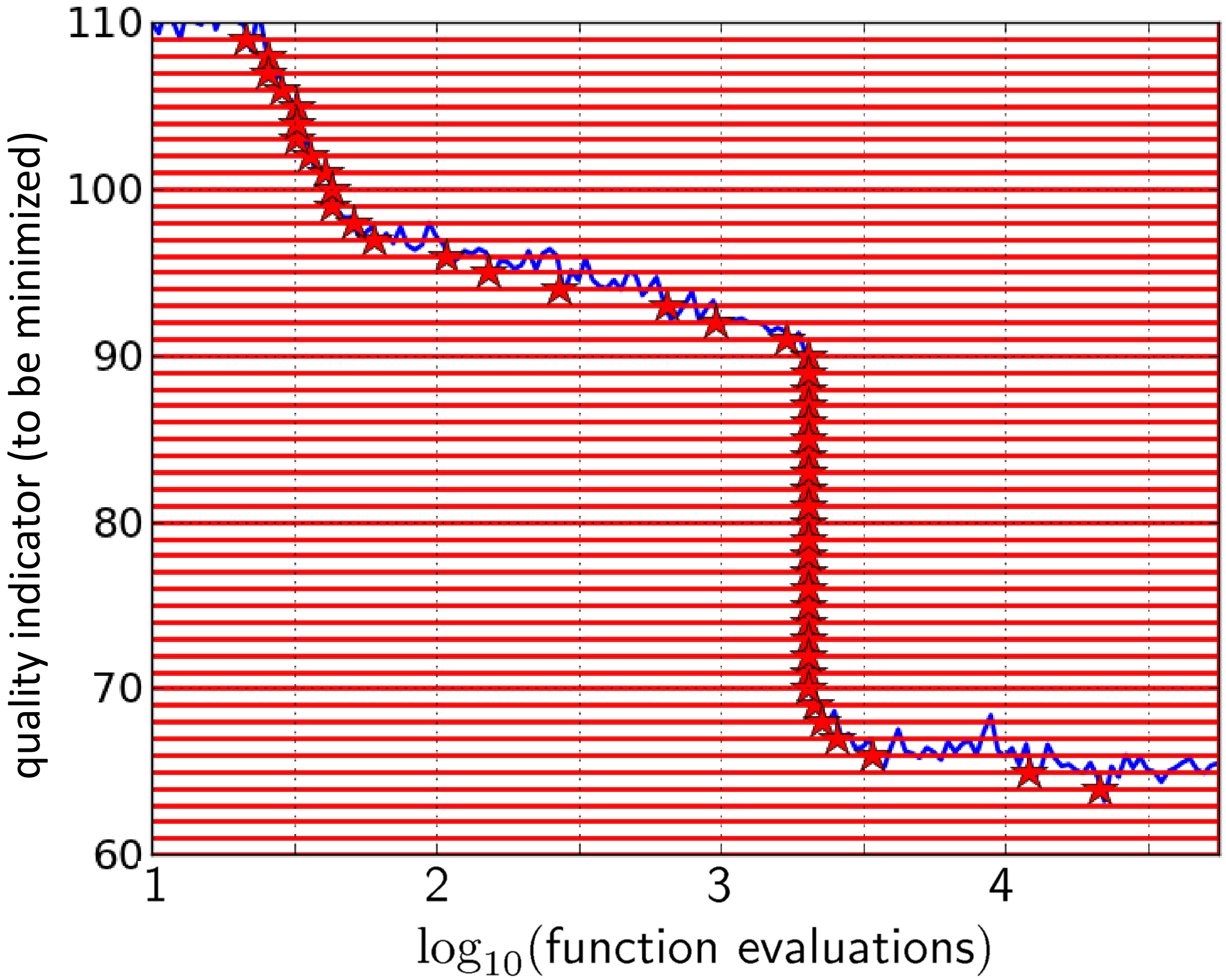
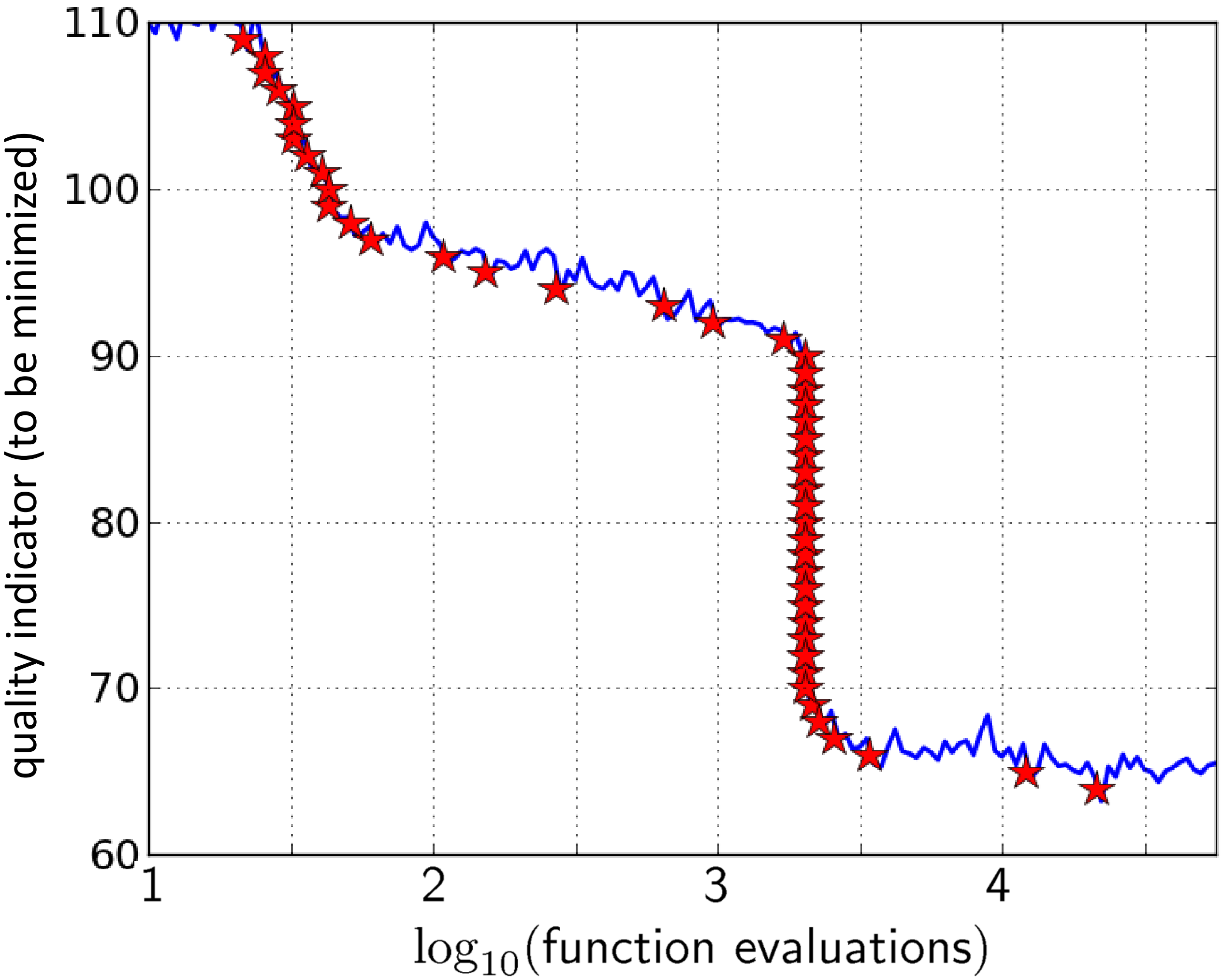


50 equally spaced targets

Reconstructing A Single Run



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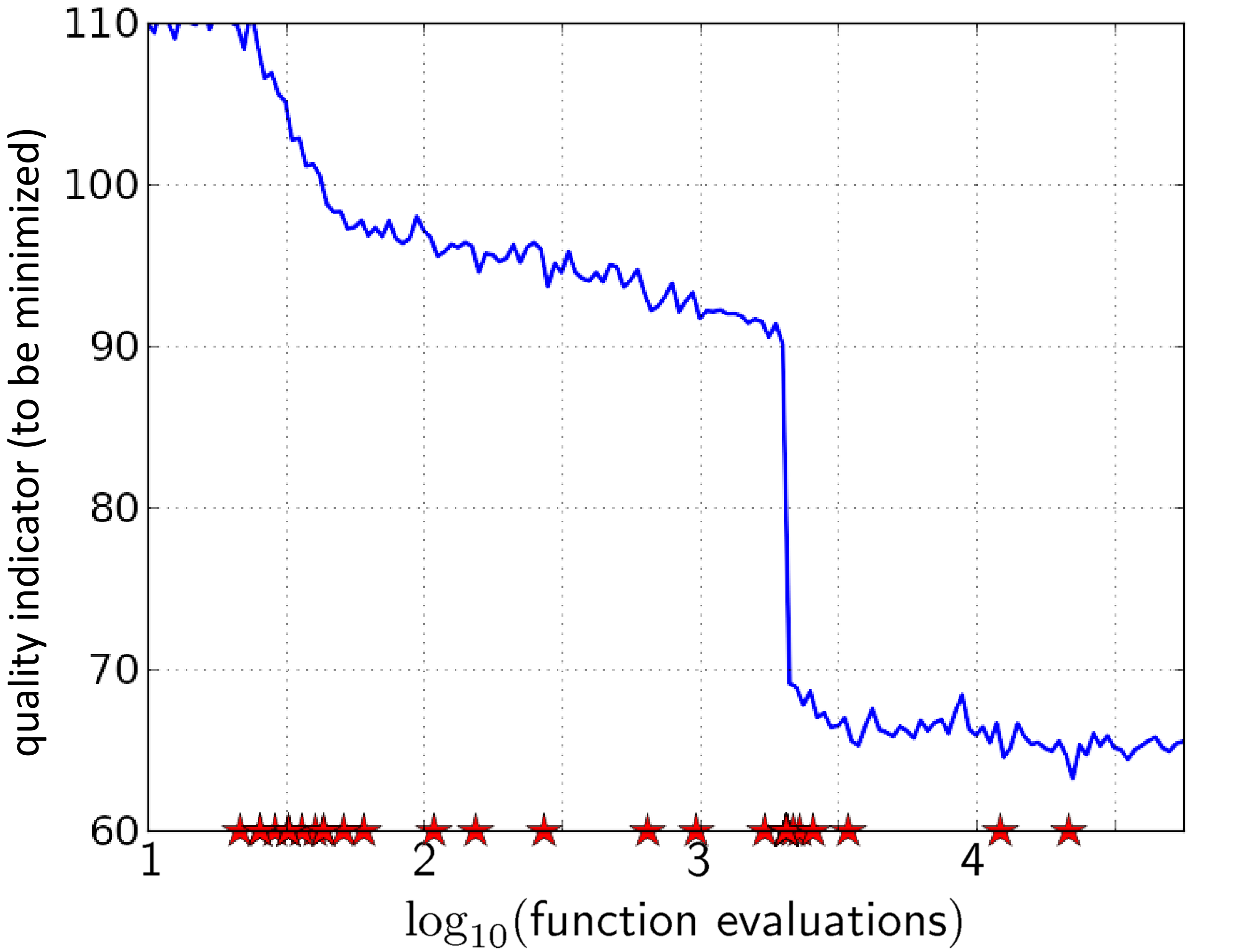
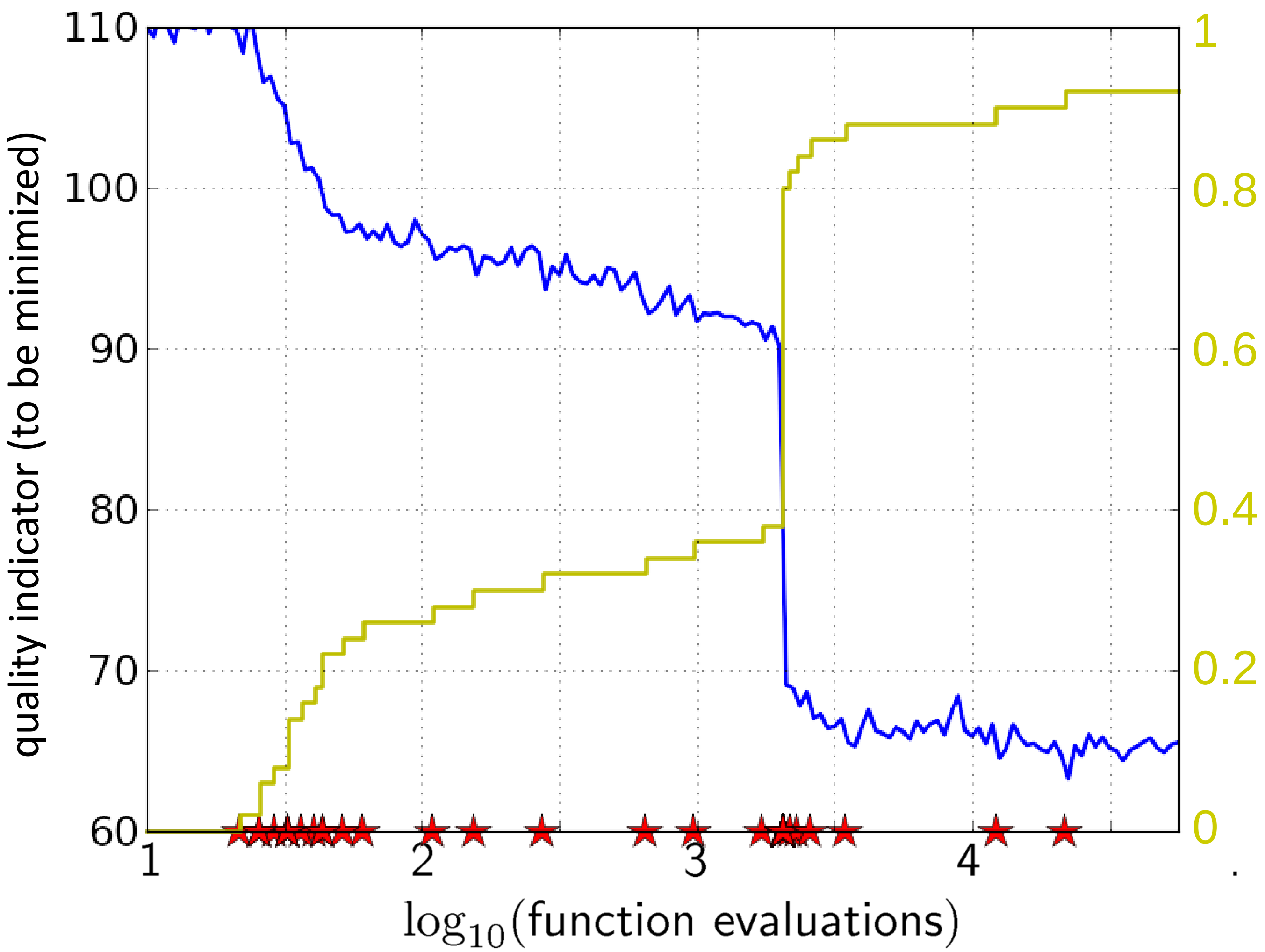
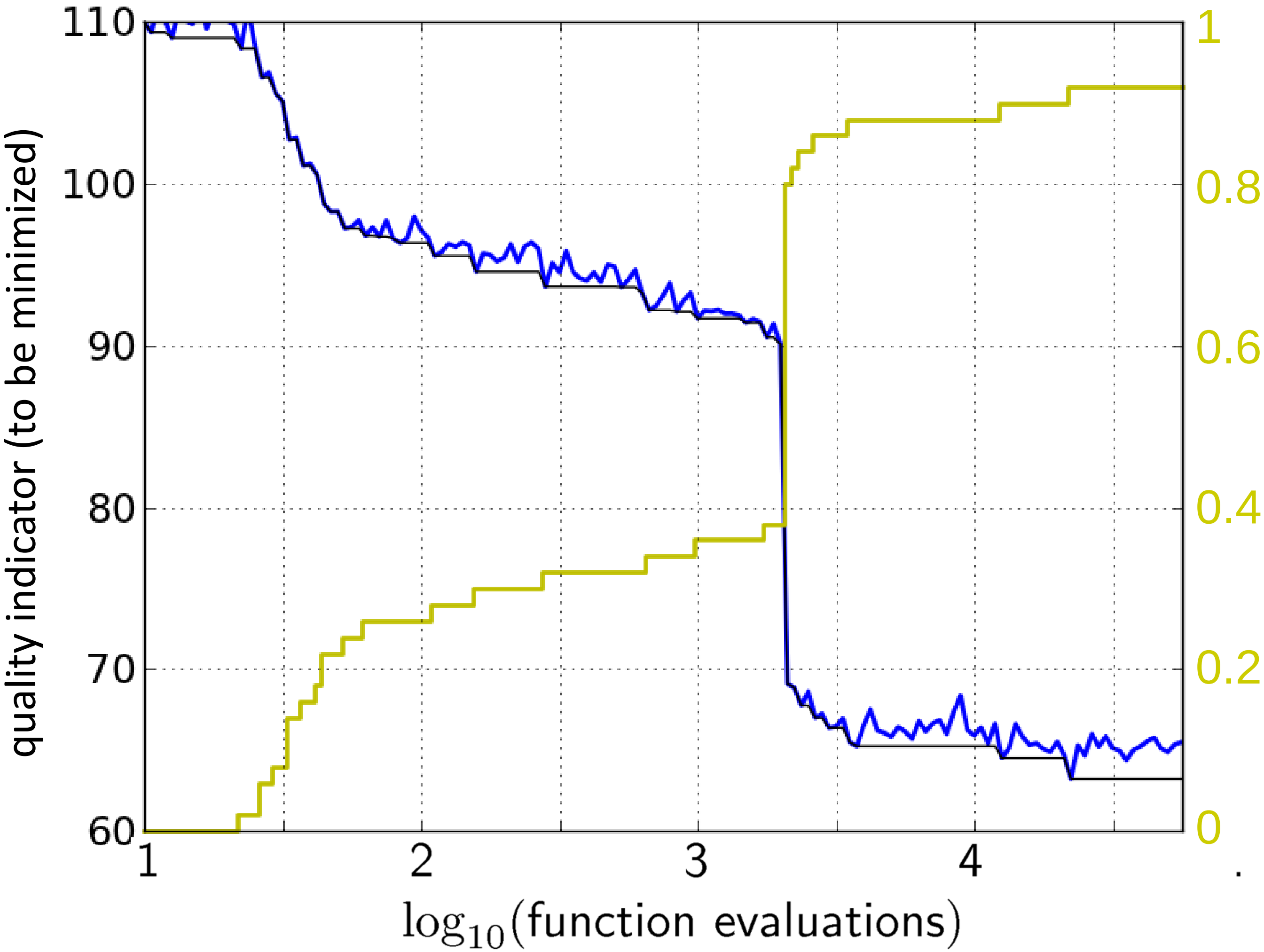


