Reinforcement Learning
Sequential Decisions, MDP and Policies

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ÉCOLE POLYTECHNIQUE

M2DS - Reinforcement Learning – Fall 2023

Outline

Decision or Decisions

Source: W. Powell

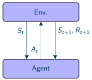
Sequential Decision Setting

Sequential Decision Setting

- In many (most?) settings, not a single decision but a sequence of decisions.
- Need to take into account the (not necessarily immediate) consequences of the sequence of decisions/actions rather than of each decisions.
- Different framework than supervised learning (no immediate feedback here) and unsupervised learning (well defined goal here).

Source: W. Powell

From Sequential Decision to Reinforcement Learning



Sequential Decision

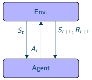
Sequential Decision

- Sequence of action A_t as a response of an environment S_t
- Feedback through a reward R_t

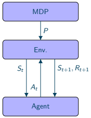
Actions?

- Is my current way of choosing actions good?
- How to make it better?

From Sequential Decision to Reinforcement Learning



Sequential Decision



MDP Modeling

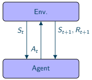
Markov Decision Process Modeling

- Specific modeling of the environment.
- Goal as as a (weighted) sum of a scalar reward.

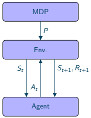
Actions?

- Is my current way of choosing actions good?
- How to make it better?

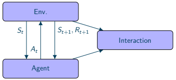
From Sequential Decision to Reinforcement Learning



Sequential Decision



MDP Modeling



Reinforcement Learning

Reinforcement Learning

- Same modeling. . .
- But no direct knowledge of the MDP.

Actions?

- Is my current way of choosing actions good?
- How to make it better?

Sequential Decision Settings

- MDP / Reinforcement Learning:

$$\max_{\pi} \mathbb{E}_{\pi}\left[\sum_t R_t\right]$$

- Optimal Control:

$$\min_{u} \mathbb{E}\left[\sum_t C(x_t, u_t)\right]$$

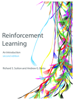
- (Stochastic) Search:

$$\max_{\theta} \mathbb{E}[F(\theta, W)]$$

- Online Regret:

$$\max \sum_k \mathbb{E}[F(\theta_k, W)]$$

References



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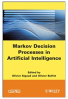

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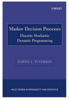

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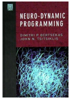

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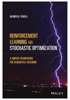

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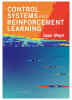

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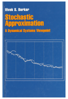

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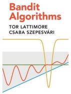

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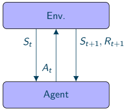
Outline

Environnement

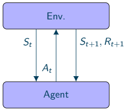


Decision Process and Environment

- At time step $t \in \mathbb{N}$:
  - State $S_t \in \mathcal{S}$: representation of the environment
  - Action $A_t \in \mathcal{A}(S_t)$: action chosen
  - Reward $R_{t+1} \in \mathcal{R}$: instantaneous reward
  - New state S_{t+1}

- Focus on the discrete setting, i.e. $\mathcal{S}$ finite, $\mathcal{A}(s)$ finite and $\mathcal{R}$ finite.
- Extension: Non finite bounded $\mathcal{R}$: easy / Non finite $\mathcal{S}$: hard / Non finite $\mathcal{A}$: harder.

Environnement



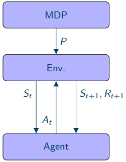
Stochastic Model

- Dynamic defined by:

$$\mathbb{P}\left(S_{t+1} = s', R_{t+1} = r \middle| (S_{t'}, A_{t'}, R_{t'}), t' \leq t\right)$$
$$= \mathbb{P}\left(S_{t+1} = s', R_{t+1} = r \middle| S_t = s, A_t = a, H_t\right)$$

where $H_t = (R_t, S_{t-1}, A_{t-1}, R_{t-1}, S_{t-2}, \dots)$ is the past and (S_t, A_t) the present.

Markov Decision Process and Environment



Markovian Environment

- Markovian Dynamic Assumption: S_{t+1} and R_{t+1} are independent of the past $H_t = (R_t, S_{t-1}, A_{t-1}, R_{t-1}, S_{t-2}, \dots)$ conditionally to the present (S_t, A_t).
- Dynamic entirely defined by state-reward transition probabilities

$$\begin{aligned}\mathbb{P}(S_{t+1} = s', R_{t+1} = r \big| S_t = s, A_t = a, H_t) &= \mathbb{P}(S_{t+1} = s', R_{t+1} = r \big| S_t = s, A_t = a) \\ &= p(s', r | s, a)\end{aligned}$$

in the discrete setting.

