QUASILINEAR EQUATIONS, INVERSE PROBLEMS AND THEIR APPLICATIONS

November 30 - December 2

Conference handbook and proceedings

2015

Laboratory Building

Plenar section (30.11-01.12)

Section «Inverse problems and integral geometry»
Chair: M. I. Belishev
Section «Nonlinear PDEs and integrability»
Chair: A. K. Pogrebkov
Section «Numerical methods» (school for young scientists)
Chair: A. A. Shananin
Section «Inverse problems of mathematical physics»
Chair: L. N. Pestov

BioPharm Building

Plenar section (02.12)

Section «Inverse problems of mathematical physics»
Chair: V. A. Sharafutdinov
Section «Nonlinear PDEs and dynamical systems»
Chair: J.-C. Saut
Section «Nonlinear PDEs and dynamical systems»
Chair: S. Kuksin

QUASILINEAR EQUATIONS, INVERSE PROBLEMS AND THEIR APPLICATIONS

Moscow Institute of Physics and Technology, Dolgoprudny
30 Nov. 2015 - 2 Dec. 2015

General information

The conference is devoted to modern mathematical methods of analysis of wave propagation and non-destructive testing.

Related mathematical problems arise within different tomographies (acoustic tomography, tomographies using elementary particles, electro-magnetic tomographies), in analysis of Schumpeter dynamics in economic growth models, in analysis of wave motion, in general, and analysis of shock wave dynamics, in particular.

In the framework of the conference the current state of the theory of quasi-linear equations and inverse problems will be discussed , as well as possible applications in medicine, geophysics, nano-physics, modeling of economic dynamics and traffic flows, and in physics of dispersive mediums.

Organizing committee

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J.-C. Saut		Université Paris-Sud, France
I.A. Taimanov		Institute of Mathematics, Russia

Organazed by

Faculty of Applied Mathematics and Control of MIPT

The Eurasian Association on Inverse Problems (EAIP)

Moscow Institute of Physics and Technology

Russian Foundation for Basic Research (RFBR)

Ecole Polytechnique

Conference program

Monday, November 30th

9:30:	Welcome	
10:00:	Plenar section. Chair: I. A. Taimanov	*(Assembly Hall, Laboratory Building)*
12:20:	Coffee break	*(Lobby of Assembly Hall, Laboratory Building)*
13:20:	Section «Inverse problems and integral geometry»	
	Chair: M. I. Belishev	*(Assembly Hall, Laboratory Building)*
	Section «Nonlinear PDEs and integrability»	
	Chair: A. K. Pogrebkov	*(202, Laboratory Building)*
15:40:	Coffee break	*(Lobby of Assembly Hall, Laboratory Building)*
16:40:	Section «Inverse problems and integral geometry»	
	Chair: M. I. Belishev	*(Assembly Hall, Laboratory Building)*
	Section «Nonlinear PDEs and integrability»	
	Chair: A. K. Pogrebkov	*(202, Laboratory Building)*

Tuesday, December 1st

10:00:	Plenar section. Chair: S. I. Kabanikhin	*(Assembly Hall, Laboratory Building)*
12:20:	Coffee break	*(Lobby of Assembly Hall, Laboratory Building)*
13:20:	Section «Inverse problems of mathematical physics»	
	Chair: V. A. Sharafutdinov	*(BioPharm Building)*
	Section «Nonlinear PDEs and dynamical systems»	
	Chair: J.-C. Saut	*(BioPharm Building)*
	Section «Numerical methods» (school for young scientists)	
	Chair: A. A. Shananin	*(202, Laboratory Building)*
14:30:	Coffee break	*(112, 118, BioPharm Building)*
14:50:	Section «Inverse problems of mathematical physics»	
	Chair: V. A. Sharafutdinov	*(BioPharm Building)*
	Section «Nonlinear PDEs and dynamical systems»	
	Chair: J.-C. Saut	*(BioPharm Building)*
	Section «Numerical methods» (school for young scientists)	
	Chair: A. A. Shananin	*(202, Laboratory Building)*
17:00	Cultural program	

Wednesday, December 2nd

10:00:	Plenar section. Chair: R.Novikov	*(BioPharm Building)*
11:45:	Coffee break	*(112, 118, BioPharm Building)*
12:00:	Introductory lecture for students «Inverse scattering and applications»	
	by R. G. Novikov	*(BioPharm Building)*
14:00:	Coffee break	*(112, 118, BioPharm Building)*
15:00:	Section «Modeling in Continuum Mechanics and Inverse Problems using HPC»	
	Chair: I. B. Petrov	*(910, Applied Mathematics Building)*
	Section «Nonlinear PDEs and dynamical systems»	
	Chair: S. Kuksin	*(BioPharm Building)*
	Section «Inverse problems of mathematical physics»	
	Chair: L. N. Pestov	*(202, Laboratory Building)*
17:40	Coffee break	
18:00	Closing	