Reconstructing A Single Run

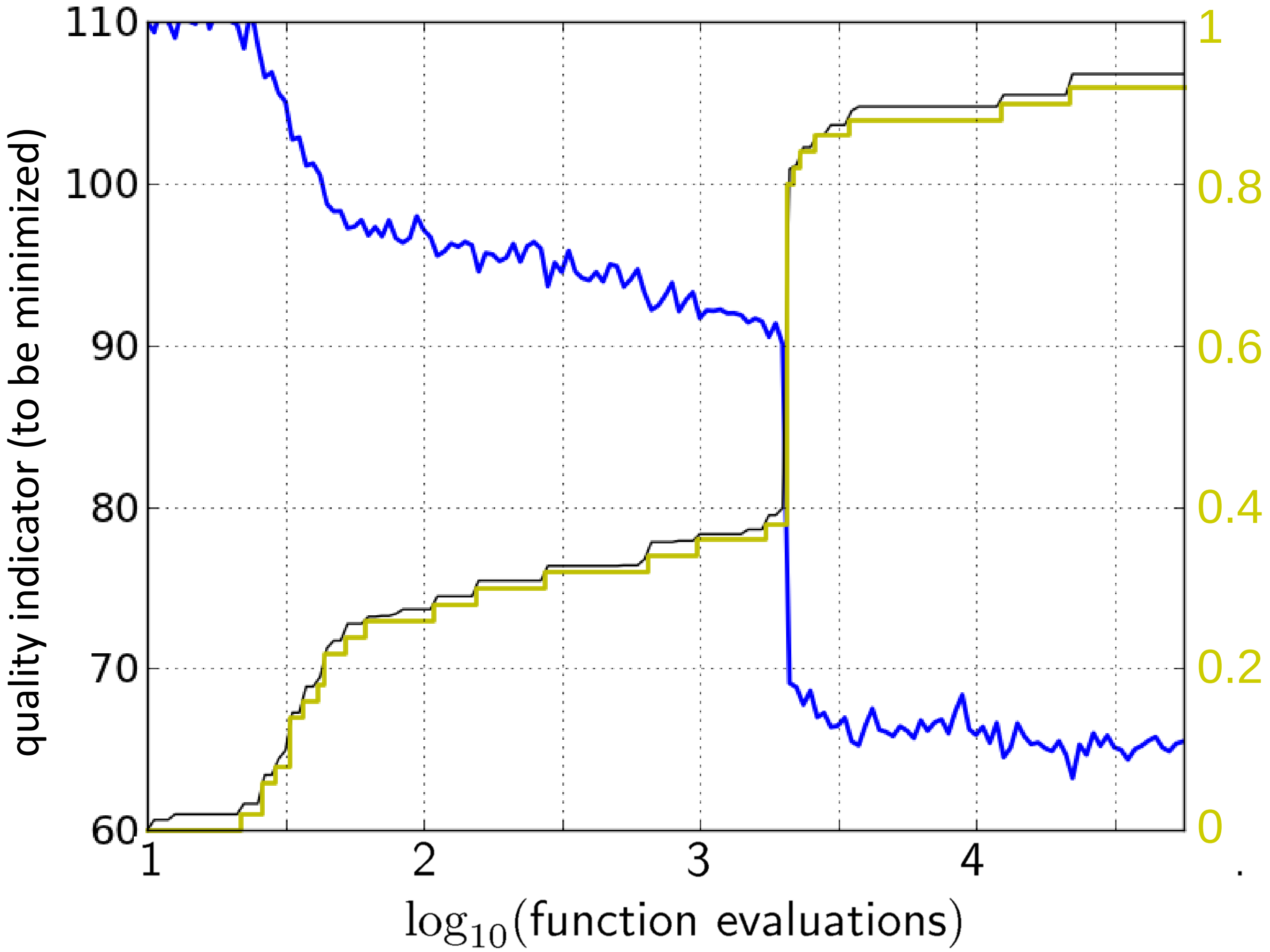


the empirical CDF makes a step for each star, is monotonous and displays for each budget the fraction of targets achieved within the budget

Reconstructing A Single Run



the ECDF
- recovers the monotonous graph, discretised and flipped



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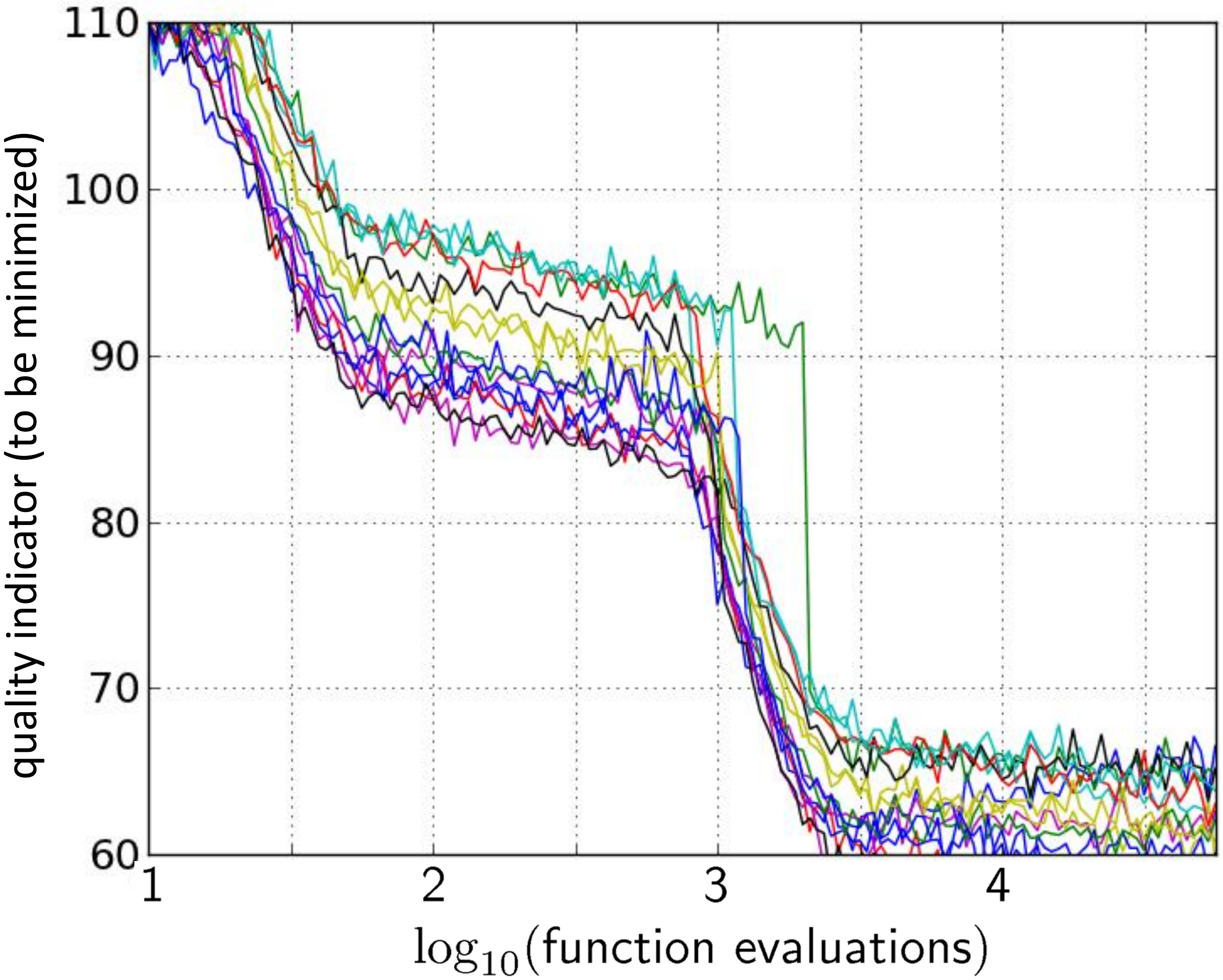
Aggregation



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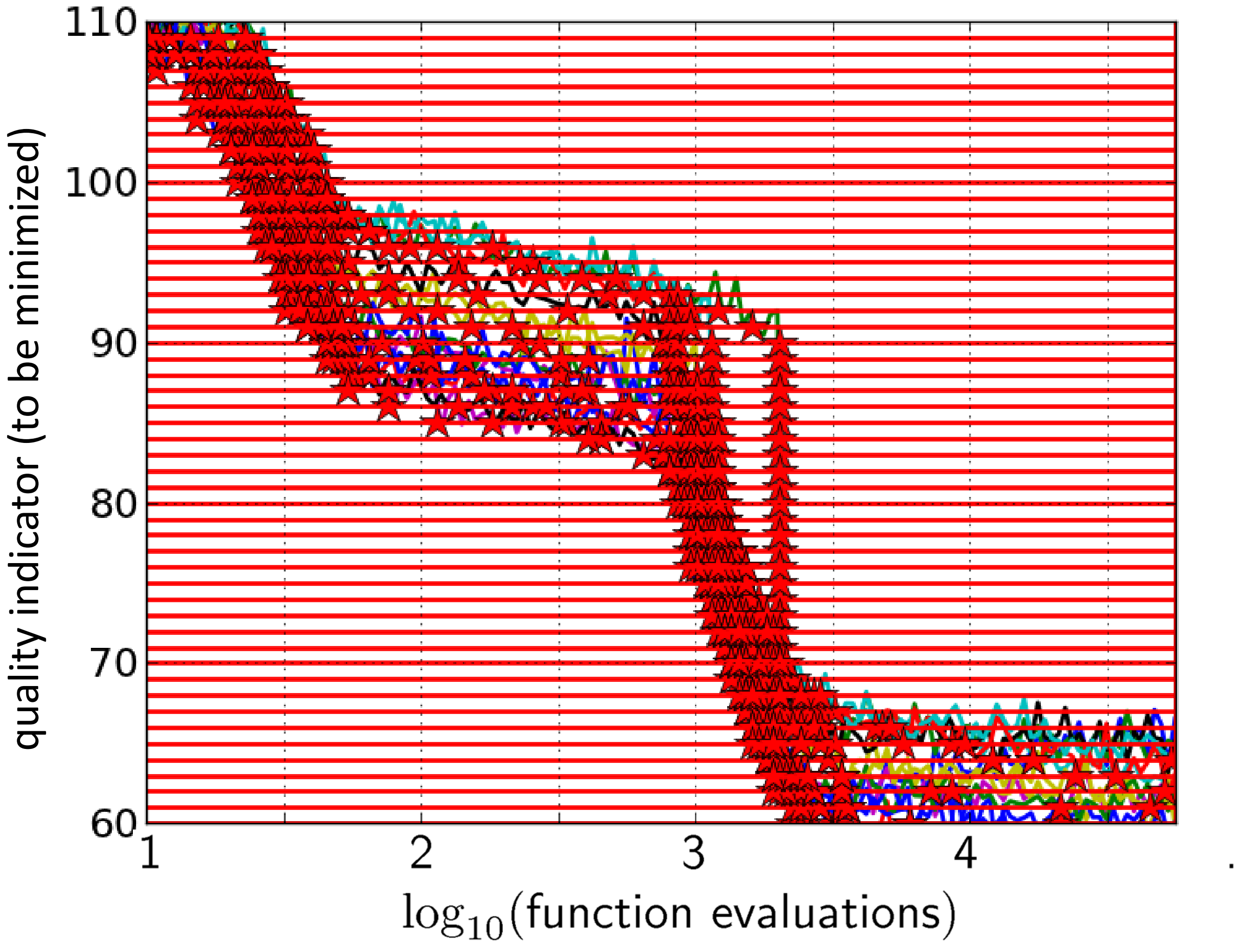

15 runs

50 targets

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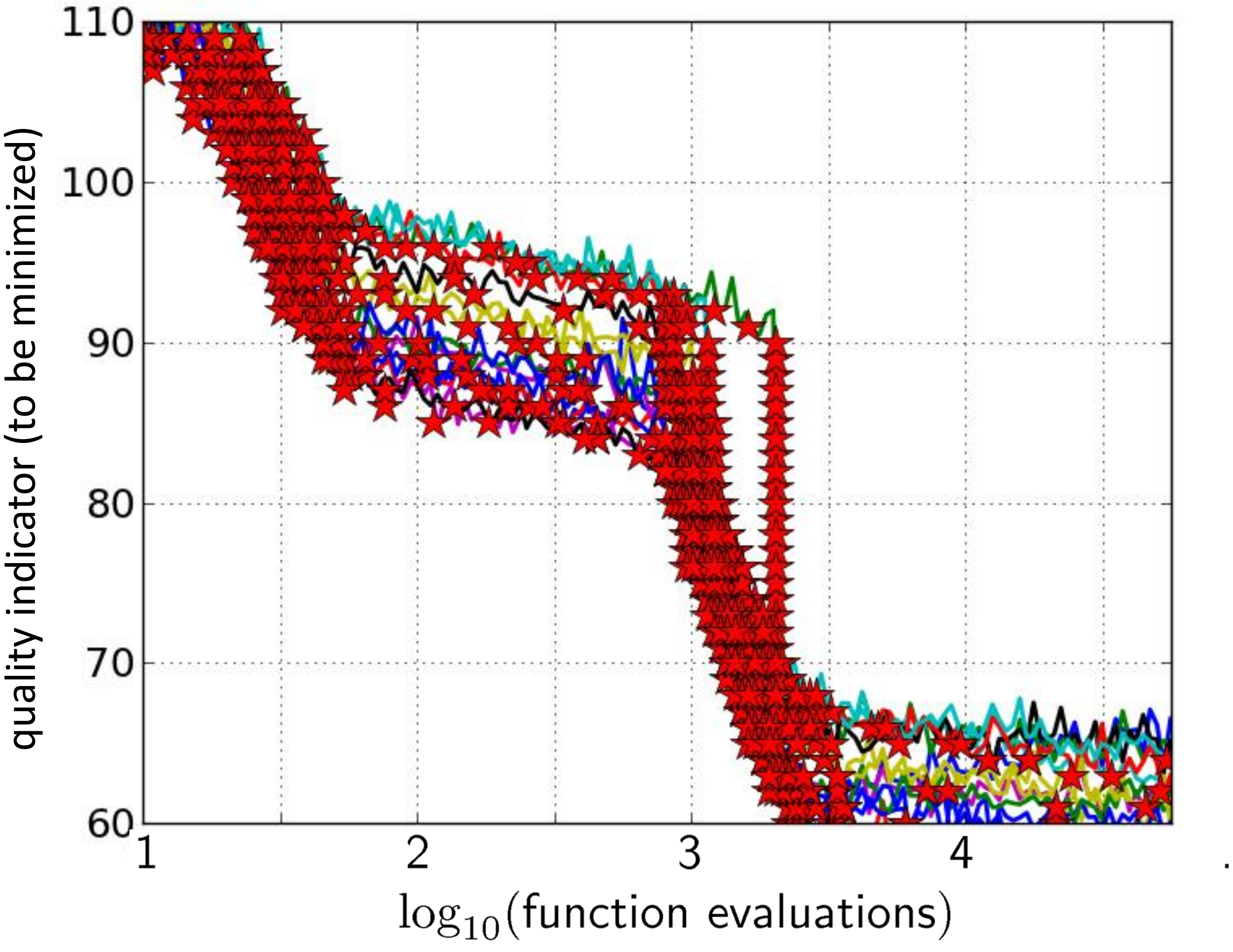

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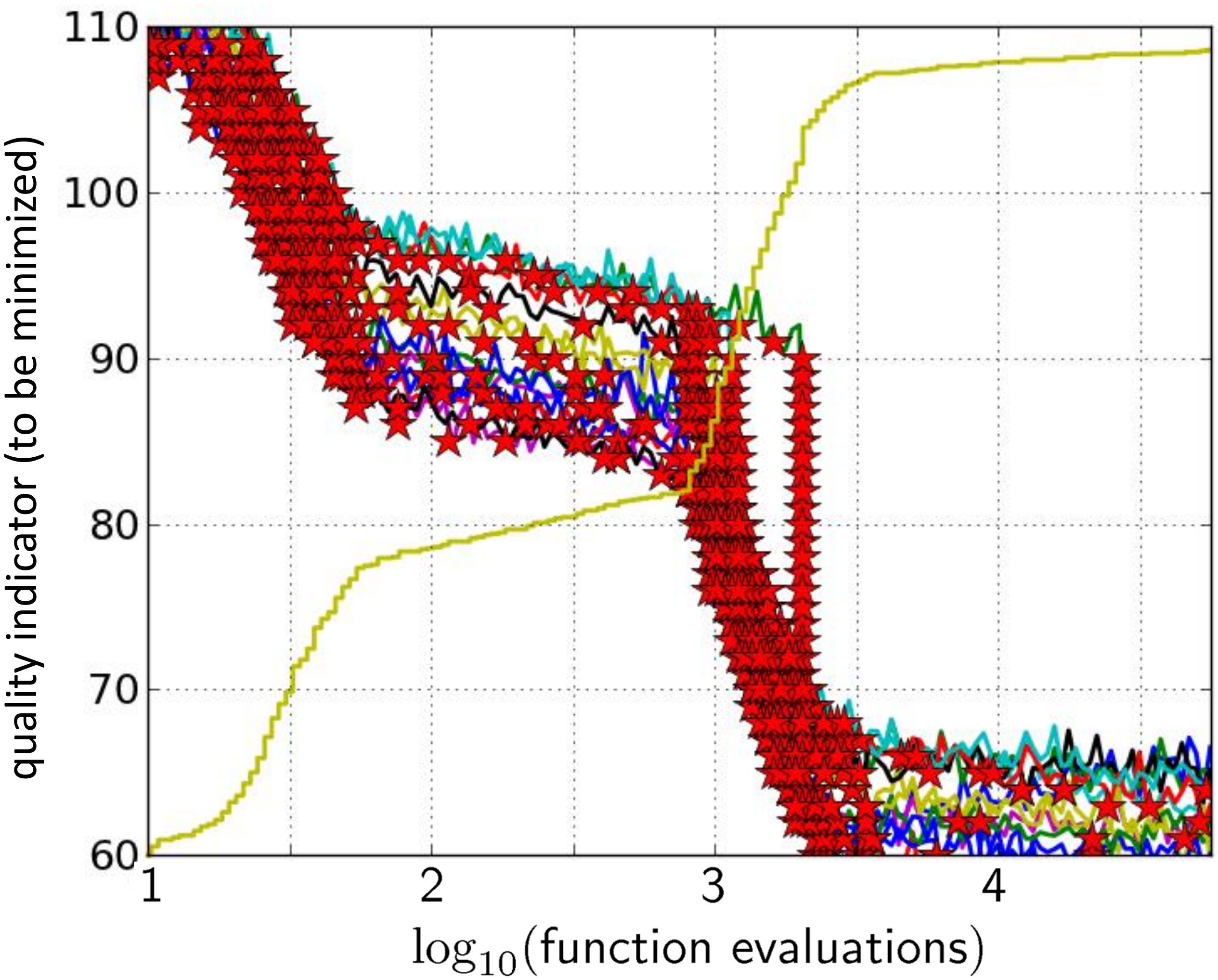
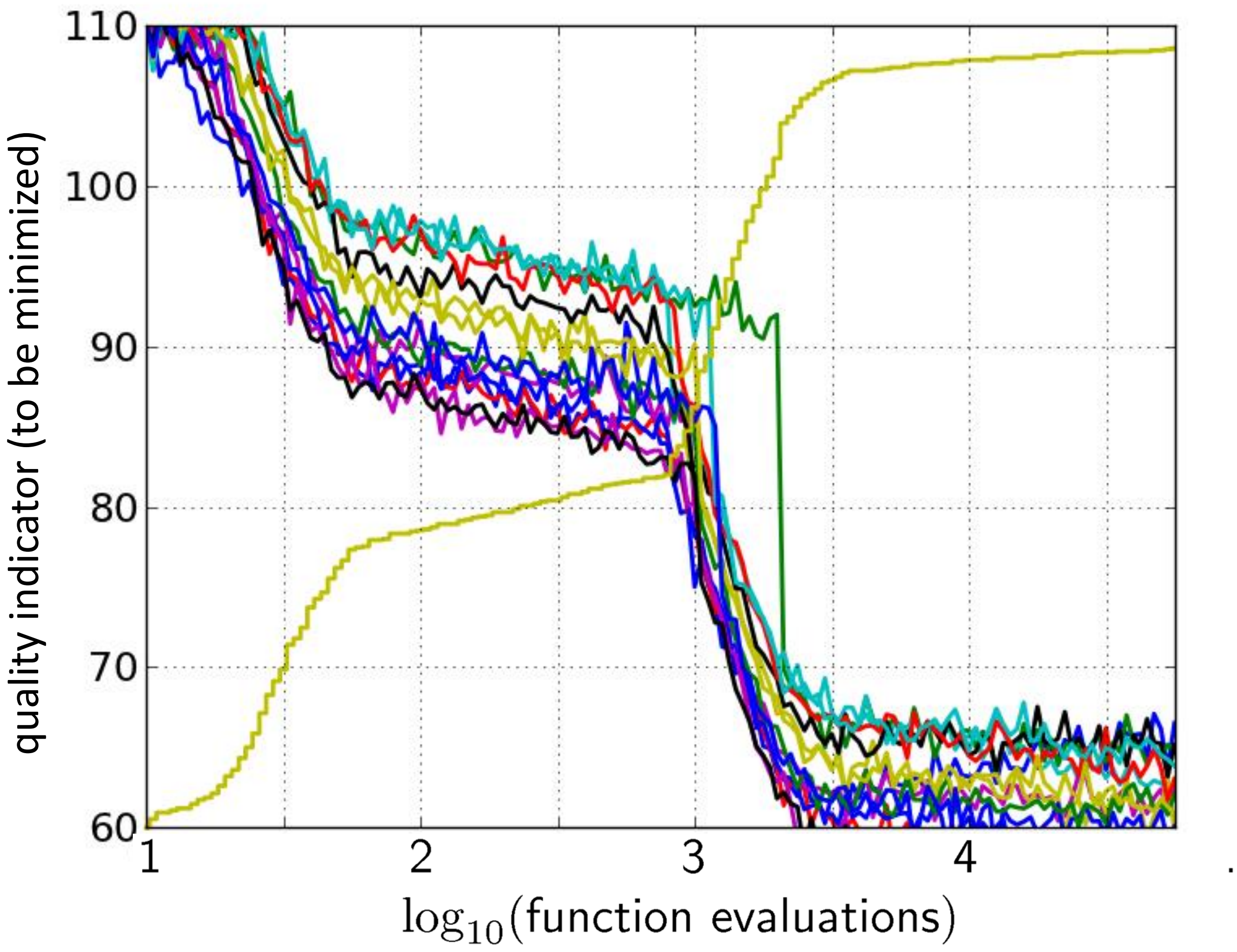


