Introduction to Reinforcement Learning

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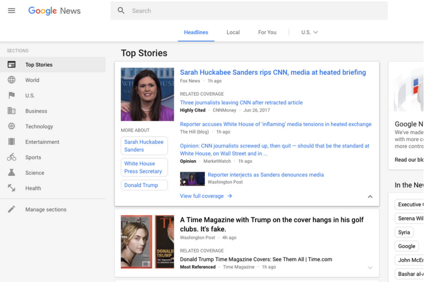
M2 DS - Fall 2021

Outline

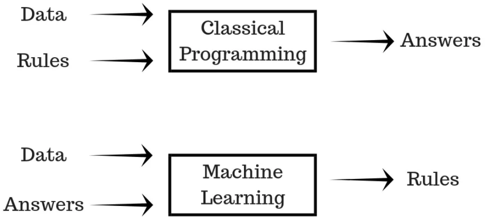
Outline

Machine Learning



Sources: MyCarDoesWhat.org/theverge.com/Rutgers Prep School

Machine Learning



A definition by Tom Mitchell (`http://www.cs.cmu.edu/~tom/`)

A computer program is said to learn from **experience E** with respect to some **class of tasks T** and **performance measure P**, if its performance at tasks in T, as measured by P, improves with experience E.

Source: C. Barabasi

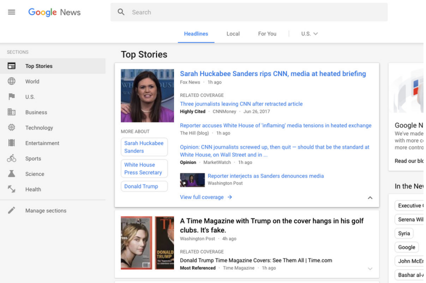
Object Detection

Source: MyCarDoesWhat.org

A detection algorithm:

- **Task**: say if an object is present or not in the image
- **Performance**: number of errors
- **Experience**: set of previously seen labeled images

Article Clustering



An article clustering algorithm:

- **Task**: group articles corresponding to the same news
- **Performance**: quality of the clusters
- **Experience**: set of articles

Source: theverge.com

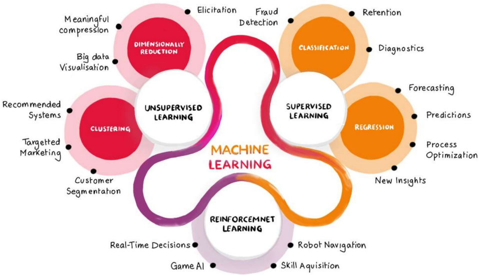
A Robot that Learns

Source: Rutgers Prep School

A robot endowed with a set of sensors playing football:

- **Task**: play football
- **Performance**: score evolution
- **Experience**:
  - past games
  - current environment and action outcome,

Three Kinds of Learning



Unsupervised Learning

- **Task:**
  Clustering/DR
- **Performance:**
  Quality
- **Experience:**
  Raw dataset
  (No Ground Truth)

Supervised Learning

- **Task:**
  Prediction/Classification
- **Performance:**
  Average error
- **Experience:**
  Good Predictions
  (Ground Truth)

Reinforcement Learning

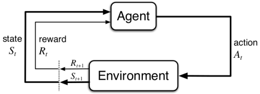
- **Task:**
  Action
- **Performance:**
  Total reward
- **Experience:**
  Reward from env.
  (Interact. with env.)

- **Timing:** Offline/Batch (learning from past data) vs Online (continuous learning)

Source: BCG

Outline

Reinforcement Learning



Reinforcement Learning Setting

- **Env.:** provides a reward and a new state for any action.
- **Agent policy π:** choice of an action A_t from the state S_t.
- **Total reward:** (discounted) sum of the rewards.

Questions

- **Policy evaluation:** how to evaluate the expected reward of a policy knowing the environment?
- **Planning:** how to find the best policy knowing the environment?
- **Reinforcement Learning:** how to find the best policy without knowing the environment?