- Informally, this means that S_t encodes all the information related to the past.

Markov Decision Process and State-Action

- State-Reward transition probabilities for a given state-action:

$$\mathbb{P}(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a, H_t) = \mathbb{P}(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a)$$
$$= p(s', r | s, a)$$

Induced State-action laws

- State transition probabilities for a given state-action:

$$\mathbb{P}(S_{t+1} = s' | S_t = s, A_t = a, H_t) = \mathbb{P}(S_{t+1} = s' | S_t = s, A_t = a)$$
$$= p(s' | s, a) = \sum_r p(s', r | s, a)$$

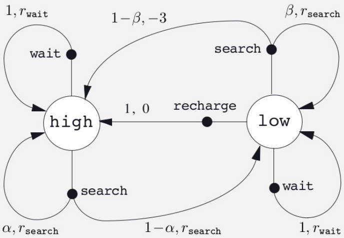
- Expected reward for a given state-action:

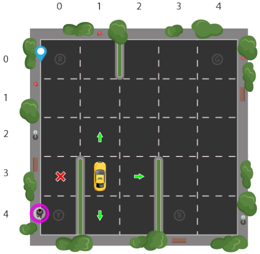
$$\mathbb{E}[R_{t+1} | S_t = s, A_t = a, H_t] = \mathbb{E}[R_{t+1} | S_t = s, A_t = a]$$
$$= r(s, a) = \sum_r r \sum_{s'} p(s', r | r, a)$$

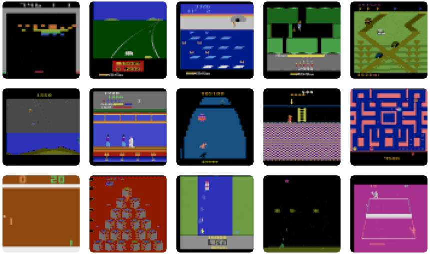
- From now on, we will always assume that the Markovian property holds for the environment.

Examples

s	a	s'	$p(s' \mid s, a)$	$r(s, a, s')$
high	search	high	α	$r_{\texttt{search}}$
high	search	low	$1-\alpha$	$r_{\texttt{search}}$
low	search	high	$1-\beta$	-3
low	search	low	β	$r_{\texttt{search}}$
high	wait	high	1	$r_{\texttt{wait}}$
high	wait	low	0	-
low	wait	high	0	-
low	wait	low	1	$r_{\texttt{wait}}$
low	recharge	high	1	0
low	recharge	low	0	-

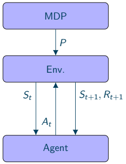






Sources:: Sutton and Barto / learndatasci.com / danijar.com

Decision Process, Agent and Policy



Agent

- Interact with the environment by choose the action given the past.

Policy Π : specification of how to choose the action

- General stochastic policy $\Pi = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$:

$$\Pi_t(A_t = a) = \pi_t(A_t = a | S_t = a, A_t = a, H_t)$$

- General deterministic policy $\Pi = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$ (with as slight abuse of notation):

$$\Pi_t(A_t = a) = \mathbf{1}_{A_t = \pi_t(S_t = a, A_t = a, H_t)}$$

Markov Decision Process, Agent and Policy

Agent

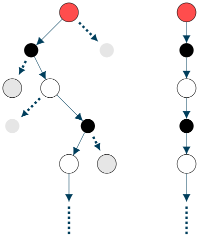
- Interact with the environment by choose the action given the past.

Policy Π: specification of how to choose the action

- History dependent stochastic policy $\Pi = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$:
$$\Pi_t(A_t = a) = \pi_t(A_t = a | S_t = s, H_t)$$
- Markovian stochastic policy $\Pi = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$:
$$\Pi_t(A_t = a) = \pi_t(A_t = a | S_t = s) = \pi_t(a|s)$$
- Stationary Markovian stochastic policy $\Pi = (\pi, \pi, \ldots, \pi, \ldots)$:
$$\Pi_t(A_t = a) = \pi(A_t = a | S_t = s) = \pi(a|s)$$

- Similar deterministic policy definition.
- Partially Observed Markov Decision Process extension: the Agent has only access to a partial observation O_t at each time step... (not the focus of the lectures)