15 runs

50 targets

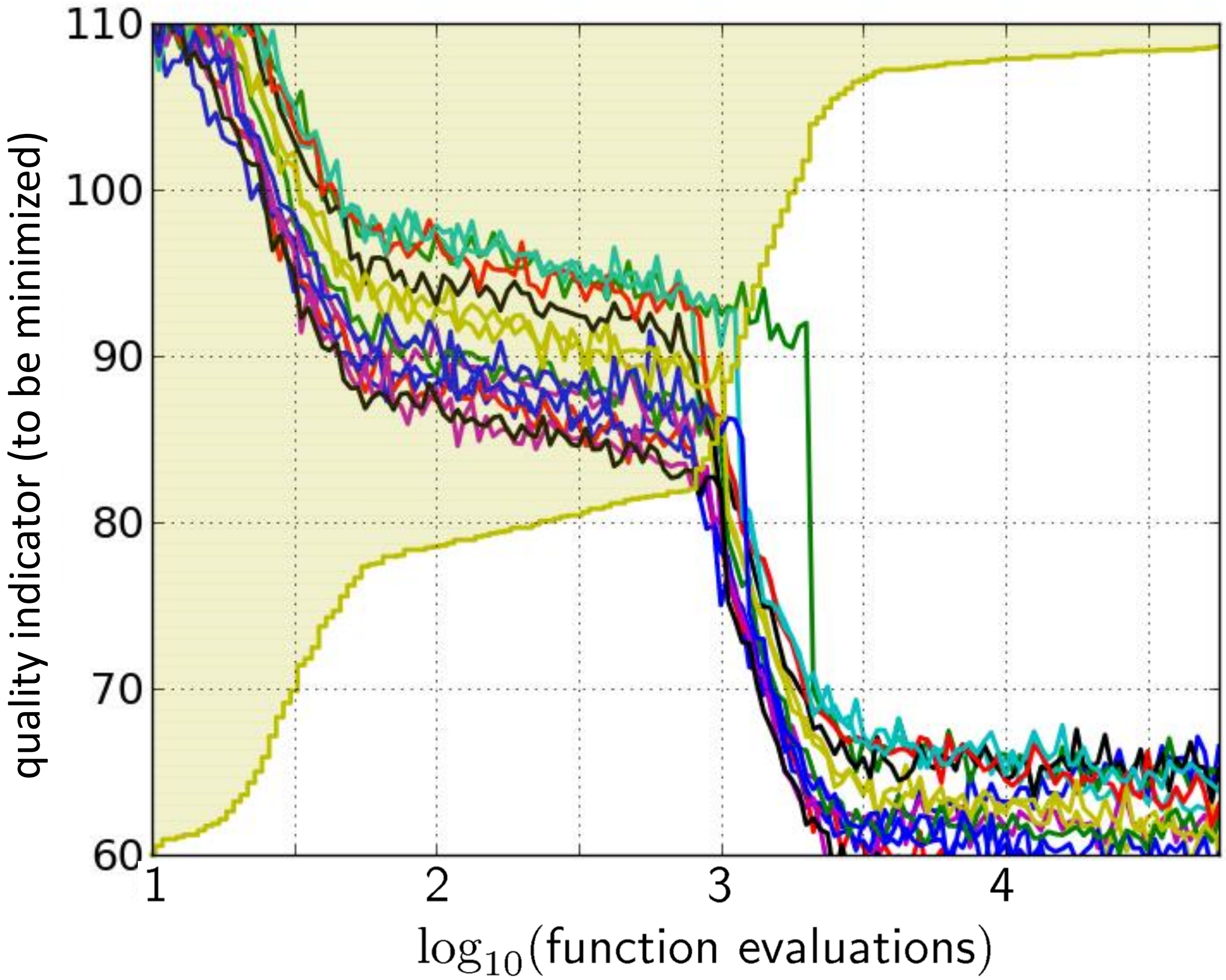
ECDF with 750 steps

Aggregation



50 targets from
15 runs
integrated in a
single graph

Interpretation



50 targets from 15 runs integrated in a single graph

area over the ECDF curve

=

average log runtime

(or geometric avg. runtime) over all targets (difficult and easy) and all runs

Worth to Note

ECDF graphs

- should never aggregate over dimension

dimension is input parameter to algorithm

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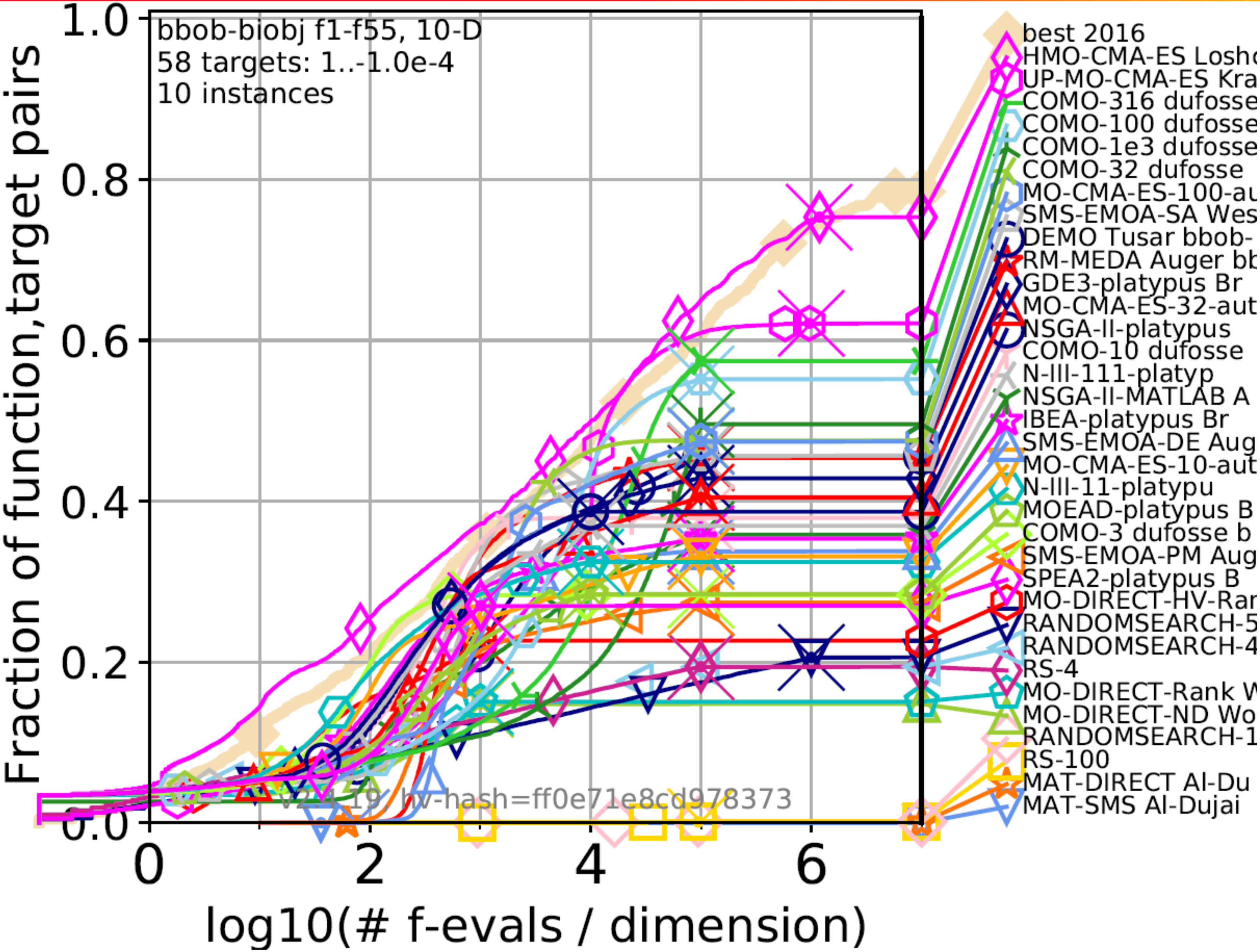
  - introduced by Moré and Wild [Moré and Wild 2009]
  - but for multiple and absolute targets
- are COCO’s main performance visualization tool

  https://github.com/numbbo/coco

Example ECDF (later more)



Mostly Overlooked: Scaling with Dimension

- In single-objective optimization: scaling behavior mandatory to investigate

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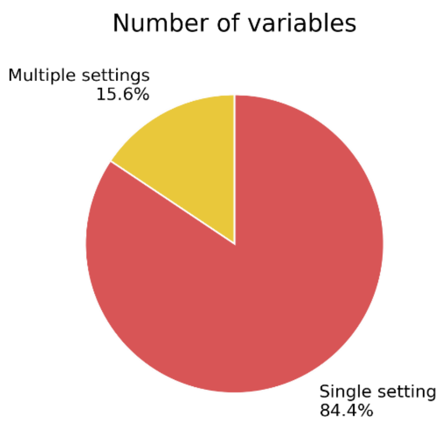

~10 papers from EMO’21 and PPSN/GECCO/CEC’21 change dimension

but 50+ papers have a “fixed” dimension

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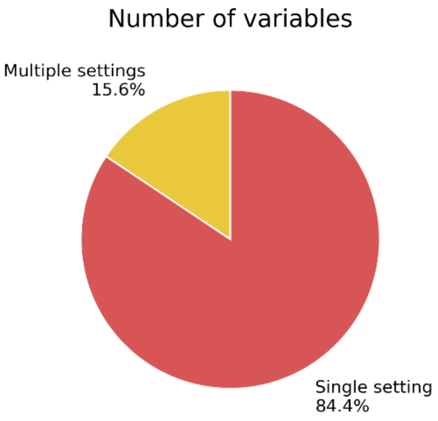

~10 papers from EMO'21 and PPSN/GECCO/CEC'21 change dimension

but 50+ papers have a "fixed" dimension

  - but in practice search space scalability almost more important

number of objectives often fixed

A Few General Recommendations

- always display everything you have
- look at single runs
- do each experiment at least twice

(= look at the *variance* of your results)

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  or any indicator which is at least monotone
- see also the tutorial slides by Nikolaus Hansen on this topic (not restricted to single-objective optimization!)

  http://www.cmap.polytechnique.fr/~nikolaus.hansen/gecco2018-experimentation-guide-slides.pdf

Recommended Experimental Setup (w/ or w/o COCO)

❶ **Benchmarking Experiment**

❷ **Choosing Algorithms for Comparison**

see `https://numbbo.github.io/data-archive/`

❸ **Postprocessing**

```
python -m cocopp resultfolder/ ALG2 ALG3
```

❹ **Displaying and Discussing Summary Results**

❺ **Investigating and Discussing Complementary Results**

❻ **Processed Data Sharing**

provide html output somewhere

❼ **Raw Data Sharing**

easy with COCO archive module & through issue tracker

Overview

❶ Performance Assessment

❷ Test Problems and Their Visualizations

❸ Recommendations from Numerical Results

Introduction

Test Problems (1)

Artificial problems (continuous and unconstrained)

v0.1: Individual problems

v0.2: MOP suite (unscalable problems)

v0.5: ZDT suite (scalable number of variables)

v1.0: DTLZ suite (scalable number of variables and objectives)

v1.2: WFG suite

v1.3: Other suites with a bottom-up construction

v1.5: Suites of distance-based problems

v2.0: The bbob-biobj(-ext) suite

Visualization of multiobjective landscapes

Low-dimensional search spaces

- Dominance ratio
- Local dominance
- Gradient path length
- PLOT

Any-dimensional search spaces

- Line cuts
- Optima network

Test Problems (2)

Artificial problems (other)

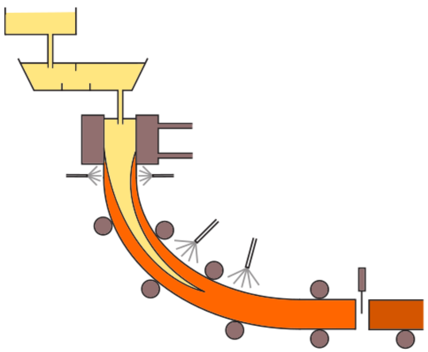
- Constrained problems
- Mixed-integer problems

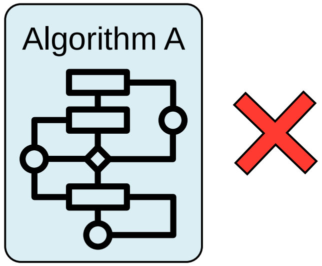
Real-world problems

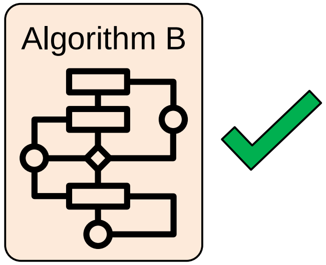
- **v0.1:** Individual problems
- **v0.2:** Suites of unscalable problems
- **v0.5:** Suites of scalable problems (number of variables)

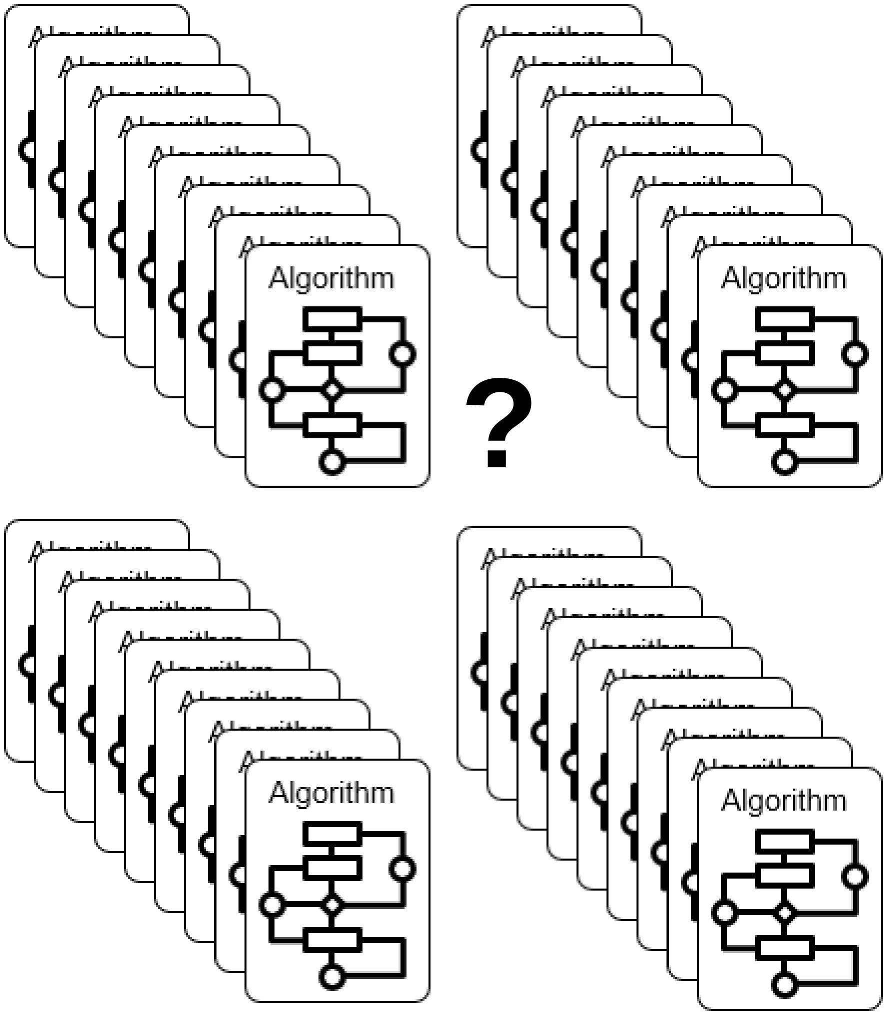
Conclusions

Why use test problems?

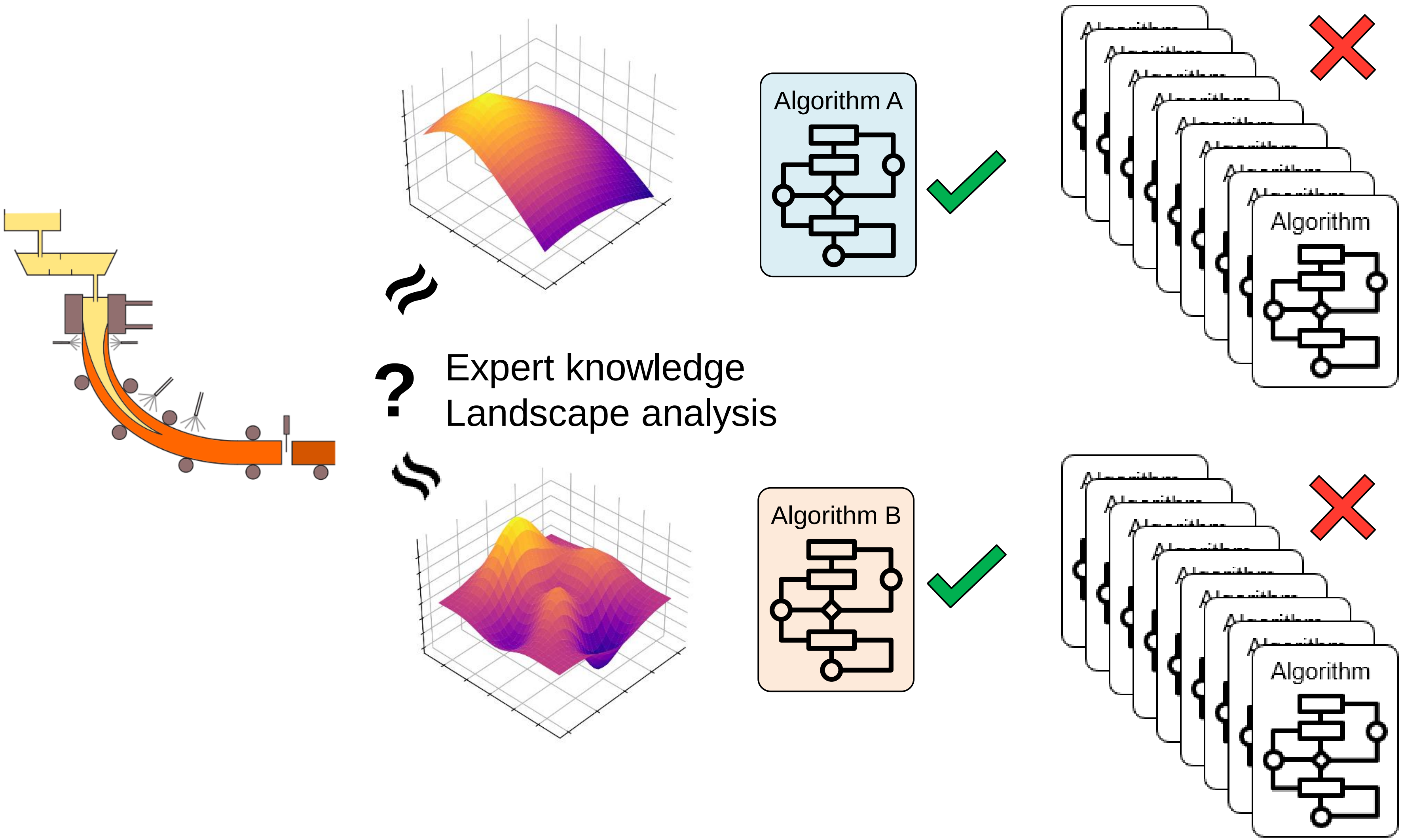








Why use test problems?