Outline

The Agent-Environment Interface

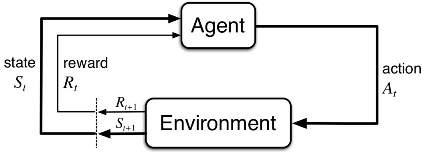


Figure 3.1: The agent–environment interaction in a Markov decision process.

MDP

- At time step $t \in \mathcal{N}$:
  - State $S_t \in \mathcal{S}$: representation of the environment
  - Action $A_t \in \mathcal{A}(S_t)$: action chosen
  - Reward $R_{t+1} \in \mathcal{R}$: instantaneous reward
  - New state S_{t+1}
- Dynamic entirely defined by

$$\mathbb{P}\left(S_{t+1} = s', R_{t+1} = r \middle| S_t = s, A_t = a\right) = p(s', r|s, a)$$

- Finite MDP: $\mathcal{S}$, $\mathcal{A}$ and $\mathcal{R}$ are finite.

Returns and Episodes

Return

- (Discounted) Return:

$$G_t = \sum_{t'=t+1}^{T} \gamma^{t'-(t+1)} R_{t'}$$

- Recursive property

$$G_t = R_{t+1} + \gamma G_{t+1}$$

- Finiteness if $|R| \leq M$

$$|G_t| \leq \begin{cases} (T-(t+1))M & \text{if } T < \infty \\ M\frac{1}{1-\gamma} & \text{otherwise} \end{cases}$$

- Not well defined if $T = \infty$ and $\gamma = 1$.

Policies and Value Functions

Policy and Value Functions

- Policy: $\pi(a|s)$
- Value function:

$$v_\pi(s) = \mathbb{E}_\pi\left[G_t | S_t = s\right] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \middle| S_t = s\right]$$

- Action value function:

$$q_\pi(s,a) = \mathbb{E}_\pi\left[G_t | S_t = s, A_t = a\right]$$

Two natural problems

- Policy evaluation: compute v_π given π.
- Planning: find $\pi^\star$ such that $v_{\pi^\star}(s) \geq v_\pi(s)$ for all s and π.

- Those objects may not exist in general!
- Can be traced back to the 50's!

Outline

Policy Evaluation by Bellman Backup

Fixed Point Property

- Bellman Equation

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a) \left[r + \gamma v_\pi(s')\right] = \mathcal{T}_\pi(v_\pi)(s)$$

- Linear equation that can be solved.

Policy Evaluation by Dynamic Programming

- Fixed point iterative algorithm: $v_{k+1}(s) = \mathcal{T}_\pi(v_k)(s)$

- Converge if $T < \infty$ or $\gamma < 1$.

Planning by Policy Improvement

Policy Improvement Property

- If π' is such that $\forall s, q_\pi(s, \pi'(s)) \geq v_\pi(s)$ then $v_{\pi'} \geq v_\pi$.

- ϵ-greedy improvement among ϵ-policy: classical improvement degraded by picking uniformly the action with probability ϵ

Policy Iteration Algorithm

- Compute v_{π_k}
- Greedy update:

$$\begin{aligned}\pi_{k+1}(s) &= \operatorname*{argmax}_a q_{\pi_k}(s, a) \\ &= \operatorname*{argmax}_a \sum_{s', r} p(s', r | s, a) \left(r + \gamma v_{\pi_k}(s') \right)\end{aligned}$$

- If $\pi' = \pi$ after a greedy update $v_{\pi_{k+1}} = v_{\pi_k} = v_*$.

- Convergence in finite time in the finite setting.

Planning by Bellman Backup

Fixed Point Property

- Bellman Equation

$$v_*(s) = \max_a \sum_{s'} \sum_r p(s', r|s, a) \left[r + \gamma v_*(s') \right] = \mathcal{T}_*(v_*)(s)$$

- Linear programming problem that can be solved.