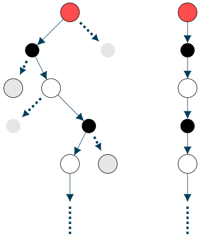
Decision Process and Trajectories



Trajectories

- Trajectory $(S_0, A_0, R_1, S_1, A_1, \ldots)$ defined by
  - an initial distribution $\mathbb{P}_0$ for S_0,
  - a policy $\Pi = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$ specifying
  $$\Pi_t(A_t = a) = \pi_t(A_t = a | S_t, H_t)$$
  - an environment specifying
  $$\mathbb{P}(S_{t+1}, R_{t+1} | S_t, A_t, H_t)$$

Decision Process and Trajectories

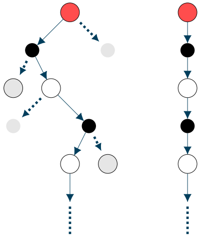


Trajectories

- Induced probability:

$$\begin{aligned}
&\mathbb{P}(S_0 = s_0, A_0 = a_0, R_1 = r_1, S_1 = s_1, A_1 = a_1, \ldots S_t = s_t, R_t = r_t) \\
&\quad = \mathbb{P}_0(S_0 = s_0) \\
&\qquad \times \pi_0(A_0 = a_0 | S_0)\, \mathbb{P}(S_1, R_1 | S_0, A_0)\, \pi_1(A_1 = a_1 | S_1 = s_1, H_1) \\
&\qquad\quad \times \cdots \times \mathbb{P}(S_t = s_t, R_t = r_t | S_{t-1} = s_{t-1}, A_{t-1} = a_{n-1}, H_{t-1})
\end{aligned}$$

Markov Decision Process and Trajectories



Trajectories

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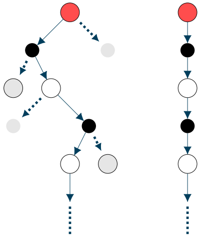
  - an initial distribution $\mathbb{P}_0$ for S_0,
  - a policy $\Pi = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$ specifying
$$\Pi_t(A_t = a) = \pi_t(A_t = a | S_t, H_t)$$
  - a Markovian environment specifying
$$\mathbb{P}(S_{t+1}, R_{t+1} | S_t, A_t)$$

Markov Decision Process and Trajectories

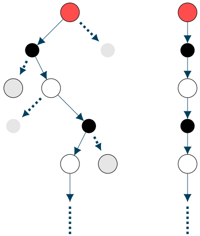


Trajectories

- Induced probability:

$$\begin{aligned}
&\mathbb{P}(S_0 = s_0, A_0 = a_0, R_1 = r_1, S_1 = s_1, A_1 = a_1, \dots S_t = s_t, R_t = r_t) \\
&\quad = \mathbb{P}_0(S_0 = s_0) \\
&\qquad \times \pi_0(A_0 = a_0 | S_0)\, \mathbb{P}(S_1, R_1 | S_0, A_0)\, \pi_1(A_1 = a_1 | S_1 = s_1, H_1) \\
&\qquad\quad \times \cdots \times \mathbb{P}(S_t = s_t, R_t = r_t | S_{t-1} = s_{t-1}, A_{t-1} = a_{t-1})
\end{aligned}$$

Markov Decision Process and Trajectories



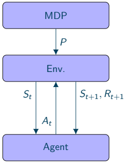
Markovian Trajectories only if the policy is Markovian

- $$\begin{aligned}&\mathbb{P}(R_{t+1}, S_{t+1}, A_{t+1}, R_{t+2}, S_{t+2}, \dots R_{t+k}, S_{t+k} | S_t, A_t, H_t)\\ &= \mathbb{P}(R_{t+1}, S_{t+1}, A_{t+1}, R_{t+2}, S_{t+2}, \dots R_{t+k}, S_{t+k} | S_t, A_t)\\ &= \mathbb{P}(S_{t+1}, R_{t+1} | S_t, A_t)\, \pi_{t+1}(A_{t+1} | S_{t+1})\\ &\quad \times \cdots \times \mathbb{P}(S_{t+k}, R_{t+k} | S_{t+k-1}, A_{t+k-1})\end{aligned}$$

- Stationary if the policy is stationary.