Desirable Characteristics of a Problem Set

[Bartz-Beielstein et al. 2020]

1. Diverse
2. Representative
3. Scalable and tunable
4. Known optima / best performance
5. [Continually updated]

Artificial problems (continuous and unconstrained)

v0.1

Individual problems

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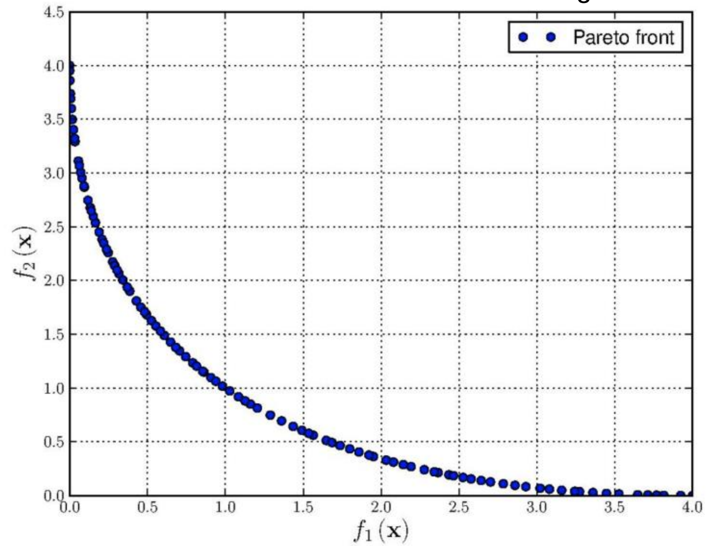
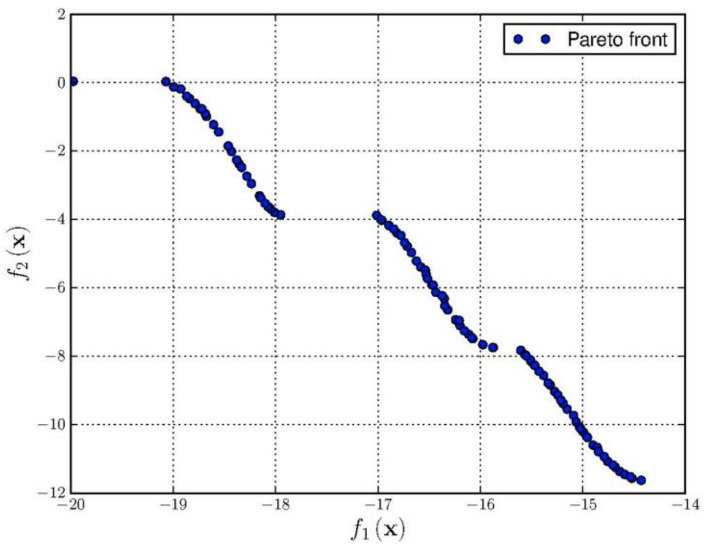
$$\text{Minimize} = \begin{cases} f_1(x) = x^2 \\ f_2(x) = (x-2)^2 \end{cases}$$

[Schaffer 1985]





$$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = \sum_{i=1}^{2}\left[-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right] \\ f_2(\boldsymbol{x}) = \sum_{i=1}^{3}\left[|x_i|^{0.8} + 5\sin\left(x_i^3\right)\right] \end{cases}$$

[Kursawe 1991]

Individual problems

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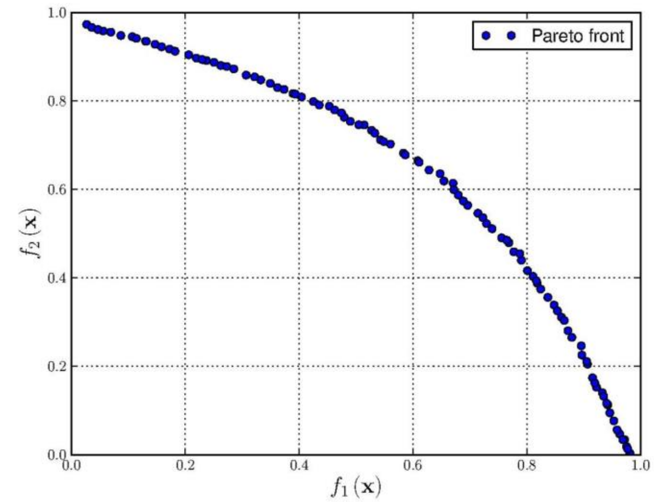
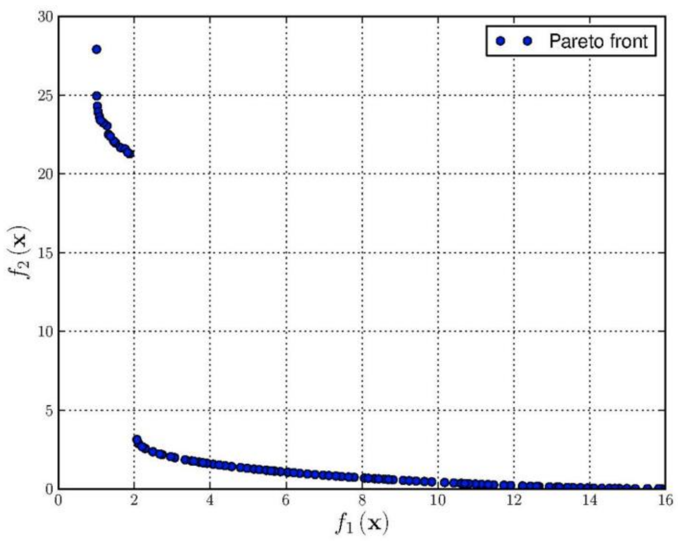
$$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i - \frac{1}{\sqrt{n}}\right)^2\right] \\ f_2(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i + \frac{1}{\sqrt{n}}\right)^2\right] \end{cases}$$

[Fonseca and Fleming 1995]





$$\text{Minimize} = \begin{cases} f_1(x, y) = \left[1 + (A_1 - B_1(x, y))^2 + (A_2 - B_2(x, y))^2\right] \\ f_2(x, y) = (x + 3)^2 + (y + 1)^2 \end{cases}$$

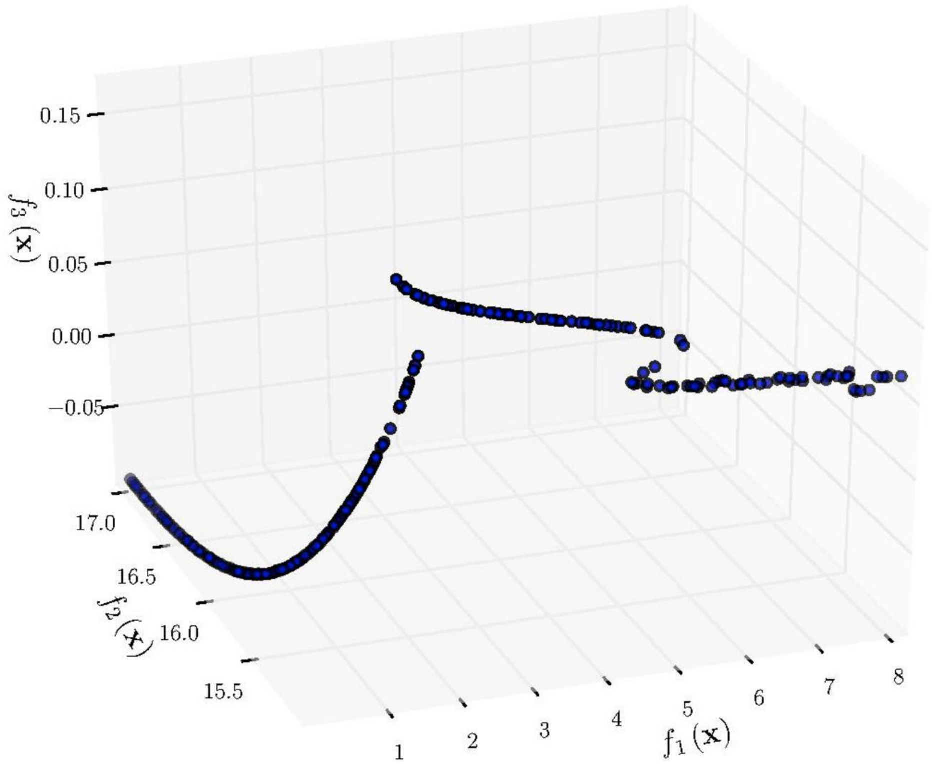
$$\text{where} = \begin{cases} A_1 = 0.5\sin(1) - 2\cos(1) + \sin(2) - 1.5\cos(2) \\ A_2 = 1.5\sin(1) - \cos(1) + 2\sin(2) - 0.5\cos(2) \\ B_1(x, y) = 0.5\sin(x) - 2\cos(x) + \sin(y) - 1.5\cos(y) \\ B_2(x, y) = 1.5\sin(x) - \cos(x) + 2\sin(y) - 0.5\cos(y) \end{cases}$$

[Poloni et al. 1996]

Images licensed under CC BY 2.0

$$\text{Minimize} = \begin{cases} f_1(x,y) = 0.5\left(x^2+y^2\right) + \sin\left(x^2+y^2\right) \\ f_2(x,y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15 \\ f_3(x,y) = \frac{1}{x^2+y^2+1} - 1.1\exp\left(-\left(x^2+y^2\right)\right) \end{cases}$$

[Viennet et al. 1996]



v0.2

MOP = Multi-Objective Problem

[Van Veldhuizen 1999]

Properties

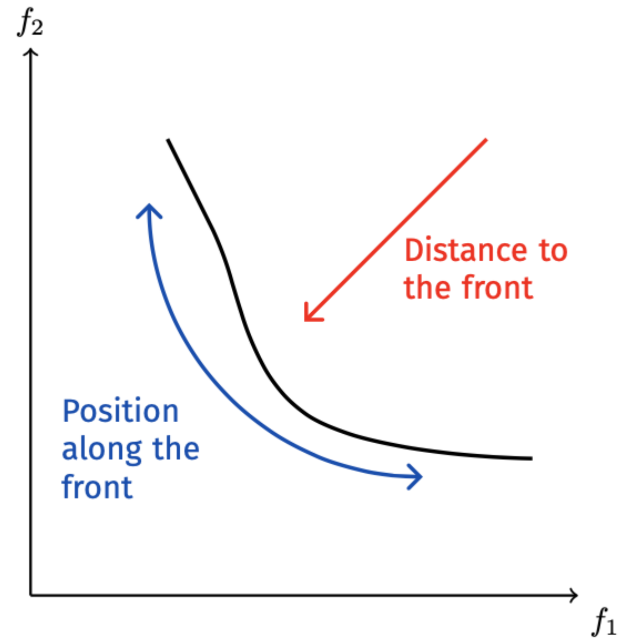
- A collection of 7 test problems from the literature (including the 5 shown before)
- Most problems have 2 or 3 variables
- Not scalable in the number of objectives
- Many problems have optimal solutions on the boundary or middle of the search space
- Some problems are both nonseparable and multimodal
- A collection of various Pareto front geometries
- The Pareto set is hard to compute for some problems

v0.5

ZDT = Zitzler, Deb, Thiele

[Zitzler et al. 2000]

The same construction for all problems
(following Deb's toolkit [Deb 1999])



Given $\mathbf{x} = \{x_1, \ldots, x_n\}$

Minimise $\overset{\text{Distribution f.}}{\boxed{f_1}}(\mathbf{y})$

$$f_2(\mathbf{y}, \mathbf{z}) = \overset{\text{Distance f.}}{\boxed{g}}(\mathbf{z})\overset{\text{Front shape}}{\boxed{h}}(f_1(\mathbf{y}), g(\mathbf{z}))$$

where $\mathbf{y} = \{x_1, \ldots, x_j\}$ Position variable(s) (j = 1 for ZDT)

$\mathbf{z} = \{x_{j+1}, \ldots, x_n\}$ Distance variables

The separation of variables was done to simplify problem construction

ZDT Suite

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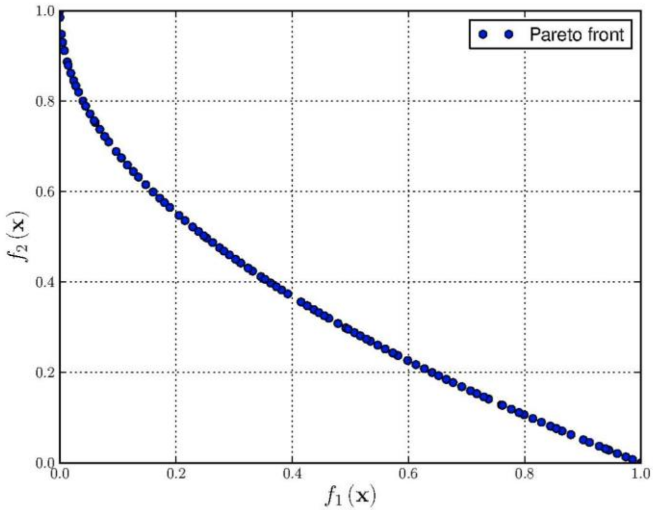
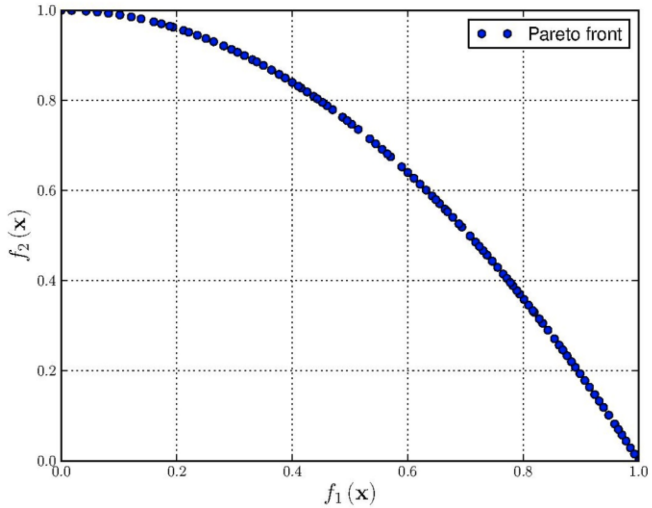
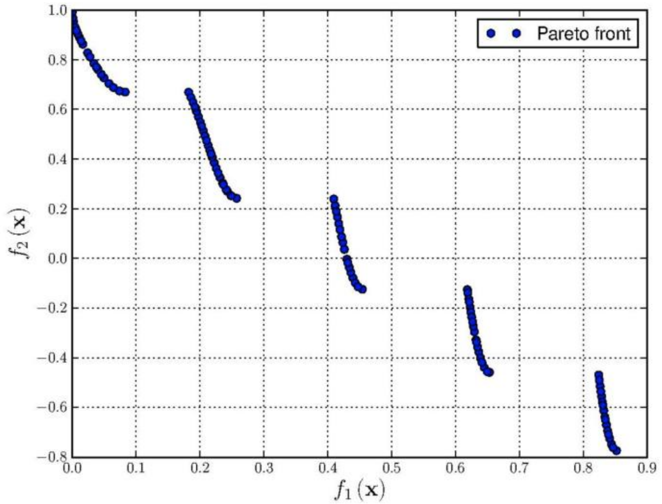


ZDT1



ZDT2



ZDT3

Properties

- 6 test problems, but ZDT5 is regularly omitted, because it has a binary encoding
- Scalable in the number of (distance) variables
- All problems have 2 objectives
- 4 problems have optimal solutions on the boundary and 1 in the middle of the search space
- All problems are separable (the first objective depends only on the first variable)
- Some problems are multimodal
- Convex, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known

v1.0

DTLZ Suite

DTLZ = Deb, Thiele, Laumanns, Zitzler

[Deb et al. 2005]

Desired Features of Test Problems

1. Have controllable difficulty to converge to the Pareto front, a widely-distributed set of Pareto-optimal solutions
2. Scalable number of variables
3. Scalable number of objectives
4. Simple to construct
5. Pareto front easy to comprehend, both the Pareto set and front known
6. Similar difficulties to those present in real-world problems

Problem Design Approaches

1. Multiple single-objective functions approach
2. Bottom-up approach
    1. Choose a Pareto front
    2. Build the objective space
    3. Construct the search space (add difficulties using the function *g*)
3. Constraint surface approach (for constrained problems)

DTLZ Suite

Properties

- Originally 9 problems, but then 2 were dropped
- Scalable number of distance variables, $M - 1$ position variables
- Scalable number of objectives
- Objectives separable in practice (optimizing one variable at a time will yield at least one global optimum)
- Linear, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known
- Most problems have the Pareto set in the middle of the search space

Note that although the suite is scalable in the number of variables, this is rarely used in benchmark studies

v1.2

WFG Suite

WFG = Walking Fish Group

[Huband et al. 2006]

Recommendations for multiobjective test problems

1. No extremal variables
2. No medial variables
3. Scalable number of variables
4. Scalable number of objectives
5. Dissimilar variable domains
6. Dissimilar objective ranges
7. Pareto set and front known

Recommendations for multiobjective test suites

[Huband et al. 2006]

1. A few unimodal test problems to test convergence velocity relative to different Pareto optimal geometries and bias conditions
2. Cover the three core types of geometries: degenerate Pareto fronts, disconnected Pareto fronts, and disconnected Pareto sets
3. The majority of problems should be multimodal with a few deceptive problems
4. The majority of problems should be nonseparable
5. Contain problems that are both nonseparable and multimodal to be representative of real-world problems

WFG Suite

Properties

- 9 problems constructed from a combination of shape functions and several transformations
- Scalable number of variables (2 variables are not supported for some of the problems)
- Scalable number of objectives
- Includes also nonseparable, multimodal, deceptive and biased problems
- Convex, linear, concave, mixed, disconnected and degenerate Pareto fronts
- The Pareto sets and fronts are known
- Optimal solutions do not lie on the boundary or the middle of the search space, but the Pareto set is linear for 8 of the 9 problems
- Still rely on distance and position variables

v1.3

Other Suites and Problems

Problems constructed with the bottom-up approach

[Zapotecas et al. 2019]

- LZ test suite of 9 problems with complicated Pareto sets [Li and Zhang 2009]
- SZDT test suite of 7 scalable problems with complicated Pareto sets [Saxena et al. 2011]
- Convex DTLZ problem [Deb and Jain 2014]
- Inverted DTLZ problem [Jain and Deb 2014]
- MNI test suite of 2 test problems with diverse shapes of the Pareto front [Masuda et al. 2016]
- LSMOP test suite of 9 test problems for large-scale optimization with variable dependencies [Cheng et al. 2017b]
- Minus-DTLZ and Minus-WFG test suites [Ishibuchi et al. 2017]
- MMF test problems with diverse landscapes [Yue et al. 2019]

CEC Competition Suites

Information about all CEC competitions:
https://www3.ntu.edu.sg/home/EPNSugan/index_files/cec-benchmarking.htm

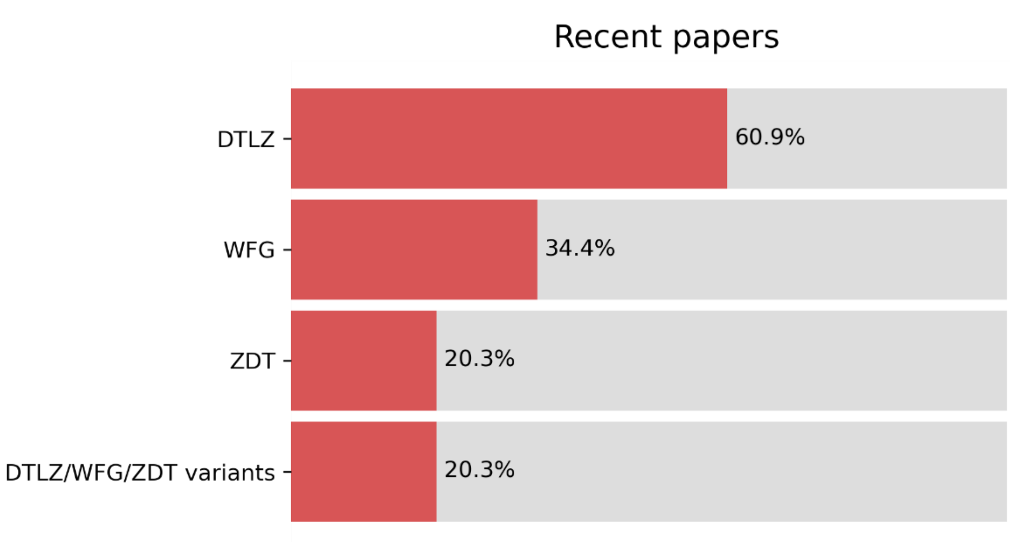
- 13 test problems for CEC 2007 [Huang et al. 2007]
  - OKA [Okabe et al. 2004], SYM-PART [Rudolph et al. 2007]
  - 4 shifted ZDT, 1 rotated ZDT
  - 2 shifted DTLZ, 1 rotated DTLZ
  - 3 WFG
- 13 test problems for CEC 2009 (UF suite) [Zhang et al. 2009]
  - 10 with complicated Pareto sets (4 from the LZ suite)
  - 2 extended rotated DTLZ
  - 1 WFG

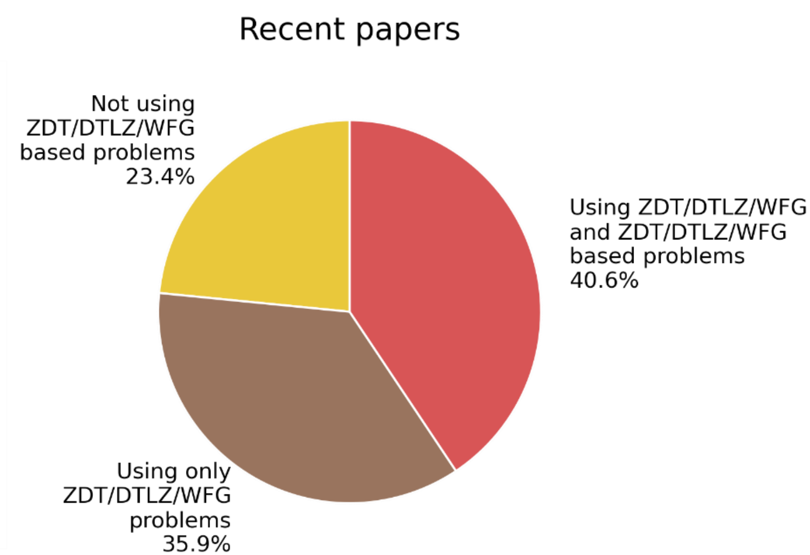
CEC Competition Suites

- 15 test problems for CEC 2017 (MaF suite) [Cheng et al. 2017a]
  - 7 modified DTLZ problems
  - 2 distance minimization problems
  - 3 WFG problems
  - 1 SZDT problem
  - 2 LSMOP problem
- 22 test problems for CEC 2019 [Liang et al. 2019]
  - 2 SYM-PART
  - Omni-test [Deb and Tiwari 2008]
  - 19 MMF problems
- 24 test problems for CEC 2020 [Liang et al. 2020]
  - 24 MMF problems