Policy Evaluation by Dynamic Programming

- Iterative algorithm: $v_{k+1}(s) = \mathcal{T}_*(v_k)(s)$

- Converge if $T < \infty$ or $\gamma < 1$.
- Amount to improve the policy after only one step of policy evaluation.

Planning by Bellman Backup

Q-value and enhancement

- Q-value:

$$q_\pi(s,a) = \sum_{s'} \sum_{r} p(s',r|s,a) \left[r + \gamma \sum_{a'} \pi(a'|s') q_\pi(s',a') \right]$$

- Easy policy enhancement: $\pi'(s) = \operatorname*{argmax}_a q(s,a)$

Fixed Point Property

- Bellman Equation

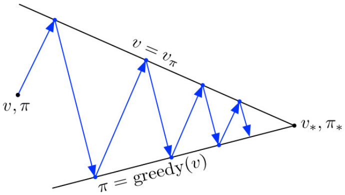
$$q_*(s,a) = \sum_{s'} \sum_{r} p(s',r|s,a) \left[r + \gamma \max_{a'} q_*(s',a') \right] = \mathcal{T}_*(q_*)(s,a)$$

- Linear programming problem that can be solved.

Policy Evaluation by Dynamic Programming

- Iterative algorithm: $q_{k+1}(s,a) = \mathcal{T}_*(q_k)(s,a)$

Generalized Policy Iteration



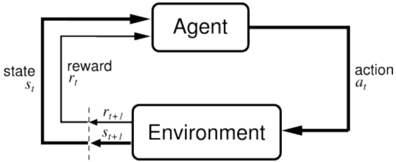
Generalized Policy Iteration

- Consists of two simultaneous interacting processes:
  - one making a value function consistent with the current policy (policy evaluation)
  - one making the policy greedy with respect to the current value function (policy improvement)

- Stabilizes only if one reaches the optimal value/policy pair.
- Asynchronous update are possible provided every state(/action) is visited infinitely often.
- Very efficient but requires the knowledge of the transition probabilities.

Outline

Reinforcement Learning



Reinforcement Learning - Sutton (98)

- An agent takes actions in a sequential way, receives rewards from the environment and tries to maximize his long-term (cumulative) reward.

Reinforcement Learning

- MDP setting with cumulative reward.
- Planning problem.
- Environment known only through interaction, i.e. some sequences $\cdots S_t A_t R_{t+1} S_{t+1} A_{t+1} \cdots$.

Monte Carlo

MC Methods

- Back to $v_\pi(s) = \mathbb{E}_\pi [G_t | S_t = s]$.
- Monte Carlo:
  - Play several episodes using policy π.
  - Average the returns obtained after any state s.

- Good theoretical properties provided every states are visited asymptotically *infinitely often*.

Extensions

- Extension to off-policy setting (behavior policy $b \neq$ target policy π) with importance sampling.
- Extension to planning with policy improvement steps

- No theoretical results for the last case.
- Need to wait until the end of an episode to update anything...

Bootstrap and TD Prediction

Bootstrap and TD

- Rely on
$$\begin{aligned} v_\pi(s) &= \mathcal{T}_\pi v_\pi(s) \\ &= \mathbb{E}\left[R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s\right] \end{aligned}$$
- Temporal Difference: stochastic approximation scheme
$$V(S_t) \leftarrow V(S_t) + \alpha\left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right)$$
- Update occurs at each time step.
- Can be proved to converge (under some assumption on α)!

- Combine the best of Dynamic Programing and MC.
- Can be written in term of Q:
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\left(R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right)$$

SARSA and Q Learning

- How to use this principle to obtain the best policy?

SARSA: Planning by Prediction and Improvement (online)

- Update Q following the current policy π
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left(R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right)$$
- Update π by policy improvement.

- May not converge if one use a greedy policy update

Q Learning: Planning by Bellman Backup (off-line)

- Update Q following the behavior policy b
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left(R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)\right)$$
- No need to use importance sampling correction for depth 1 update.

- Proof of convergence in both cases.