Outline

Rewards and Total Return



Rewards and Total Returns

- MDP: Rewards R_t encode all the feedbacks!
- Quality of a policy Π measured from the remaining total return:

$$G_t = \sum_{t'=t+1}^{\infty} R_{t'}$$

- Expected total return following Π starting from s:

$$\mathbb{E}_{\Pi}[G_t | S_t = s] = \sum_{t'=t+1}^{\infty} \mathbb{E}[R_{t'} | S_t = s]$$

Total Return: Issue and Fixes

Issues

- G_t is a limiting process and thus may not be defined!
- Can diverge to $\pm\infty$ and not converge at all.

Fixes?

- Finite horizon: $G_t^T = \sum_{t'=t+1}^{T} R_{t'}$
- Episodic setting: it exists a random T such that $\forall t' \geq R, R_{t'} = 0$ and $\mathbb{E}[T] < \infty$ so that $G_t = \sum_{t'=t+1}^{\infty} R_{t'}$ is well defined.
- Discounted setting: for $0 < \gamma < 1$, $G_t^\gamma = \sum_{t'=t+1}^{\infty} \gamma^{t'-(t+1)} R_{t'}$
- Average return: $\overline{G}_t = \lim \frac{1}{T} \sum_{t'=t+1}^{t+T} R_{t'}$

Finite Horizon

$$G_t^T = \sum_{t'=t+1}^{T} R_{t'}$$

Finite Horizon Setting

- Always well defined and easy to interpret.
- Loss of Markovianity as we need to know the time step...

- Can be put in a classical Markov framework!
  - Define an absorbing state s_{abs} (a state that cannot be escaped and from which the reward is always 0).
  - Extend the state space $\mathcal{S}$ to $(\mathcal{S} \times \{0, \dots, T\}) \cup \{s_{\text{abs}}\}$.
  - Define an state reward transition probability:

$$p\left(\tilde{s}', r | \tilde{s}, a\right) = \begin{cases} p(s', t | s, a) & \text{if } \tilde{s} = (s, t),\ t < T \text{ and } \tilde{s'} = (s', t+1) \\ 1 & \text{if } \tilde{s} = (s, t),\ t = T,\ \tilde{s'} = s_{\text{abs}} \text{ and } r = 0 \\ 1 & \text{if } \tilde{s} = s_{\text{abs}}\ ,\ \tilde{s'} = s_{\text{abs}} \text{ and } r = 0 \\ 0 & \text{otherwise} \end{cases}$$

Episodic Setting

$$G_t = \sum_{t'=t+1}^{\infty} R_{t'}$$

Episodic Setting

- Assumption: for any policy Π, the average duration before $R_t = 0$ is smaller than a finite horizon H: $$\mathbb{E}_\Pi\left[\min_{t, R_{t'}=0, \forall t' \geq t} t\right] \leq H < +\infty$$
- Strong assumption...
- Easy to interpret.

- Equivalent def.:
  - Replace all the states from which R_t remains equal to 0 whatever the policy by a single absorbing state s_{abs},
  - Assumption: for any policy Π, the average duration to reach this state is smaller than a finite horizon H: $$\mathbb{E}_\Pi\left[\min_{t, S_t = s_{\text{abs}}} t\right] \leq H < +\infty$$

Discounted

$$G_t^\gamma = \sum_{t'=t+1}^{T} \gamma^{t'-(t+1)} R_{t'}$$

Discounted

- Always defined but not that easy to interpret.
- Easiest theoretical setting!

- Equivalent to an episodic setting if one adds an absorbing state s_{abs} and changes all state-reward transition probabilities to:

$$p(s', r|s, a) = \begin{cases} \gamma p(s', r|s, a) & \text{if } s' \neq s_{\text{abs}}, s \neq s_{\text{abs}} \\ (1-\gamma) & \text{if } s' = s_{\text{abs}}, r = 0, s \neq s_{\text{abs}} \\ 1 & \text{if } s' = s_{\text{abs}}, r = 0, s = s_{\text{abs}} \\ 0 & \text{otherwise} \end{cases}$$

- Horizon $H = 1/(1-\gamma)$.

Average Return Setting

$$\overline{G}_t = \lim \frac{1}{T} \sum_{t'=t+1}^{t+T} R_{t'}$$

Average Return

- Not always defined. (Cesaro Average)
- Always equal to 0 in the episodic setting!
- Natural definition in a *stationary* setting...
- Complex theoretical analysis!