Survey of Recent Papers

- 64 papers on unconstrained continuous multiobjective optimization from recent conferences (without application papers)
  - CEC 2020
  - GECCO 2020
  - PPSN 2020
  - EMO 2021





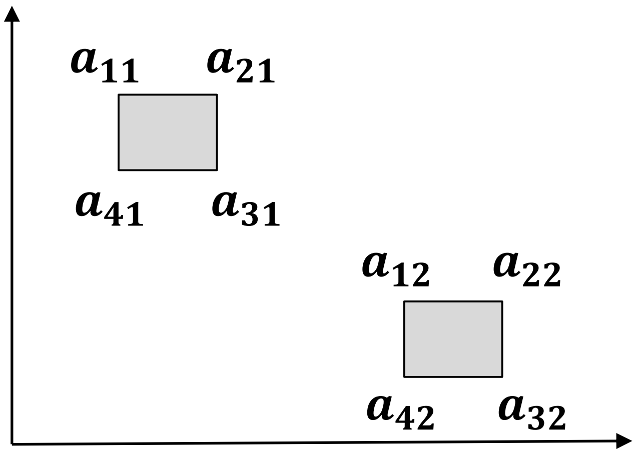
v1.5

General idea

[Ishibuchi et al. 2010]

$$\text{Minimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ \ldots,\ f_k(\mathbf{x}))$$

$$f_i(\mathbf{x}) = \min\{\mathrm{dis}(\mathbf{x}, \mathbf{a}_{i1}), \mathrm{dis}(\mathbf{x}, \mathbf{a}_{i2}), \ldots, \mathrm{dis}(\mathbf{x}, \mathbf{a}_{im})\}$$



- 2-D test problems that are inherently visualizable
- Pareto set easy to characterize
- Scalable in the number of objectives
- Useful for visualizing the distribution of solutions
- Unlikely to be relevant for real-world problems
- Based on earlier work [Köppen et al. 2005, Rudolph et al. 2007]

Distance-Based Problems

Extensions

- High-dimensional search spaces [Masuda et al. 2014]
- Distance to lines (instead of points) [Li et al. 2014, 2018]
- Dominance resistance regions [Fieldsend 2016]
- Local Pareto fronts [Liu et al. 2018]
- Problem generator for scalable problems with various properties (local fronts, disconnected Pareto sets and fronts, dominance resistance regions, uneven ranges of objective values, varying density of solutions) [Fieldsend et al. 2019]

v2.0

Motivation

[Brockhoff et al. 2016]

- Most other suites are constructed based on the desired Pareto front properties
- Consequently, problems have artificial properties not likely to exist in real-world problems
  - Distance and position parameters (DTLZ-like problems)
  - Linear objectives (distance-based problems)
- In real-world problems each objective is a separate function
- Go back to basics – use single-objective functions for each objective
  - Idea not new [Schaffer 1985, Igel et al. 2007, Emmerich and Deutz 2007, Kerschke et al. 2016]

Construction

- Use the functions from the **bbob** suite
  - Well-understood
  - Scalable in the number of variables and *parametrized*
  - 24 functions categorized in 5 groups based on their properties
    - Separable
    - Low or moderate conditioning
    - High conditioning and unimodal
    - Multimodal with global structure
    - Multimodal with weak global structure
- How to avoid an explosion in the number of problems?

bbob-biobj Suite

Column groups: Separable (f_1, f_2) | Low or moderate conditioning (f_6, f_8) | High conditioning and unimodal (f_{13}, f_{14}) | Multimodal with global structure (f_{15}, f_{17}) | Multimodal with weak global structure (f_{20}, f_{21})

	f_1	f_2	f_6	f_8	f_{13}	f_{14}	f_{15}	f_{17}	f_{20}	f_{21}
f_1	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
f_2		F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}	F_{17}	F_{18}	F_{19}
f_6			F_{20}	F_{21}	F_{22}	F_{23}	F_{24}	F_{25}	F_{26}	F_{27}
f_8				F_{28}	F_{29}	F_{30}	F_{31}	F_{32}	F_{33}	F_{34}
f_{13}					F_{35}	F_{36}	F_{37}	F_{38}	F_{39}	F_{40}
f_{14}						F_{41}	F_{42}	F_{43}	F_{44}	F_{45}
f_{15}							F_{46}	F_{47}	F_{48}	F_{49}
f_{17}								F_{50}	F_{51}	F_{52}
f_{20}									F_{53}	F_{54}
f_{21}										F_{55}

Row groups:
- Separable (f_1, f_2)
- Low or moderate conditioning (f_6, f_8)
- High conditioning and unimodal (f_{13}, f_{14})
- Multimodal with global structure (f_{15}, f_{17})
- Multimodal with weak global structure (f_{20}, f_{21})

f_1 Sphere
f_2 Ellipsoid separable

f_6 Attractive sector
f_8 Rosenbrock original

f_{13} Sharp ridge
f_{14} Sum of different powers

f_{15} Rastrigin
f_{17} Schaffer F7 (condition 10)

f_{20} Schwefel x*sin(x)
f_{21} Gallagher 101 peaks

bbob-biobj-ext Suite

	Separable					Low or moderate conditioning				High conditioning and unimodal					Multimodal with global structure					Multimodal with weak global structure					
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	
Separable	F_1	F_2	F_{56}	F_{57}	F_{58}	F_3		F_4					F_5	F_6	F_7		F_8			F_9	F_{10}				f_1
		F_{11}	F_{59}	F_{60}	F_{61}	F_{12}		F_{13}					F_{14}	F_{15}	F_{16}		F_{17}			F_{18}	F_{19}				f_2
				F_{62}	F_{63}																				f_3
					F_{64}																				f_4
																									f_5
Low or moderate conditioning						F_{20}	F_{65}	F_{21}	F_{66}				F_{22}	F_{23}	F_{24}		F_{25}			F_{26}	F_{27}				f_6
								F_{67}	F_{68}																f_7
								F_{28}	F_{69}				F_{29}	F_{30}	F_{31}		F_{32}			F_{33}	F_{34}				f_8
																									f_9
High conditioning and unimodal											F_{70}	F_{71}	F_{72}	F_{73}											f_{10}
												F_{74}	F_{75}	F_{76}											f_{11}
													F_{77}	F_{78}											f_{12}
													F_{35}	F_{36}	F_{37}		F_{38}			F_{39}	F_{40}				f_{13}
														F_{41}	F_{42}		F_{43}			F_{44}	F_{45}				f_{14}
Multimodal with global structure															F_{46}		F_{47}	F_{79}	F_{80}	F_{48}	F_{49}				f_{15}
																									f_{16}
																	F_{50}	F_{81}	F_{82}	F_{51}	F_{52}				f_{17}
																			F_{83}						f_{18}
																									f_{19}
Multimodal with weak global structure																				F_{53}	F_{54}	F_{84}	F_{85}	F_{86}	f_{20}
																					F_{55}	F_{87}	F_{88}	F_{89}	f_{21}
																							F_{90}	F_{91}	f_{22}
																								F_{92}	f_{23}
																									f_{24}

f_1 Sphere
f_2 Ellipsoid separable
f_3 Rastrigin separable
f_4 Skew Rastrigin-Bueche
f_5 Linear slope

f_6 Attractive sector
f_7 Step-ellipsoid
f_8 Rosenbrock original
f_9 Rosenbrock rotated

f_{10} Ellipsoid
f_{11} Discus
f_{12} Bent cigar
f_{13} Sharp ridge
f_{14} Sum of different powers

f_{15} Rastrigin
f_{16} Weierstrass
f_{17} Schaffer F7 (condition 10)
f_{18} Schaffer F7 (condition 1000)
f_{19} Griewank-Rosenbrock F8F2

f_{20} Schwefel x*sin(x)
f_{21} Gallagher 101 peaks
f_{22} Gallagher 21 peaks
f_{23} Katsuuras
f_{24} Lunacek bi-Rastrigin

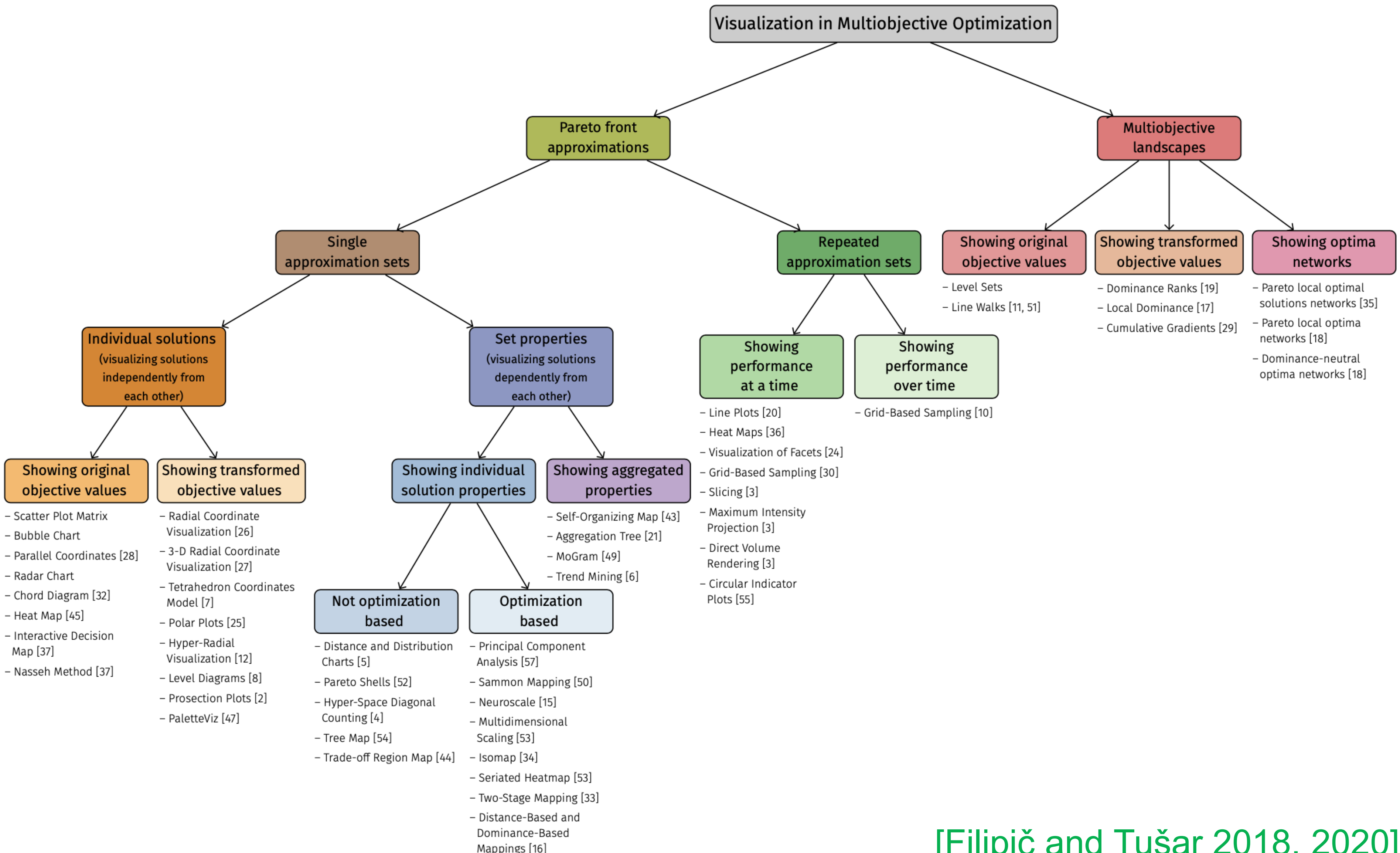
bbob-biobj(-ext) Suite

Properties

- Construction similar as in real-world problems
- Scalability in the number of variables
- Various problem properties (more diverse than existing multiobjective test suites)
- Problem instances more diverse than for the single-objective suite
- Currently limited to 2 objectives
- Unknown Pareto set and front (but known single-objective optima)
- Available approximations of the Pareto fronts (and sets for lower-dimensional problems)

Visualization of multiobjective landscapes

Visualization in Multiobjective Optimization



[Filipič and Tušar 2018, 2020]

Low-dimensional search spaces

- Dominance ratio [Fonseca 1995]
- Gradient path length (inspired by gradient plots [Kerschke and Grimme 2017])
- Local dominance [Fieldsend et al. 2019]
- PLOT [Shaepermeier et al. 2020]

Any-dimensional search spaces

- Line cuts [Brockhoff et al. 2016, Volz et al. 2019]
- Optima network [Liefooghe et al. 2018, Fieldsend and Alyahya 2019]

Various visualizations of bbob-biobj-ext problems:
https://numbbo.github.io/bbob-biobj/

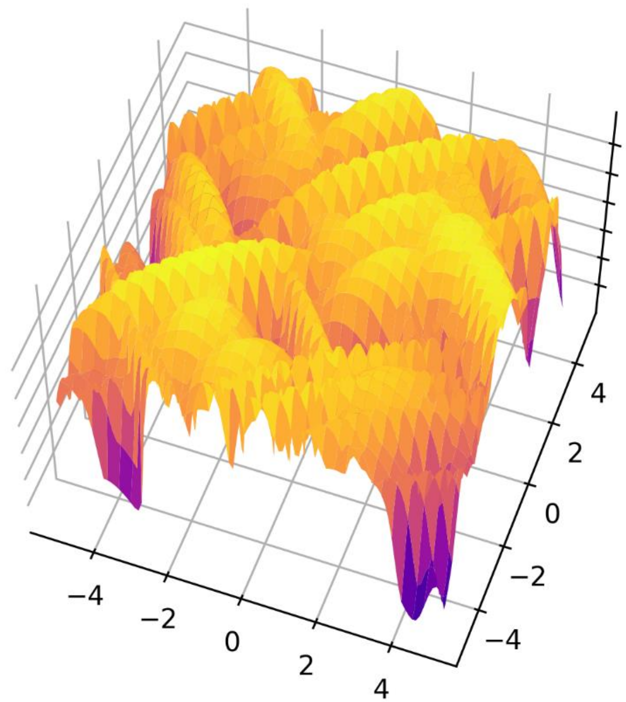
Visualizations of bbob-biobj and other multi-objective suites using PLOT:
https://schaepermeier.shinyapps.io/moPLOT/

Visualization of Multiobjective Problem Landscapes

Problems for demonstration

- Double sphere problem bbob-biobj $F_1 = (f_1, f_1)$, instance 1
- Sphere-Gallagher problem bbob-biobj $F_{10} = (f_1, f_{21})$, instance 1
- Double Gallagher problem bbob-biobj $F_{55} = (f_{21}, f_{21})$, instance 1

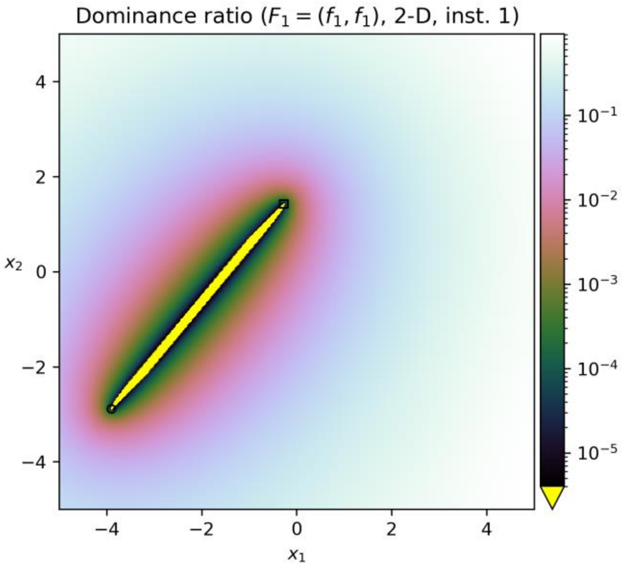
Gallagher = Gallagher’s Gaussian
101-me Peaks Function

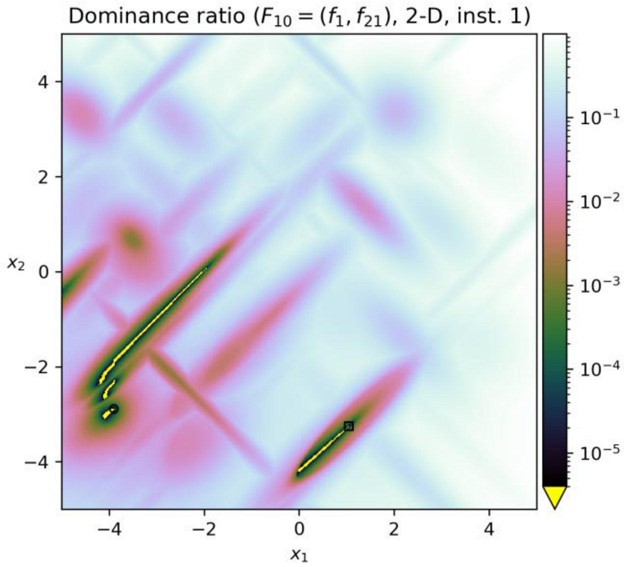


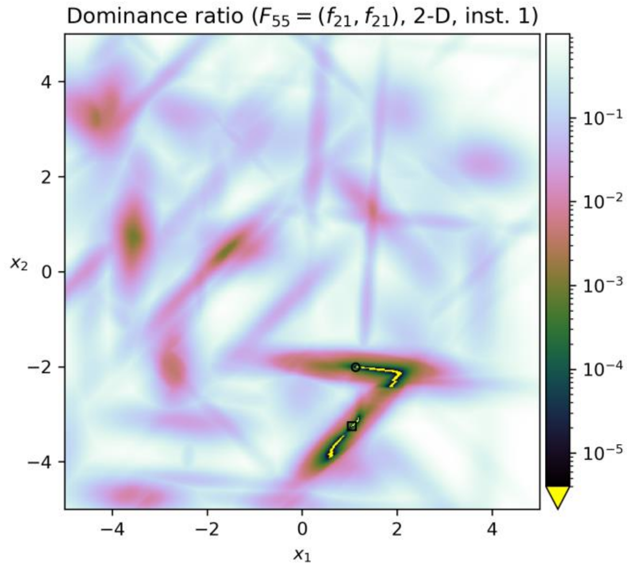
Dominance Ratio

[Fonseca 1995]

- Discretized search space (501 x 501 grid)
- Dominance ratio = the ratio of grid points that dominate the current point
- All nondominated points have a ratio of zero
- Visualize dominance ratios in logarithmic scale







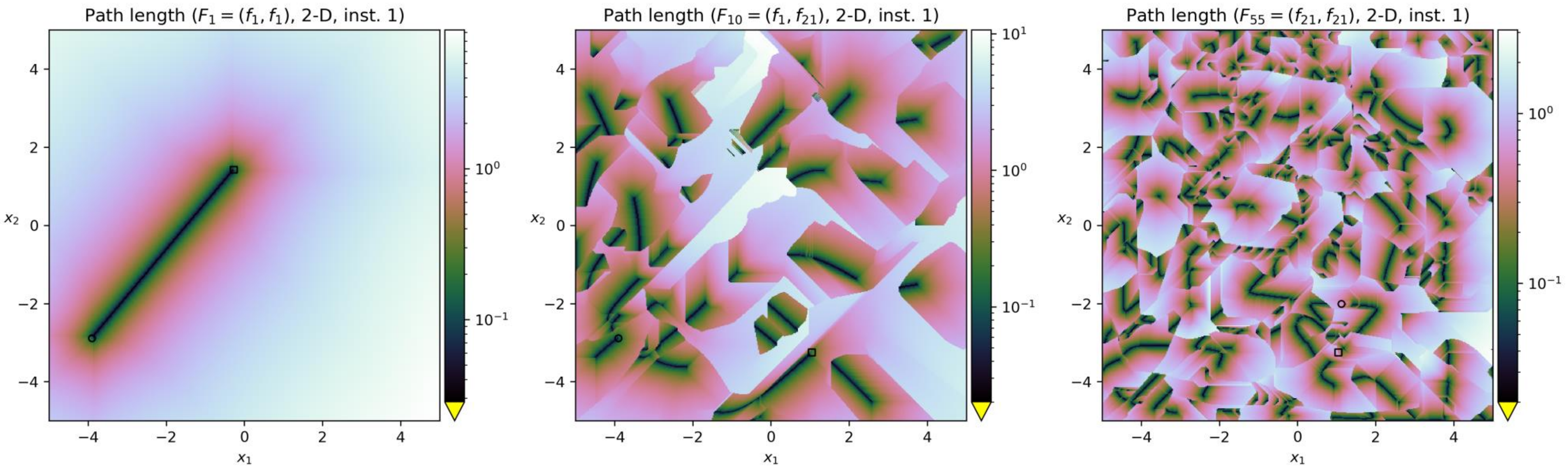
Gradient Path Length

Adjusted from [Kerschke and Grimme 2017]