Variations

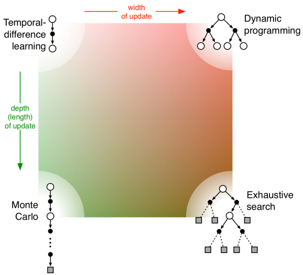


Figure 8.11: A slice through the space of reinforcement learning methods, highlighting the two of the most important dimensions explored in Part I of this book: the depth and width of the updates.

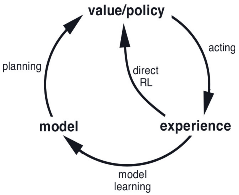
Depth

- Number of steps in the update.

Width

- Number of states/actions considered at each step.

Planning and Learning



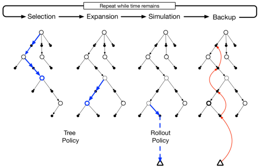


Figure 8.10: Monte Carlo Tree Search. When the environment changes to a new state, MCTS executes as many iterations as possible before an action needs to be selected, incrementally building a tree whose root node represents the current state. Each iteration consists of the four operations **Selection**, **Expansion** (though possibly skipped on some iterations), **Simulation**, and **Backup**, as explained in the text and illustrated by the bold arrows in the trees. Adapted from Chaslot, Bakkes, Szita, and Spronck (2008).

Planning and Models

- Planning can combine a model estimation (DP) and direct learning (RL).

Real Time Planning

- Planning can be made online starting from the current state.

- Curse of dimensionality: methods are hard to use when the cardinality of the states and the actions are large!

Outline

Value Function Approximation

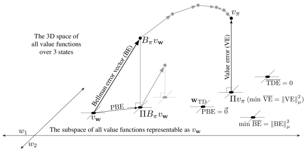
Value Function Approximation

- **Idea:** replace $v(s)$ by a parametric $\hat{v}(s, \boldsymbol{w})$.
- **Issues:**
  - Which approximation functions?
  - How to define the quality of the approximation?
  - How to estimate $\boldsymbol{w}$?

Approximation functions

- Any parametric (or kernel based) approximation could be used.
- Most classical choice:
  - Linear approximation.
  - Deep Neural Nets...

Approximation Quality



- How define when $\hat{v}(\cdot, \boldsymbol{w})$ is close to v_π (or v_*)

Prediction(/Control)

- Prediction objective:

$$\sum_s \mu(s)(v_\pi(s) - \hat{v}(s, \boldsymbol{w}))^2$$

- Bellman Residual:

$$\sum_s \mu(s)(\mathcal{T}_\pi \hat{v}(s, \boldsymbol{w}) - \hat{v}(s, \boldsymbol{w}))^2$$

  or its projection...

- **Issue:** Neither v_π or $\mathcal{T}_\pi$ are known...

Online Gradient and Semi-Gradient

Online Prediction

- SGD algorithm on $\boldsymbol{w}$:
$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \alpha \left(v_\pi(S_t) - \hat{v}(S_t, \boldsymbol{w}) \right) \nabla \hat{v}(S_t, \boldsymbol{w})$$
- MC approximation (still SGD):
$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \alpha \left(G_t - \hat{v}(S_t, \boldsymbol{w}) \right) \nabla \hat{v}(S_t, \boldsymbol{w})$$
- TD approximation (not SGD anymore):
$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \alpha \left(R_{t+1} + \gamma \hat{v}(S_{t+1}, \boldsymbol{w}_t) - \hat{v}(S_t, \boldsymbol{w}) \right) \nabla \hat{v}(S_t, \boldsymbol{w})$$

- Deeper or wider scheme possible.

Online Control

- SARSA-like algorithm:
  - Prediction step as previously with the current policy
  $$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \alpha \left(R_{t+1} + \gamma \hat{q}(S_{t+1}, A_{t+1}, \boldsymbol{w}) - \hat{q}(S_t, A_t, \boldsymbol{w}) \right) \nabla \hat{q}(S_t, A_t, \boldsymbol{w})$$
  - ϵ-greedy update of the current policy

Offline Control with Approximation

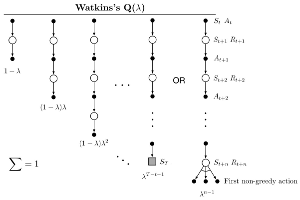


Figure 12.12: The backup diagram for Watkins's Q(λ). The series of component updates ends either with the end of the episode or with the first nongreedy action, whichever comes first.