- Under a strict stationarity assumption ($R_t \sim R_{t'}$), link with discounted setting as

$$\mathbb{E}_\Pi[G_t^\gamma] = \sum_{t=0}^{\infty} \gamma^t \mathbb{E}_\Pi[R_{t+1}] = \frac{1}{1-\gamma} \mathbb{E}_\Pi[R_t] = \frac{1}{1-\gamma} \mathbb{E}_\Pi\left[\overline{G}_t\right]$$

State Value Functions

State Value Functions

- Return expectation for a policy Π starting from s at time t
  - Finite horizon setting:
$$v_{t,\Pi}^T(s) = \mathbb{E}_\Pi\left[G_t^T | S_t = s\right] = \sum_{t'=t+1}^{T} \mathbb{E}_\Pi[R_{t'} | S_t = s]$$
  - Episodic setting:
$$v_{t,\Pi}(s) = \mathbb{E}_\Pi[G_t | S_t = s] = \sum_{t'=t+1}^{\infty} \mathbb{E}_\Pi[R_{t'} | S_t = s]$$
  - Discounted:
$$v_{t,\Pi}^\gamma(s) = \mathbb{E}_\Pi[G_t^\gamma | S_t = s] = \sum_{t'=t+1}^{\infty} \gamma^{t'-(t+1)} \mathbb{E}_\Pi[R_{t'} | S_t = s]$$
  - Average return setting:
$$\overline{v}_{t,\Pi}(s) = \mathbb{E}_\Pi\left[\overline{G}_t | S_t = s\right] = \lim_{T \to \infty} \frac{1}{T} \sum_{t'=t+1}^{t+T} \mathbb{E}_\Pi[R_{t'} | S_t = s]$$

- Depends on t for a history dependent policy!

Markovian Policy and State Value Functions

State Value Functions

- Return expectation for a Markovian policy Π starting from s at time t.
  - Finite horizon setting (with time extended state space):
$$v^T_{t,\Pi}(s) = \mathbb{E}_\Pi\left[G^T_t | S_t = s\right] = \sum_{t'=t+1}^{T} \mathbb{E}_\Pi[R_{t'} | S_t = s]$$
  - Episodic setting:
$$v_{t,\Pi}(s) = \mathbb{E}_\Pi[G_t | S_t = s] = \sum_{t'=t+1}^{\infty} \mathbb{E}_\Pi[R_{t'} | S_t = s]$$
  - Discounted:
$$v^\gamma_{t,\Pi}(s) = \mathbb{E}_\Pi[G^\gamma_t | S_t = s] = \sum_{t'=t+1}^{\infty} \gamma^{t'-(t+1)} \mathbb{E}_\Pi[R_{t'} | S_t = s]$$
  - Average return setting:
$$\overline{v}_{t,\Pi}(s) = \mathbb{E}_\Pi\left[\overline{G}_t | S_t = s\right] = \lim_{T \to \infty} \frac{1}{T} \sum_{t'=t+1}^{t+T} \mathbb{E}_\Pi[R_{t'} | S_t = s]$$

- Becomes independent on t if the policy is stationary and Markovian the generic case (except in the finite horizon setting).

State-Action Value Functions

State Value Functions

- Return expectation for a policy Π starting from s and an action a at time t.
  - Finite horizon setting:
  $$q^T_{t,\Pi}(s,a) = \mathbb{E}_\Pi\left[G^T_t | S_t = s, A_t = a\right] = \sum_{t'=t+1}^{T} \mathbb{E}_\Pi[R_{t'} | S_t = s, A_t = a]$$
  - Episodic setting:
  $$q_{t,\Pi}(s,a) = \mathbb{E}_\Pi[G_t | S_t = s, A_t = a] = \sum_{t'=t+1}^{\infty} \mathbb{E}_\Pi[R_{t'} | S_t = s, A_t = a]$$
  - Discounted:
  $$q^\gamma_{t,\Pi}(s,a) = \mathbb{E}_\Pi[G^\gamma_t | S_t = s, A_t = a] = \sum_{t'=t+1}^{\infty} \gamma^{t'-(t+1)} \mathbb{E}_\Pi[R_{t'} | S_t = s, A_t = a]$$
  - Average return setting:
  $$\overline{q}_{t,\Pi}(s,a) = \mathbb{E}_\Pi\left[\overline{G}_t | S_t = s, A_t = a\right] = \lim_{T\to\infty} \frac{1}{T} \sum_{t'=t+1}^{t+T} \mathbb{E}_\Pi[R_{t'} | S_t = s, A_t = a]$$

- Different strategy for action at time t than after...
- Independent of t for a Markovian policy except for the finite horizon setting!