- Compute the ***bi-objective gradient*** for all grid points

$$v = \frac{\nabla f_1(x)}{\|\nabla f_1(x)\|} + \frac{\nabla f_2(x)}{\|\nabla f_2(x)\|}$$

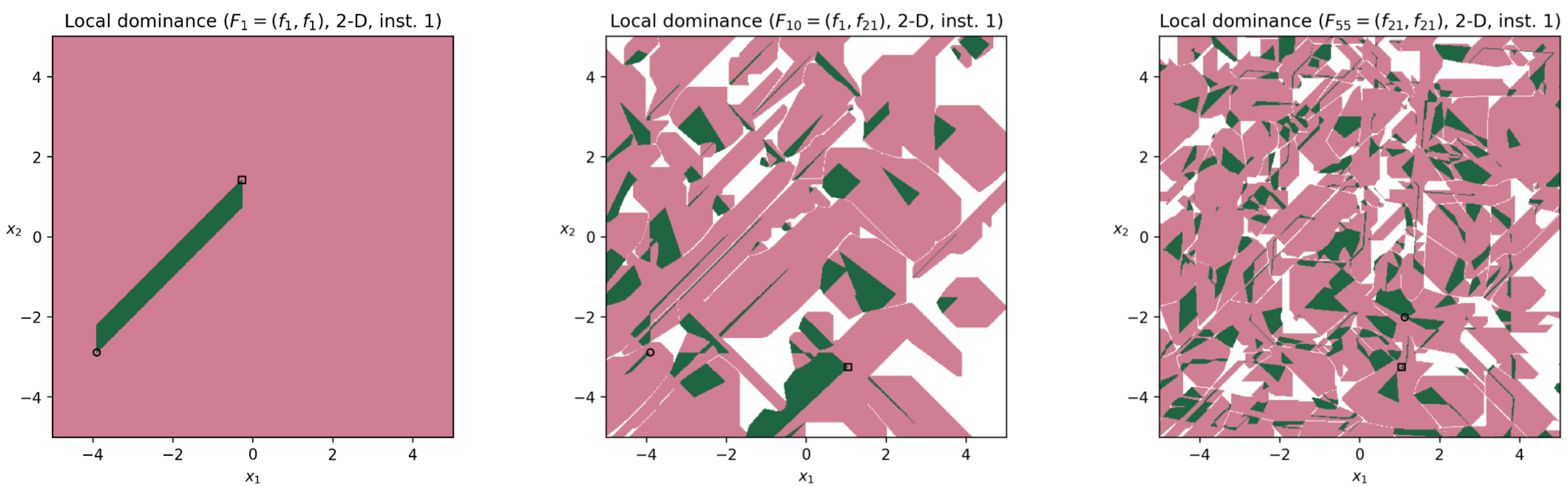
- From a grid point, follow the path in the direction of this gradient
- Visualize the length of the path to the local optimum



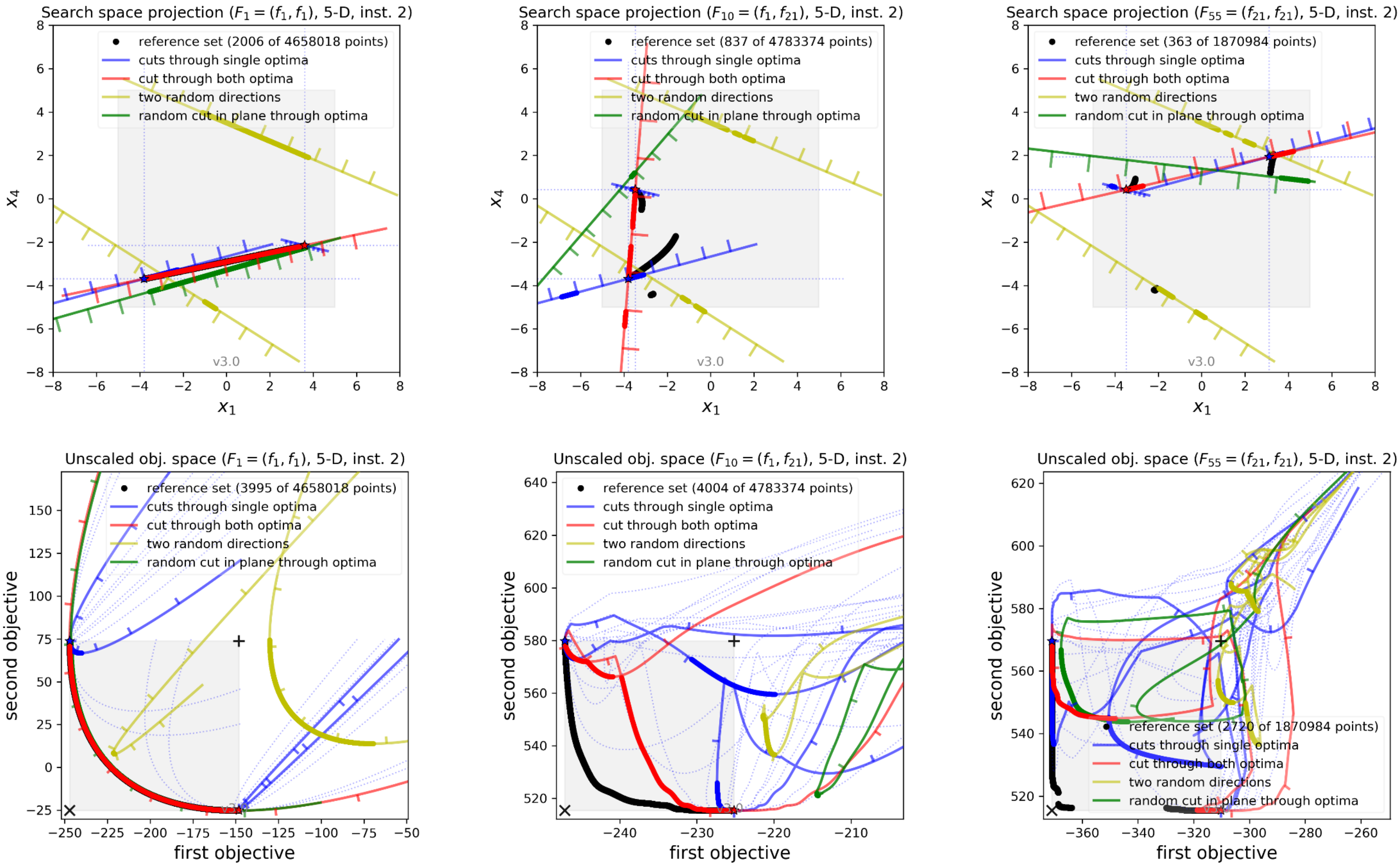
Local Dominance

[Fieldsend et al. 2019]

- Green: Dominance-neutral local optima regions
  - Points that are mutually nondominated with all their 8 neighbors (not equal to Pareto sets)
- Pink: Basins of attraction
  - Points that are dominated by at least one neighbor and whose dominating paths lead to the same green region
- White: Boundary regions
  - Points whose dominating paths lead to different green regions



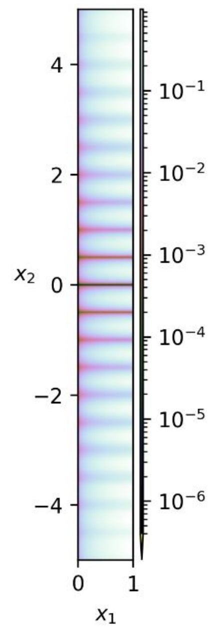
Line Cuts

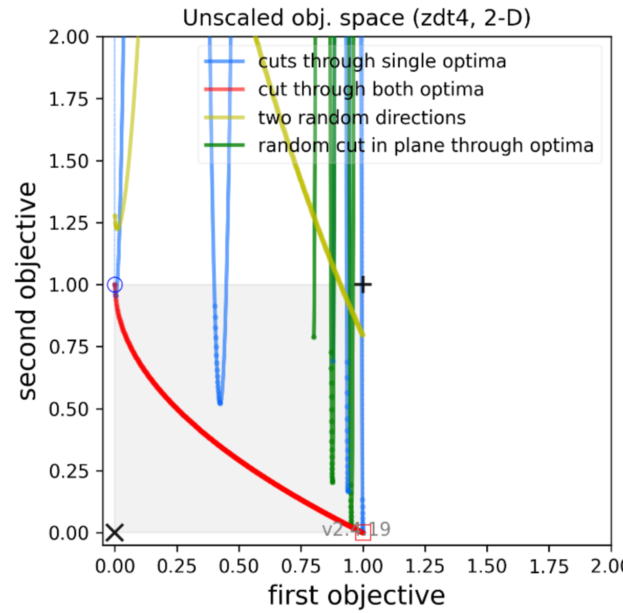


Search space projection ($F_1 = (f_1, f_1)$, 5-D, inst. 2)
reference set (2006 of 4658018 points)
cuts through single optima
cut through both optima
two random directions
random cut in plane through optima
v3.0
x_1
x_4
Search space projection ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)
reference set (837 of 4783374 points)
Search space projection ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)
reference set (363 of 1870984 points)
Unscaled obj. space ($F_1 = (f_1, f_1)$, 5-D, inst. 2)
reference set (3995 of 4658018 points)
Unscaled obj. space ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)
reference set (4004 of 4783374 points)
Unscaled obj. space ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)
reference set (2720 of 1870984 points)
first objective
second objective

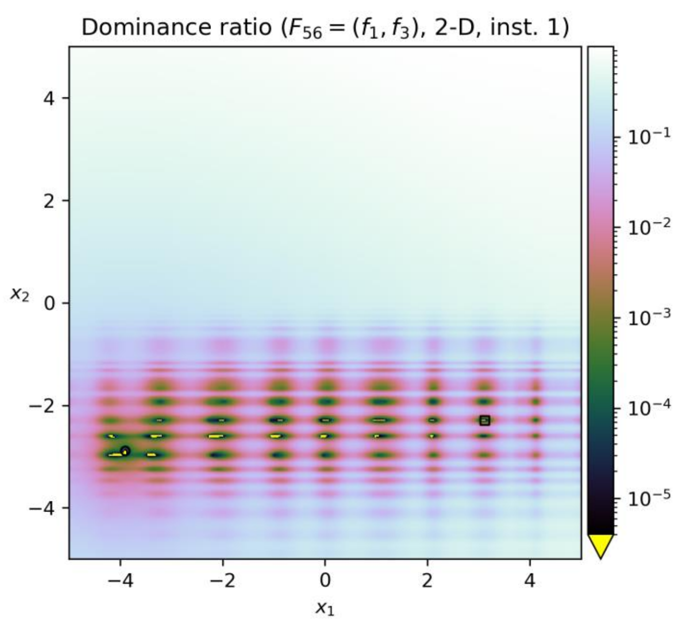
Comparison of Problem Landscapes

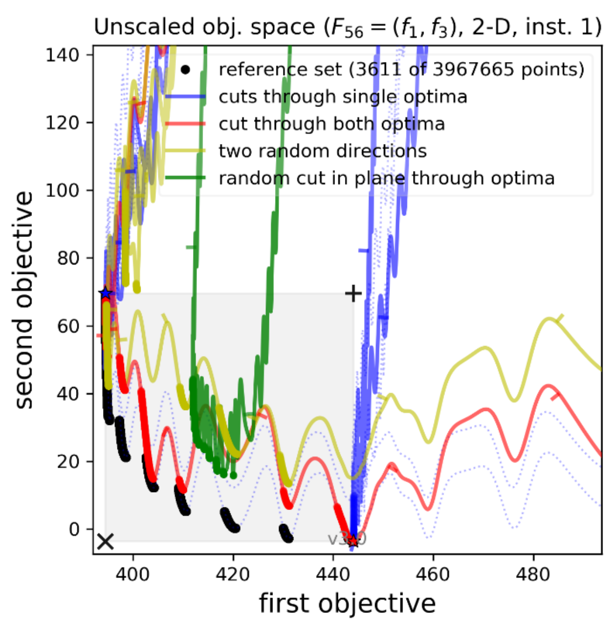
ZDT4





Two problems where both objectives are separable, first is unimodal and second is multimodal