Offline Control

- Q-Learning like algorithm:

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \alpha \left(R_{t+1} + \gamma \max_a \hat{q}(S_{t+1}, a, \boldsymbol{w}) - \hat{q}(S_t, A_t, \boldsymbol{w}) \right) \times \nabla \hat{q}(S_t, A_t, \boldsymbol{w})$$

  with an arbitrary policy b.
- Deeper formulation using importance sampling possible.

- **Issue:** Hard to make it converge in general!

Sutton-Barto's Deadly Triad

- **Function Approximation**
- **Bootstrapping**
- **Off-policy training**

Stabilization Tricks

- (Back to policy iteration),
- Memory replay: sample from a set of episodes
- Frozen Q: use the previous weights in the max
- Clip/normalize rewards. . .

Actor-Critic

- Other approach with a **parametric policy**.

Actor-Critic

- Simultaneous parameterization of
  - the policy π by $\boldsymbol{\theta}$,
  - the value function s by $\boldsymbol{w}$
- Simultaneous update:

$$\delta_t = R_t + \gamma \hat{v}(S_{t+1}, \boldsymbol{w}) - \hat{v}(S_t, \boldsymbol{w})$$

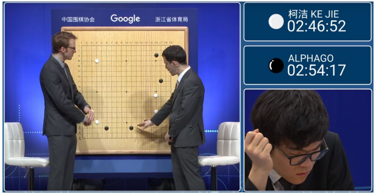
$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_{t+1} + \alpha \delta_t \frac{\nabla \pi(a|S_t, \boldsymbol{\theta})}{\pi(a|S_t, \boldsymbol{\theta})}$$

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_{t+1} + \alpha \delta_t \nabla \hat{v}(S_t, \boldsymbol{w})$$

- Online approach
- Can be adapted to continuous actions.

Outline

AlphaGo



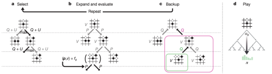
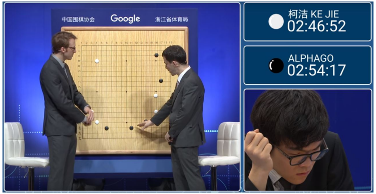


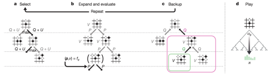
Figure 2 | MCTS in AlphaGo Zero. **a,** Each simulation traverses the tree by selecting the edge with maximum action value Q, plus an upper confidence bound U that depends on a stored prior probability P and visit count N for that edge (which is incremented once traversed). **b,** The leaf node is expanded and the associated position s is evaluated by the neural network $(P(s,\cdot),V(s)) = f_\theta(s)$; the vector of P values are stored in the outgoing edges from s. **c,** Action value Q is updated to track the mean of all evaluations V in the subtree below that action. **d,** Once the search is complete, search probabilities π are returned, proportional to $N^{1/\tau}$, where N is the visit count of each move from the root state and τ is a parameter controlling temperature.

AlphaGo

- Enhanced MCTS technique using a Deep NN for both the value function and the policy.
- Rollout policy and initial value network by supervised learning on a huge database.
- Enhancement of the value network using Actor/Critic RL on self-play.

AlphaGo





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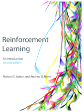
AlphaGo Zero

- No supervised initialization but only self-play.
- Alternate
  - MCTS with a current policy.
  - Gradient descent toward the resulting MCTS policy
- Much shorter training time and better performance!

Outline

References



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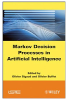

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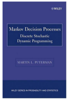

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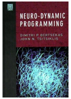

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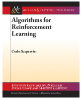

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