State Value Function vs State-Action Value Functions

$$v_{t,\Pi}(s) = \mathbb{E}_\Pi[G_t | S_t = s] \quad q_{t,\Pi}(s,a) = \mathbb{E}_\Pi[G_t | S_t = s, A_t = a]$$

State vs State-Action

- Performance measure of a policy Π:
  - starting from a state s for the state value function,
  - starting from a state s and an action a (not necessarility related to Π) for the state-action value function.
- State value function at time t from state-action value function:

$$v_{t,\Pi}(s) = \sum_a \Pi_t(a) q_t(s,a)$$

Do We Really Need The History Dependent Policies?

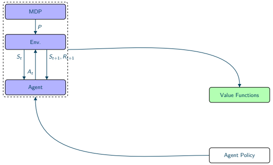
Equivalent Markovian policy in terms of value function

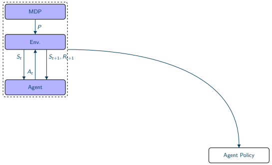
- **Thm:** For any policy Π and any initial distribution $\mathbb{P}_0(S_0)$, it exists a Markovian policy $\widetilde{\Pi}$ such that
$$\forall t, \forall s, v_{t,\Pi}(s) = v_{t,\widetilde{\Pi}}(s).$$

- Relies on the Markovian environment.
- Possible choice:
$$\widetilde{\pi}_t \{A_t = a_t | S_t = s_t\} = \mathbb{E}_{\mathbb{P},\mathbb{P}_0}[\pi_t(A_t = a_t | S_t = s_t, H_t) | S_t = s_t, S_0]$$
- **No need to consider non Markovian policy** if the goal is entirely defined in terms of value functions.

Outline

Goals





Prediction

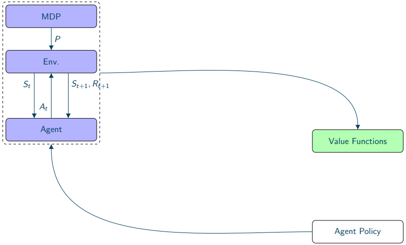
- What is the performance of a given policy?

Planning

- What is the *best* policy?

- Planning is harder than predicting.

Prediction



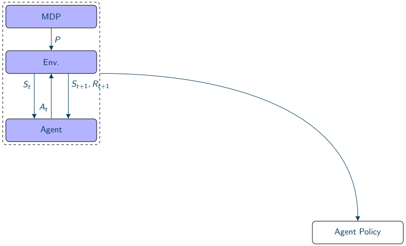
Prediction

- What is the performance of a given policy?
- Compute/Approximate/Estimate

$$v_{t,\Pi}(s) = \mathbb{E}_\Pi[G_t | S_t = s]$$

- Well defined provided the expectation exists.

Planning

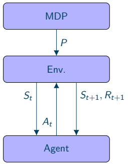


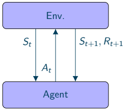
Planning

- What is the *best* policy?
- A possible definition: $\underset{\Pi}{\operatorname{argmax}} \sum_{s,t} \mu(s,t) v_{t,\Pi}(s)$
- Not necessarily well defined...
- Several choices for μ!
- More realistic goal: find a *good* policy...

Outline

What Do We Know?





Model

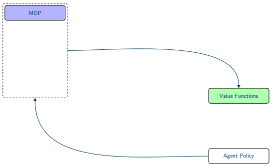
- Able to use the MDP transition probabilities.
- Markov Decision Process / Operations Research.
- Probability world.

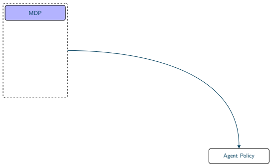
Only Observations

- No access to the MDP transition probabilities.
- Reinforcement Learning.
- Statistic world.

- Reinforcement Learning is harder than Markov Decision Process / Operations Research.

Markov Decision Process / Operations Research

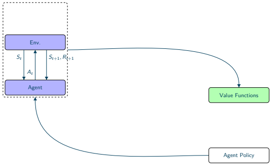


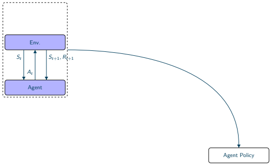


MDP / OR

- Stochastic setting in which the world is known.
- MDP model assumption.
- Probability world / Idealized setting…
- Lots of insight for the RL problem.