bbob-biobj-ext F_{56}

f_1 Sphere function

f_3 Rastrigin function

Comparison of Problem Landscapes

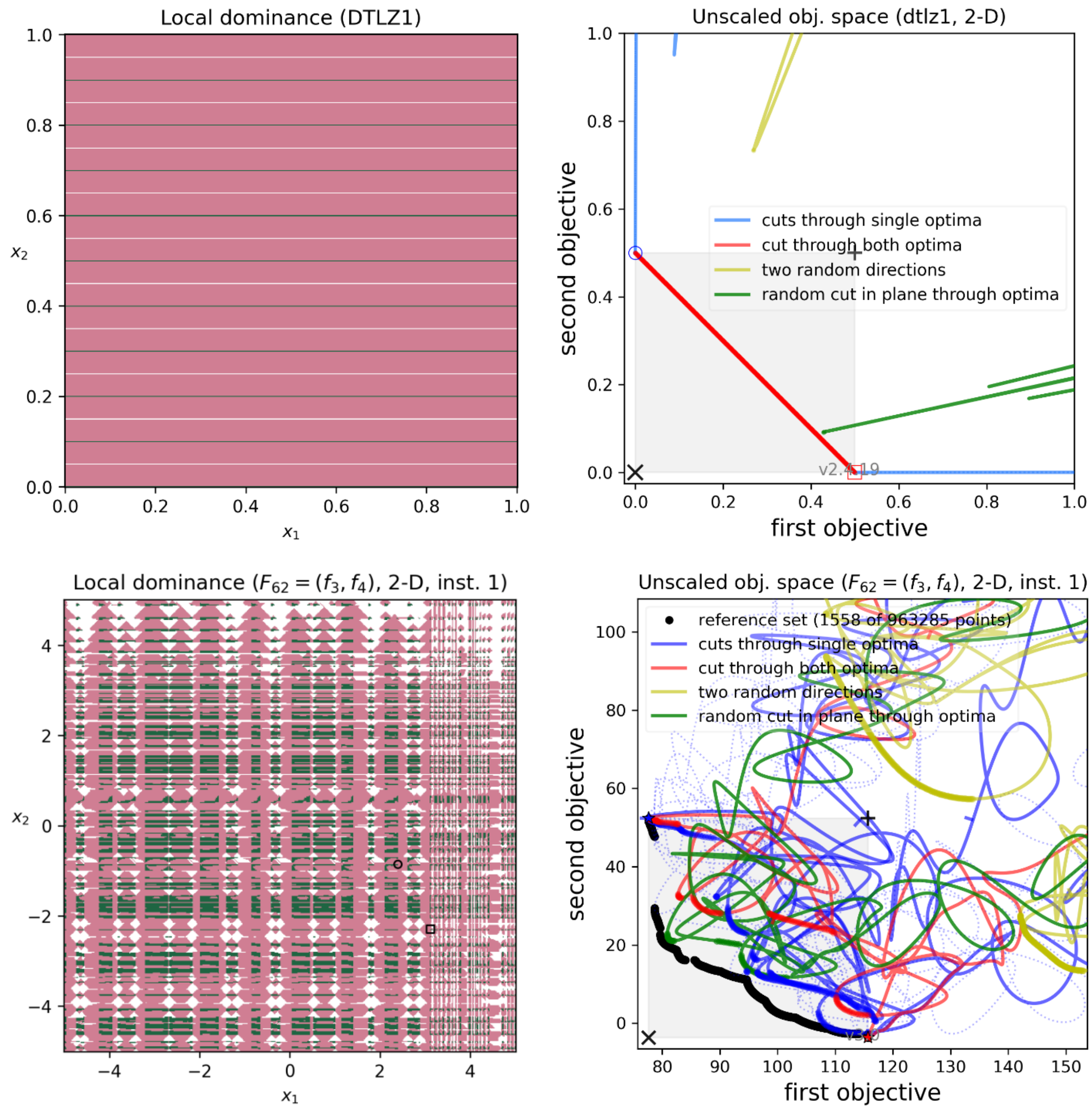
DTLZ1

Two problems where both objectives are separable and multimodal

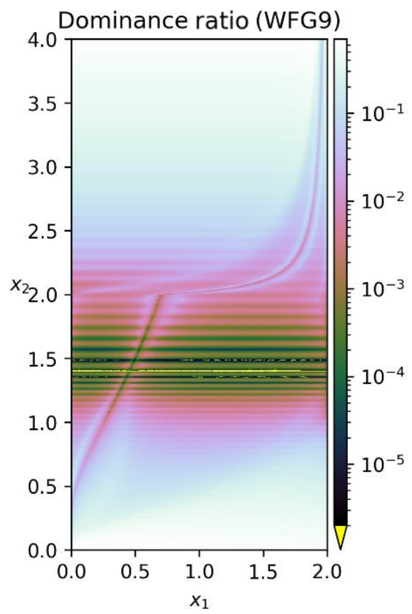
bbob-biobj-ext F_{62}

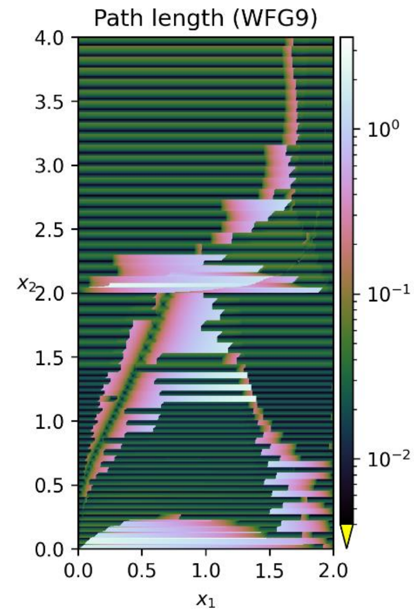
f_3 Rastrigin function

f_4 Skew Rastrigin-Bueche



WFG9

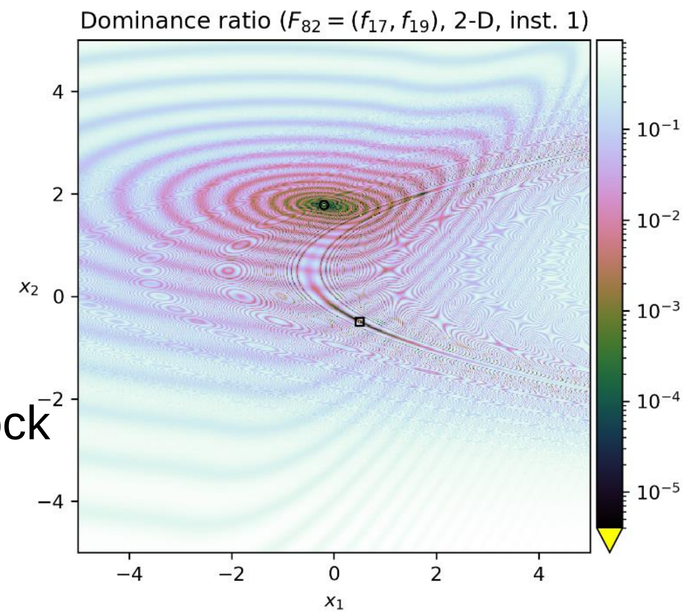


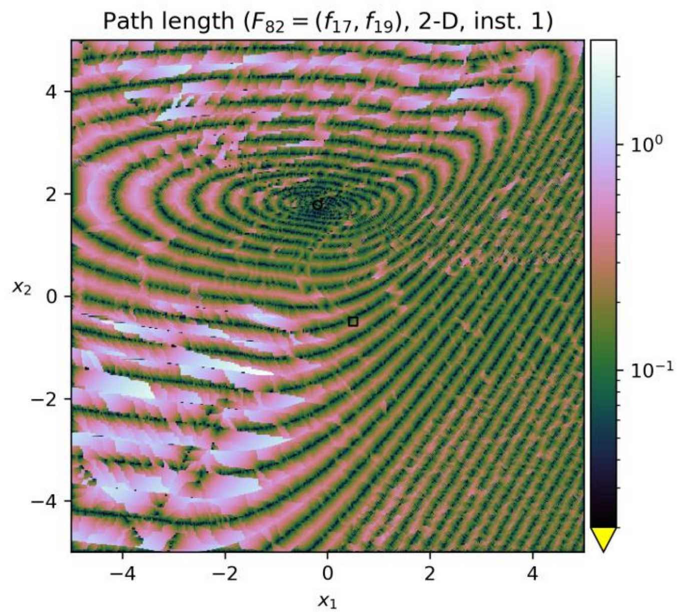


Two problems where both objectives are nonseparable and multimodal

bbob-biobj-ext F_{82}

f_{17} Schaffer F7
f_{19} Griewank-Rosenbrock

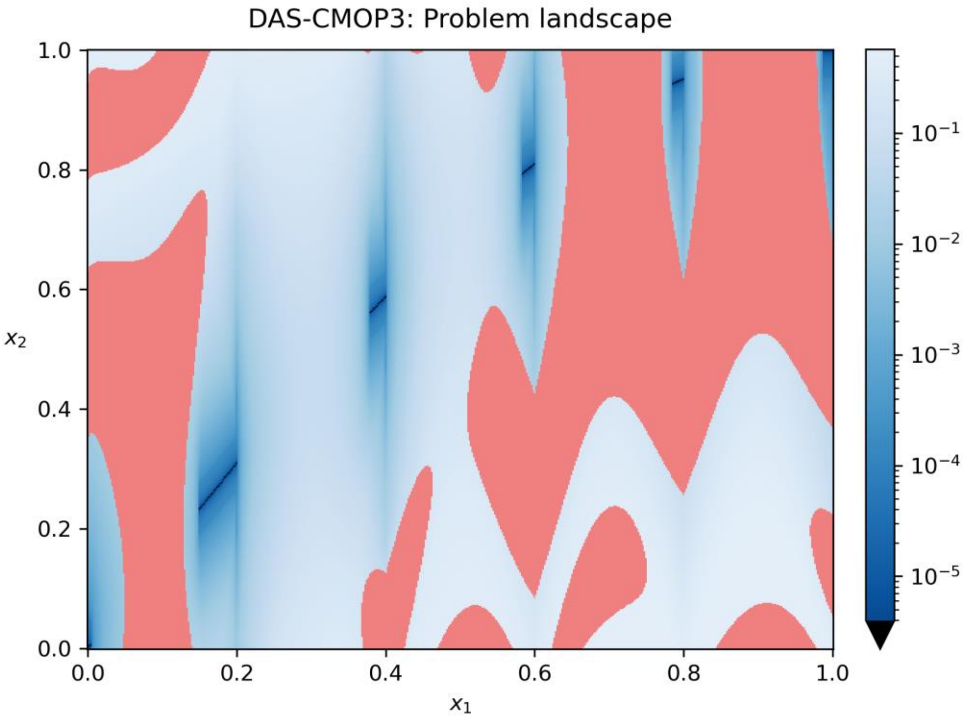




Other artificial problems

Suites of multiobjective problems with constraints

- CTP [Deb et al. 2001]
- CF [Zhang et al. 2009]
- C-DTLZ [Jain and Deb 2014]
- NCTP [Li et al. 2016]
- DC-DTLZ [Li et al. 2019]
- LIR-CMOP [Fan et al. 2019a]
- DAS-CMOP [Fan et al. 2019b]
- MW [Ma and Wang 2019]



Analysis and visualization of multiobjective problems with constraints:
https://vodopijaaljosa.github.io/cmop-web/

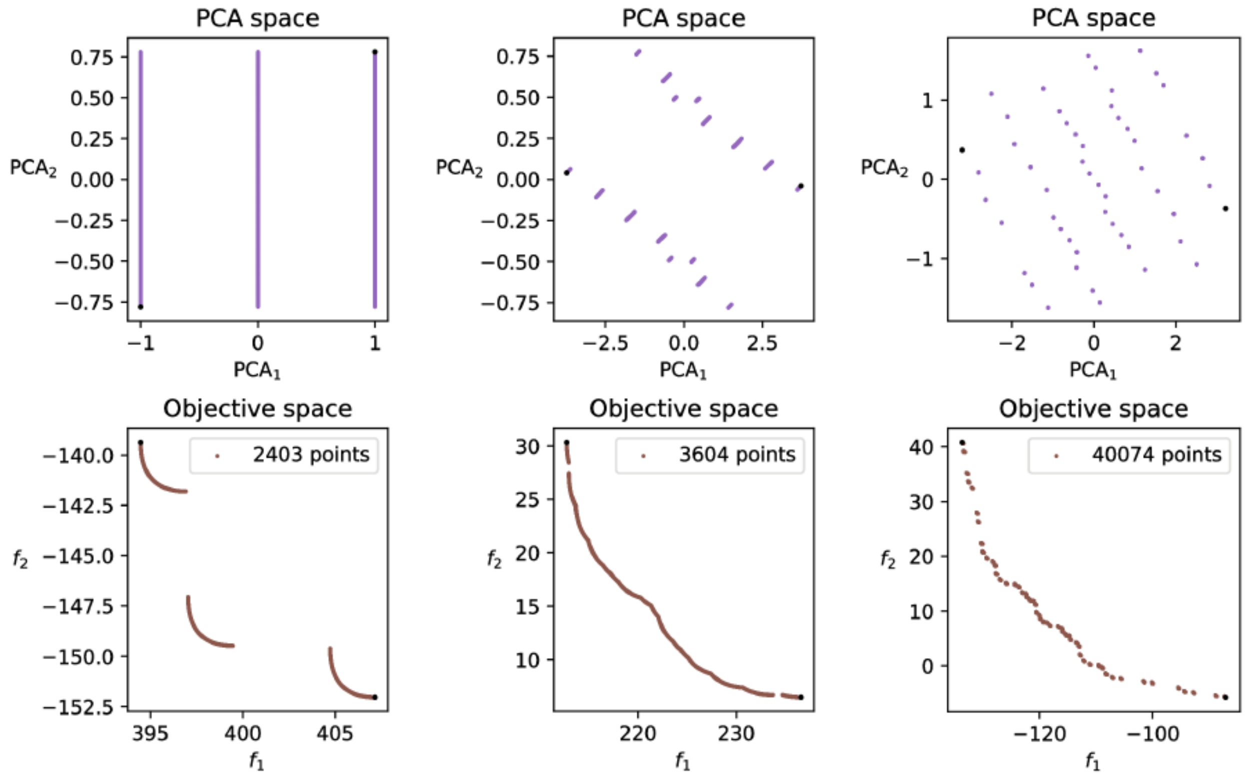
Tutorial on Multiobjective optimization in the presence of constraints:
https://dis.ijs.si/filipic/cec2021tutorial/

Other Artificial Problems

Suites of multiobjective mixed-integer problems

- Exeter suite of 6 problems constructed with the bottom-up approach [McClymont and Keedwell 2011]
- bbob-biobj-mixint suite of 92 bi-objective problems [Tušar et al. 2019]

Pareto set and front approximations for three different instances of the double sphere function



Real-world problems

v0.1

Real-World Problems

Individual problems

- Radar waveform design problem with a varying number of variables and 9 objectives [Hughes 2007]
- HBV problem of calibrating the HBV rainfall-runoff model with 14 variables and 4 objectives [Reed et al. 2013]
- MAZDA car structure design problem with 222 integer variables, 2 objectives and 54 constraints [Kohira et al. 2018]

v0.2

Suites of Real-World Problems

Suites of unscalable problems

[Tanabe and Ishibuchi 2020]

- RE suite of 16 test problems with a different number of variables (2–7) and objectives (2–9)
  - 11 continuous problems
  - 1 integer problem
  - 4 mixed-integer problems
- CRE suite of 8 test problems with constraints a different number of variables (3–7) and objectives (2–5)
  - 6 continuous problems
  - 1 integer problem
  - 1 mixed-integer problem
- Both suites consist of previously published problems

v0.5

Suites of Real-World Problems

Suites of scalable problems

- Heat exchanger design problem with scalable variables and 1 or 2 objectives [Daniels et al. 2018]
- Suite of 3 bi-objective TopTrumps problems in multiple dimensions and instances [Volz et al. 2019]
- Suite of 26 bi-objective MarioGAN problems in multiple dimensions and instances [Volz et al. 2019]

Conclusions

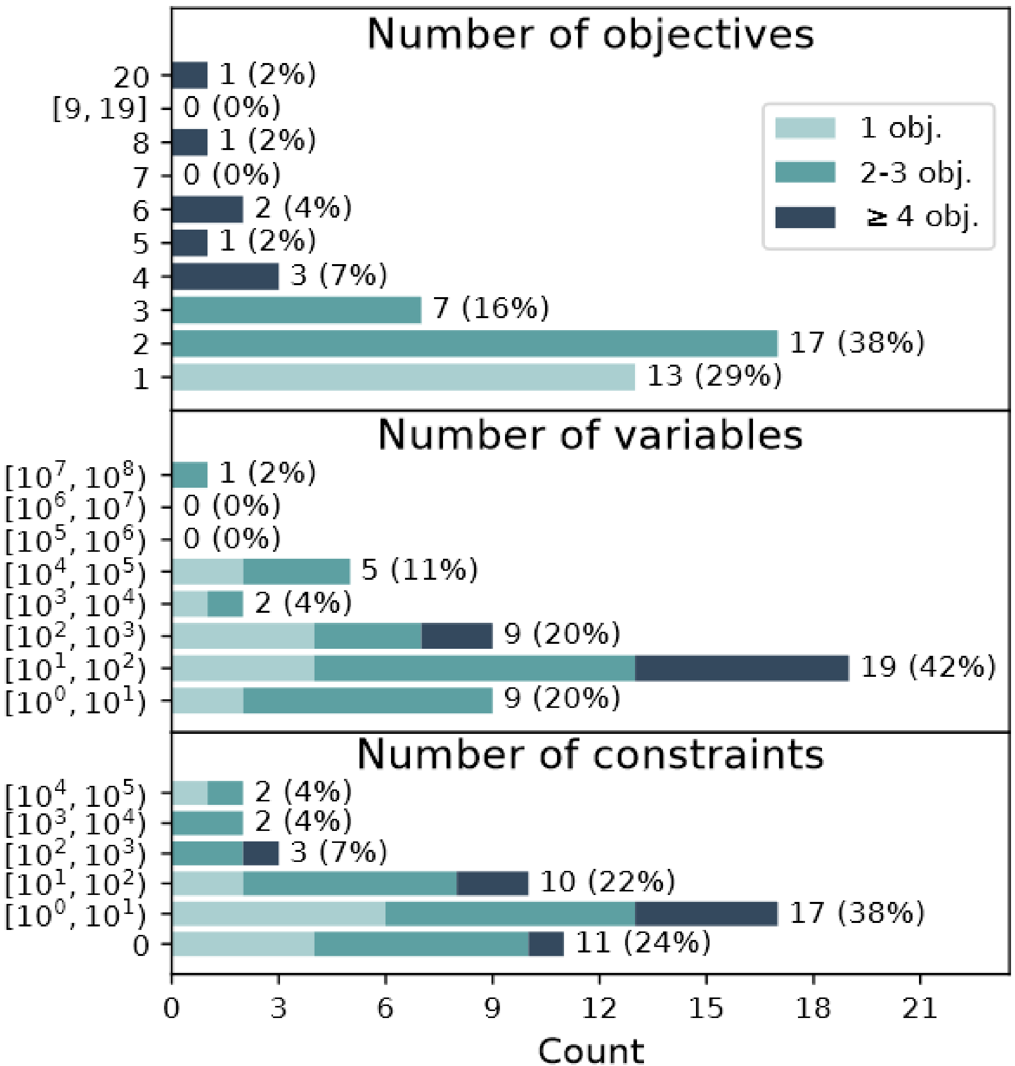
We should think about the usefulness of our research

Results of a questionnaire on the properties of real-world problems has shown their diversity [van der Blom et al. 2020]

https://sites.google.com/view/macoda-rwp/home

Most research is done on continuous unconstrained problems

Although the test problems are scalable, most studies use a fixed number of variables

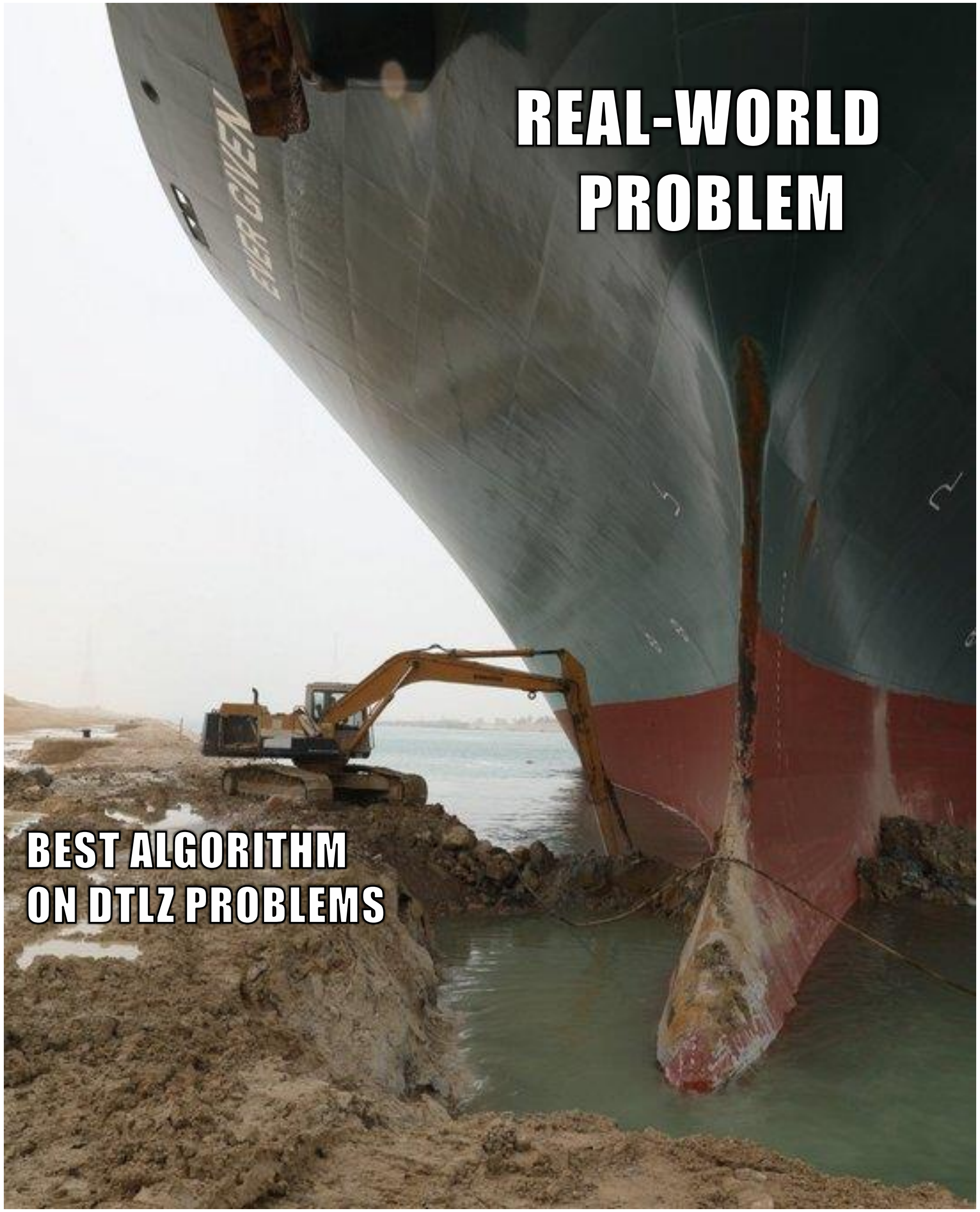


Conclusions

Problem suites constructed with the bottom-up approach have unrealistic properties

Algorithms are overfitting to these problems (especially the overused DTLZ and WFG) [Ishibuchi et al. 2017]

Using separate functions for the objectives looks like a step in the right direction



Overview

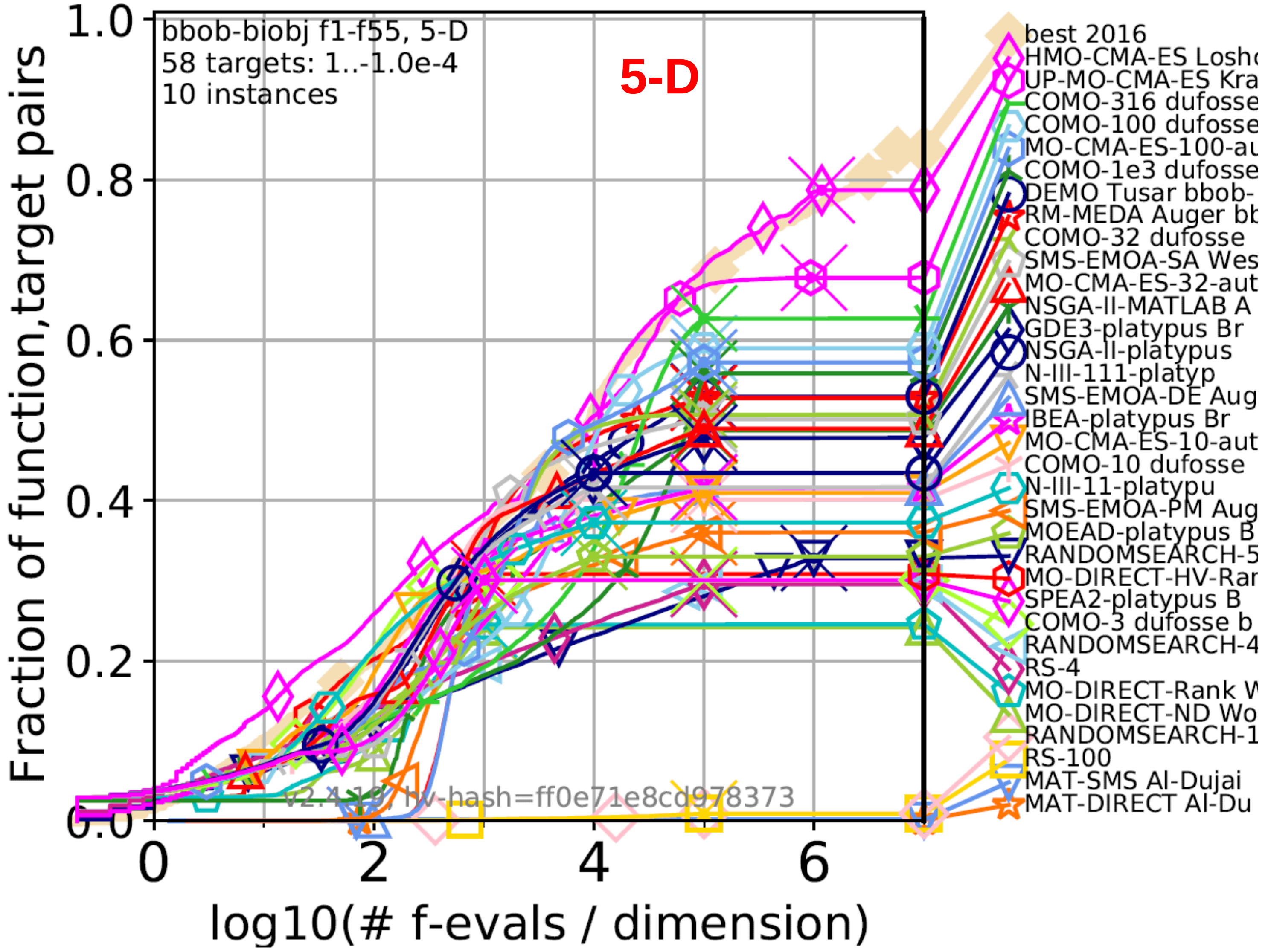
❶ Performance Assessment

❷ Test Problems and Their Visualizations

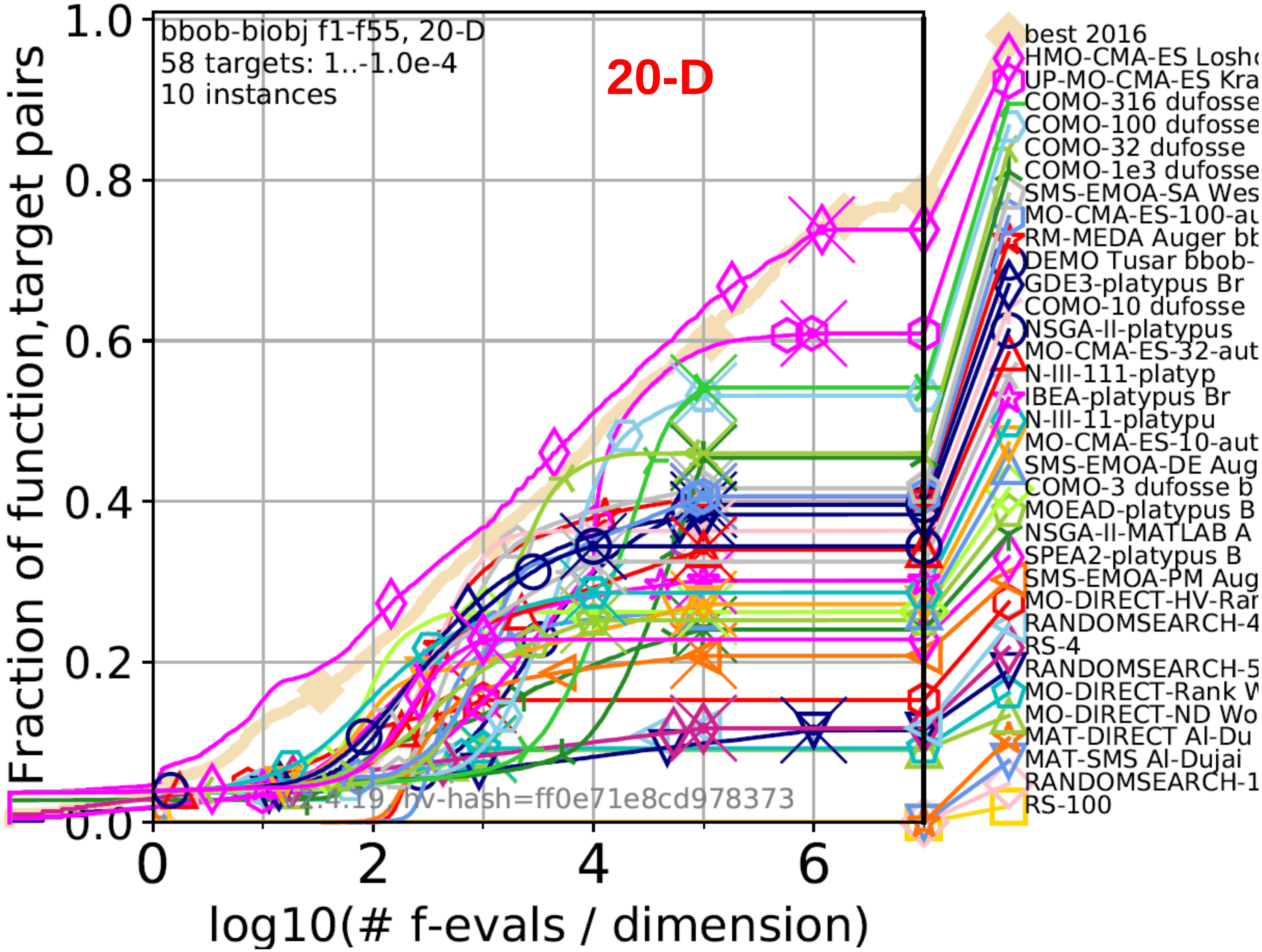
❸ Recommendations from Numerical Results

```
python -m cocopp bbob-biobj*
```