Reinforcement Learning





RL

- Stochastic setting in which the world is observed through interactions.
- Still MDP model assumption.
- More realistic setting?
- More difficult theoretical analysis.

Outline

MDP vs Discrete Control

MDP

- State s and action a
- Dynamic model:
$$\mathbb{P}(s'|s,a)$$
- Reward r defined by $\mathbb{P}(r|s',s,a)$.
- Policy Π: $a_t = \pi_t(S_t, H_t)$
- Goal:
$$\max \mathbb{E}_{\Pi}\left[\sum_t R_t\right]$$

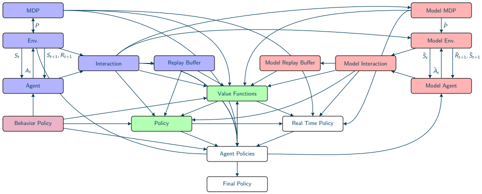
Discrete Control

- State x and control u
- Dynamic model:
$$x' = f(x,u,W)$$
with W a stochastic perturbation.
- Cost: $C(x,u,W)$.
- Control strategy U: $u_t = u(x_t, H_t)$
- Goal:
$$\min_U \mathbb{E}_U\left[\sum_t C(x_t,u_t,W_t)\right]$$

- Almost the same setting but with a different vocabulary!

Outline

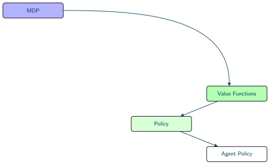
RL: What Are We Going To See?



Outline

- Operations Research and MDP.
- Reinforcement learning and interactions.
- More tabular reinforcement learning.
- Reinforcement and approximation of value functions.
- Actor/Critic: a Policy Point of View

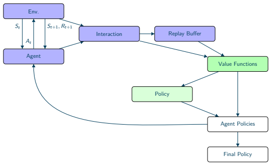
Operations Research and MDP



How to find the best policy knowing the MDP?

- Is there an optimal policy?
- How to estimate it numerically?

- Finite states/actions space assumption (tabular setting).
- Focus on interative methods using value functions (dynamic programming).
- Policy deduced by a statewise optimization of the value function over the actions.
- Focus on the discounted setting.

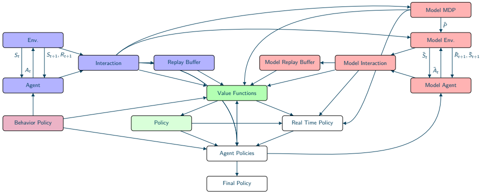


How to find the best policy not knowing the MDP?

- How to interact with the environment to learn a good policy?
- Can we use a Monte Carlo strategy outside the episodic setting?
- How to update value functions after each interaction?

- Focus on stochastic methods using tabular value functions (Q learning, SARSA. . .)
- Policy deduced by a statewise optimization of the value function over the actions.

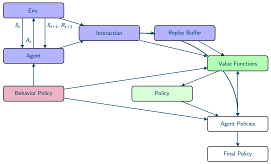
More Tabular Reinforcement Learning



Can We Do Better?

- Is there a gain to wait more than one step before updating?
- Can we interact with a different policy than the one we are estimating?
- Can we use an estimated model to plan?
- Can we plan in real time instead of having to do it beforehand?

- Finite states/actions space setting (tabular setting).

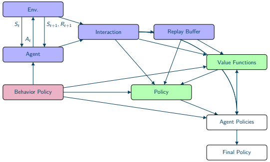


How to Deal with a Large/Infinite states/action space?

- How to approximate value functions?
- How to estimate good approximation of value functions?

- Finite action space setting.
- Stochastic algorithm (Deep Q Learning...).
- Policy deduced by a statewise optimization of the value function over the actions.

Actor/Critic: a Policy Point of View



Could We Directly Parameterized the Policy?

- How to parameterize a policy?
- How to optimize this policy?
- Can we combine parametric policy and approximated value function?

- State Of The Art Algorithms (DPG,PPO, SAC...)

Outline

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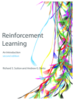

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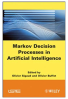

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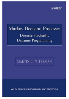

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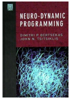

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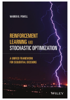

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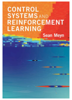

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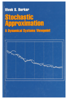

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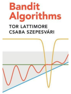

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