Aggregated Results Over All 55 Functions



Aggregated Results Over All 55 Functions



Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

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- many-objective problems
  - problems/suites
  - indicators
  - efficient implementations

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  - simulation crashes
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  - …
- benchmarking results from more classical approaches

Three “New Year” Resolutions

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❶ Show convergence graphs/ECDF

anything else than tables for fixed budget

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Thank you!

Supplementary Material

Bibliography

[Audet et al 2021] C. Audet, J. Bigeon, D. Cartier, S. Le Digabel, and L. Salomon. Performance indicators in multiobjective optimization. European Journal of Operational Research, 292(2), 2021

[Bartz-Beielstein et al. 2020] T. Bartz-Beielstein, C. Doerr, J. Bossek, S. Chandrasekaran, T. Eftimov, A. Fischbach, P. Kerschke, M. López-Ibáñez, K. M. Malan, J. H. Moore, B. Naujoks, P. Orzechowski, V. Volz, M. Wagner, T. Weise. Benchmarking in Optimization: Best Practice and Open Issues, ArXiv e-prints, arXiv:2007.03488v2, 2020

[Brockhoff et al. 2016] D. Brockhoff, T. Tušar, A. Auger, and N. Hansen. Using Well-understood single-objective functions in multiobjective black-box optimization test suites, ArXiv e-prints, arXiv:1604.00359v3, 2016

[Cheng et al. 2017a] R. Cheng, M. Li, Y. Tian, X. Zhang, S. Yang, Y. Jin, and X. Yao. A benchmark test suite for evolutionary many-objective optimization. Complex & Intelligent Systems 3:67–81, 2017

[Cheng et al. 2017b] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff. Test problems for large-scale Multiobjective and many-objective optimization, IEEE Transactions on Cybernetics, 47(12):4108-4121, 2017

[Daniels et al. 2018] S. J. Daniels, A. A. M. Rahat, R. M. Everson, G. R. Tabor, and J. E. Fieldsend. A suite of computationally expensive shape optimisation problems using computational fluid dynamics. In Parallel Problem Solving from Nature (PPSN XV), volume 11102 of LNCS, pp. 296–307. Springer, 2018

[Deb 1999] K. Deb. Multi-objective genetic algorithms: Problem difficulties and construction of test problems, Evolutionary Computation 7(3):205–230, 1999

[Deb and Jain 2014] K. Deb and H. Jain. An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: Solving problems with box constraints, IEEE Transactions on Evolutionary Computation 18(4):577–601, 2014

[Deb and Tiwari 2008] K. Deb and S. Tiwari. Omni-optimizer: A Procedure for single and multi-objective optimization. In Evolutionary Multi-Criterion Optimization (EMO 2005), volume 3410 of LNCS, pp. 47-61. Springer, 2005

Bibliography

[Deb et al. 2001] K. Deb, A. Pratap, and T. Meyarivan. Constrained test problems for multi-objective evolutionary optimization. In Evolutionary Multi-Criterion Optimization (EMO 2001), volume 1993 of LNCS, pp. 284–298. Springer, 2001

[Deb et al. 2005] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler. Scalable test problems for evolutionary multiobjective optimization. In Evolutionary Multiobjective Optimization: Theoretical Advances and Applications, pp. 105–145. Springer, 2005

[Emmerich and Deutz 2007] M. T. Emmerich and A. H. Deutz. Test problems based on Lamé superspheres. In Evolutionary Multi-Criterion Optimization (EMO 2007), volume 4403 of LNCS, pp. 922–936. Springer, 2007

[Fan et al. 2019a] Z. Fan, W. Li, X. Cai, H. Huang, Y. Fang, Y. You, J. Mo, C. Wei, and E. Goodman. An improved epsilon constraint-handling method in MOEA/D for CMOPs with large infeasible regions, Soft Computing 23(23):12491–12510, 2019

[Fan et al. 2019b] Z. Fan, W. Li, X. Cai, H. Li, C. Wei, Q. Zhang, K. Deb, and E. Goodman. Difficulty adjustable and scalable constrained multiobjective testproblem toolkit, Evolutionary Computation 28(3):339–378, 2019

[Fieldsend 2016] J. E. Fieldsend. Enabling dominance resistance in visualisable distance-based many-objective problems. In Genetic and Evolutionary Computation Conference Companion (GECCO 2016 Companion), pp. 1429–1436. ACM, 2016

[Fieldsend and Alyahya 2019] J. E. Fieldsend and K. AlYahya. Visualising the landscape of multi-objective problems using local optima networks. In Genetic and Evolutionary Computation Conference Companion (GECCO 2019 Companion), pp. 1421–1429. ACM, 2019

[Fieldsend et al. 2019] J. E. Fieldsend, T. Chugh, R. Allmendinger, and K. Miettinen A feature rich distance-based many-objective visualisable test problem generator. In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2019), pp. 541–549. ACM, 2019

[Filipič and Tušar 2018] B. Filipič and T. Tušar. A Taxonomy of Methods for Visualizing Pareto Front Approximations. In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2018), pp. 649–656. ACM, 2018

Bibliography

[Filipič and Tušar 2020] B. Filipič and T. Tušar. Visualization in multiobjective optimization. In Genetic and Evolutionary Computation Conference Companion (GECCO 2018 Companion), pp. 775–800. ACM, 2020

[Fonseca 1995] C. M. Fonseca. Multiobjective Genetic Algorithms with Application to Control Engineering Problems. Ph.D. thesis, University of Sheffield, 1995

[Fonseca and Fleming 1995] C. M. Fonseca and P. J. Fleming. An overview of evolutionary algorithms in multiobjective optimization. Evolutionary Computation 3:1-16, 1995

[Fonseca et al. 2011] C. M. Fonseca, A. P. Guerreiro, M. López-Ibáñez, and L. Paquete. On the computation of the empirical attainment function. In Evolutionary Multi-Criterion Optimization (EMO 2011), volume 6576 of LNCS, pp. 106-120. Springer, 2011

[Huang et al. 2007] V. L. Huang, A. K. Qin, K. Deb, E. Zitzler, P. N. Suganthan, J. J .Liang, M. Preuss, and S. Huband. Problem definitions for performance assessment on multi-objective optimization algorithms, Technical Report, 2007

[Huband et al. 2006] S. Huband, P. Hingston, L. Barone, and L. While. A review of multiobjective test problems and a scalable test problem toolkit, IEEE Transactions on Evolutionary Computation 10(5):477–506, 2006

[Hughes 2007] E. J. Hughes, Radar waveform optimisation as a many-objective application benchmark. In Evolutionary Multi-Criterion Optimization (EMO 2007), volume 4403 of LNCS, pp. 700–714. Springer, 2007

[Igel et al. 2007] C. Igel, N. Hansen, and S. Roth. Covariance matrix adaptation for multi-objective optimization. Evolutionary Computation, 15(1):1–28, 2007

[Ishibuchi et al. 2010] H. Ishibuchi, Y. Hitotsuyanagi, N. Tsukamoto, and Y. Nojima. Many-objective test problems to visually examine the behavior of multiobjective evolution in a decision space. In Parallel Problem Solving from Nature (PPSN XI), volume 6239 of LNCS, part II, pp. 91-100, Springer, 2010

[Ishibuchi et al. 2017] H. Ishibuchi, Y. Setoguchi, H. Masuda, and Y. Nojima. Performance of decomposition-based many-objective algorithms strongly depends on Pareto front shapes. IEEE Transactions on Evolutionary Computation 21(2):169–190, 2017

Bibliography

[Jain and Deb 2014] H. Jain and K. Deb. An evolutionary many-objective optimization algorithm using reference-point based nondominated sorting approach, part II: Handling constraints and extending to an adaptive approach, IEEE Transactions on Evolutionary Computation 18(4):602–622, 2014

[Kerschke and Grimme 2017] P. Kerschke and C. Grimme. An expedition to multi-modal multi-objective optimization landscapes. In Evolutionary Multi-Criterion Optimization (EMO 2017), volume 10173 of LNCS, pp. 329–343. Springer, 2017

[Kerschke et al. 2016] P. Kerschke, H. Wang, M. Preuss, C. Grimme, A. H. Deutz, H. Trautmann, and M. Emmerich. Towards analyzing multimodality of continuous multiobjective landscapes. In International Conference on Parallel Problem Solving from Nature (PPSN XIV), volume 9921 of LNCS, pp. 962-972. Springer, 2016

[Kohira et al. 2018] T. Kohira, H. Kemmotsu, A. Oyama, T. Tatsukawa. Proposal of benchmark problem based on real-world car structure design optimization. In Genetic and Evolutionary Computation Conference Companion (GECCO 2019 Companion), pp. 183-184. ACM, 2018

[Köppen et al. 2005] M. Köppen, R. Vicente-Garcia, and B. Nickolay. Fuzzy-Pareto-dominance and its application in evolutionary multi-objective optimization. In Evolutionary Multi-Criterion Optimization (EMO 2005), volume 3410 of LNCS, pp. 399–412. Springer, 2005

[Kursawe 1991] F. Kursawe. A variant of evolution strategies for vector optimization. In International Conference on Parallel Problem Solving from Nature (PPSN I), volume 496 of LNCS, pp. 193–197, 1991

[Li and Yao 2019] M. Li and X. Yao. Quality evaluation of solution sets in multiobjective optimisation: A survey. ACM Computing Surveys (CSUR), 52(2):1-38. 2019

[Li and Zhang 2009] H. Li and Q. Zhang. Multiobjective optimization problems with complicated Pareto sets, OEA/D and NSGA-II, IEEE Transactions on Evolutionary Computation 13(2):284–302, 2009

[Li et al. 2014] M. Li, S. Yang, and X. Liu. A test problem for visual investigation of high-dimensional multi-objective search. In Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2014), pp. 2140–2147. IEEE, 2014

Bibliography

[Li et al. 2016] J. P. Li, Y. Wang, S. Yang, and Z. Cai. A comparative study of constraint-handling techniques in evolutionary constrained multiobjective optimization. In IEEE Congress on Evolutionary Computation (pp. 4175–4182, 2016

[Li et al. 2018] M. Li, C. Grosan, S. Yang, X. Liu, and X. Yao. 2018. Multiline distance minimization: A visualized many-objective test problem suite. IEEE Transactions on Evolutionary Computation 22(1):61–78, 2018

[Liang et al. 2019] J. J. Liang, B. Y. Qu, D. W. Gong, and C. T. Yue. Problem definitions and evaluation criteria for the CEC 2019 special session on multimodal multiobjective optimization, Technical Report, 2019

[Liang et al. 2020] J. J. Liang, P. N. Suganthan, B. Y. Qu, D. W. Gong, and C. T. Yue. Problem definitions and evaluation criteria for the CEC 2020 special session on multimodal multiobjective optimization, Technical Report 201912, Zhengzhou University, China, 2020

[Liefooghe et al. 2018] A. Liefooghe, B. Derbel, S. Vérel, M. López-Ibáñez, H. E. Aguirre, and K. Tanaka. On Pareto local optimal solutions networks. In International Conference on Parallel Problem Solving from Nature (PPSN XV), volume 11102 of LNCS, pp. 232-244. Springer, 2018

[Liu et al. 2018] Y. Liu, H. Ishibuchi, Y. Nojima, N. Masuyama, and K. Shang. A Double niched evolutionary algorithm and its behavior on polygon-based problems. In International Conference on Parallel Problem Solving from Nature (PPSN XV), volume 11101 of LNCS, pp. 262–273. Springer, 2018

[López-Ibáñez et al. 2010] M. López-Ibáñez, L. Paquete, and T. Stützle. Exploratory analysis of stochastic local search algorithms in biobjective optimization. In Experimental Methods for the Analysis of Optimization Algorithms, pp. 209–222. Springer, 2010

[Ma and Wang 2019] Z. Ma and Y. Wang. Evolutionary constrained multiobjective optimization: Test suite construction and performance comparisons, IEEE Transactions on Evolutionary Computation 23(6):972–986, 2019

[Masuda et al. 2014] H. Masuda, Y. Nojima, and H. Ishibuchi. Visual examination of the behavior of EMO algorithms for many-objective optimization with many decision variables. In IEEE Congress on Evolutionary Computation (CEC 2014), pp. 2633–2640. IEEE, 2014

Bibliography

[Masuda et al. 2016] H. Masuda, Y. Nojima, and H. Ishibuchi. Common properties of scalable multiobjective problems and a new framework of test problems. In Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2016), pp. 3011-3018. IEEE, 2016

[McClymont and Keedwell 2011] K. McClymont and E. Keedwell. Benchmark multi-objective optimisation test problems with mixed encodings. In Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2011), pp. 2131–2138. IEEE, 2011

[Moré and Wild 2009] J.J. Moré and S. M. Wild. Benchmarking derivative-free optimization algorithms. SIAM Journal on Optimization, 20(1), 172-191. 2009

[Okabe et al. 2004] T. Okabe, Y. Jin, M. Olhofer, and B. Sendhoff. On test functions for evolutionary multi-objective optimization. In Parallel Problem Solving From Nature (PPSN VIII), volume 3242 of LNCS, pp. 792–802, 2004

[Poloni et al. 1996] C. Poloni, G. Mosetti, and S. Contessi. Multi objective optimization by GAs: Application to system and component design. In Proceedings of Computational Methods in Applied Sciences '96, pp. 258–264, 1996

[Reed et al. 2013] P. M. Reed, D. Hadka, J. D. Herman, J. R. Kasprzyk, and J. B. Kollat. Evolutionary multiobjective optimization in water resources: The past, present, and future. Advances in Water Resources, 51:438-456, 2013

[Rudolph et al. 2007] G. Rudolph, B. Naujoks, and M. Preuss. Capabilities of EMOA to detect and preserve equivalent Pareto subsets, In Evolutionary Multi-Criterion Optimization (EMO 2007), volume 4403 of LNCS, pp. 36–50. Springer, 2007

[Saxena et al. 2011] D. K. Saxena, Q. Zhang, J. Ao, A. Duro, and A. Tiwari. Framework for many-objective test problems with both simple and complicated Pareto set shapes. In Evolutionary Multi-Criterion Optimization (EMO 2011), volume 6576 of LNCS, pp. 197–211. Springer, 2011

Bibliography

[Shaepermeier et al. 2020] L. Schäpermeier, C. Grimme, P. Kerschke. One PLOT to show them all: Visualization of efficient sets in multi-objective landscapes. In Parallel Problem Solving From Nature (PPSN XVI), volume 12270 of LNCS, pp. 154-167, 2020

[Schaffer 1985] J. D. Schaffer. Multiple objective optimization with vector evaluated genetic algorithms. In John J. Grefenstette, editor, Conference on Genetic Algorithms and Their Applications, pp. 93–100, 1985

[Tanabe and Ishibuchi 2020] R. Tanabe and H. Ishibuchi. An easy-to-use real-world multi-objective optimization problem suite. Applied Soft Computing 89:106078, 2020

[Tušar et al. 2019] T. Tušar, D. Brockhoff, and N. Hansen. Benchmark problems for single- and bi-objective optimization. In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2019), pp. 718-726. ACM, 2019

[van der Blom et al. 2020] K. van der Blom, T. M. Deist, V. Volz, M. Marchi, Y. Nojima, B. Naujoks, A. Oyama, and T. Tušar. Identifying properties of real-world optimisation problems through a questionnaire. In Many-Criteria Optimization and Decision Analysis (To appear). Springer, 2021

[Van Veldhuizen 1999] D. A. Van Veldhuizen. Multiobjective evolutionary algorithms: Classifications, analyses, and new innovations, Ph.D. dissertation, Air Force Institute of Technology, Ohio, USA, 1999

[Viennet et al. 1996] R. Viennet, C. Fonteix, and I. Marc. Multicriteria optimization using a genetic algorithm for determining a Pareto set, International Journal of Systems Science, 27(2):255-260, 1996

[Volz et al. 2019] V. Volz, B. Naujoks, P. Kerschke, and T. Tušar. Single- and multi-objective game-benchmark for evolutionary algorithms. In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2019), pp. 647-655. ACM, 2019

[Yue et al. 2019] C. Yue, B. Qu, K. Yu, J. Liang, and X. Li. A novel scalable test problem suite for multimodal multiobjective optimization, Swarm and Evolutionary Computation, 48:62-71, 2019

[Zapotecas et al. 2019] S. Zapotecas-Martínez, C. A. Coello Coello, H. E Aguirre, and K. Tanaka. 2019. A review of features and limitations of existing scalable multi-objective test suites. IEEE Transactions on Evolutionary Computation 23(1): 130–142, 2019

Bibliography

[Zhang et al. 2009] Q. Zhang, A. Zhou, S. Zhao, P. N. Suganthan, W. Liu, and S. Tiwari. Multiobjective optimization test instances for the CEC 2009 special session and competition. Technical Report CES-487, University of Essex, UK, 2009

[Zitzler et al. 2000] E. Zitzler, K. Deb, and L. Thiele. Comparison of multiobjective evolutionary algorithms: Empirical results. Evolutionary Computation, 8(2):173–195, 2000

[Zitzler et al. 2003] E. Zitzler, L. Thiele, M. Laumanns, C.M. Fonseca, and V. Grunert Da Fonseca. Performance assessment of multiobjective optimizers: An analysis and review. IEEE Transactions on evolutionary computation, 7(2):117-132. 2003

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After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. After postdoctoral research positions at Inria Saclay Ile-de-France in Orsay and at Ecole Polytechnique in Palaiseau, both in France, Dimo has been a permanent researcher at Inria: from 2011 till 2016 with the Inria Lille - Nord Europe research center and since October 2016 with the Saclay - Ile-de-France research center, co-located with CMAP, Ecole Polytechnique, IP Paris. His most recent research interests are focused on evolutionary multiobjective optimization (EMO) and other (single-objective) blackbox optimization techniques, in particular with respect to benchmarking, theoretical aspects, and expensive optimization.

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