Conference program

Monday, November 30th

09:00-12:30 Conference registration, Laboratory building hall

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Plenar section

Chair: I. A. Taimanov

9:30: **Welcome**

10:00: **S. I. Kabanikhin (ICM&MG SB RAS, Russia)**

Multidimentional analogy of Gelfand-Levitan-Krein-Marchenko equations are investigated and applied to acoustic inverse problem (S.I. Kabanikhin, A.D. Satybaev, M.A. Shishlenin, 2004). We discuss several numerical methods for solving multidimentional analogy of GLKM equarions.

10:35: **J. Boman (Stockholm University, Sweden)**

«Tomographic region-of-interest reconstruction from incomplete data»

In applications of Computerized Tomography (CT) it is often of interest to reconstruct a function in a proper subset, the region of interest (RIO), of its support from a proper subset of a full CT-scan. In several recent works it has been shown that stable ROI reconstruction is possible in certain situations. On the other hand, I have shown that stable ROI reconstruction is impossible in certain situations where it has been conjectured to be possible.

11:10: **R. G. Novikov (Ecole Polytechnique, France)**

«Inverse scattering without phase information»

We report on nonuniqueness, uniqueness and reconstruction results in quantum mechanical and acoustic inverse scattering without phase information. We are motivated by recent and very essential progress in this domain.

11:45: **J.-C. Saut (Université Paris-Sud, France)**

«Dispersive perturbations of nonlinear hyperbolic equations»

We will present various results on the life span and the qualitative behavior of solutions to «weak» dispersive perturbations of nonlinear hyperbolic equations. We will focus in particular as a toy model on the so-called fractionary Korteweg-de Vries equation and its variants.

12:20: **Coffee break**

</td><td>Assembly Hall, Laboratory Building, MIPT</td></tr>
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Section «Inverse problems and integral geometry»

Chair: M. I. Belishev

13:20: **L. N. Pestov (Immanuel Kant Baltic Federal University, Russia)**

«Integral geometry problems on convex Riemannian manifolds»

We consider X-ray transform of scalar functions on compact Riemannian manifold (M,g) with boundary. We assume that manifold (M,g)$admits a strictly convex function which is constant at the boundary. Notice, that many geodesics may have conjugate points for such manifolds.
The uniqueness of the inversion of X-ray transform of any smooth function is proved in this talk.

</td><td>Assembly Hall, Laboratory Building, MIPT</td></tr>
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Conference program

Monday, November 30th

Section «Inverse problems and integral geometry»

13:55: **W. Lionheart (University of Manchester, UK)**

«Nonabelian tomography for polarized light and neutrons»

joint work with Naeem Desai and Soren Schmidt

In polarized light tomography for a birefringent material measuremens are made of the change of the polarization of monochromatic light as it passes through the medium. For weakly birefringent media the propagation of light can be modeled by the Rytov-Sharafutdinov law giving rise to a non-abelian ray transform. For sufficiently small deviations from an isotropic background the linearization of this problem is the Truncated Transverse Ray Transform. We review reconstructions algorithms for this linearized problem in special cases, including range conditions. Imaging magnetic fields using polarized Neutrons is a new tomographic modality. It is modelled by a non-abelian plane-by-plane ray transform, for which uniqueness of solution is known from the work of Eskin for sufficiently smooth magnetic fields. For less smooth fields a similar method to that used by Novikov for non-linear polarized light tomography, which uses repeated solution of the linear problem, may be applicable and we present some numerical results.

14:30: **A. Puro (Information Science Institute, Estonia)**

«Optical tensor field tomography of residual stresses in fibers»

Algorithms of tensor field tomography for nondestructive reconstructions of residual stresses in fibers and fibers preforms is presented. The method is based on the measurement of the change in the polarization light passed through a birefringent medium. The transparent specimen is placed in an immersion tank and a beam of polarized light propagate normal to the fiber axis. The reconstruction of stresses by means of tomographic measurements in a system of parallel planes is possible only with the use of additional information. Here, it is assumed that there exists a temperature field, the so-called fictive temperature which causes a stress field, equal stress pattern (thermoelasticity model). The problem of the reconstruction of residual stresses in glass fibers can be divided into two stages. Firstly, to transform ray tensor integrals to scalar one by using equilibrium equations and then to reconstruct stresses by using thermoelasticity equations (the inverse problem of thermoelasticity of optical tomography). We study this inverse problem in the frame of nonhomogeneous elasticity. In the theory of nonhomogeneous elasticity, exact solutions play a fundamental role for several reasons. We use semi-inverse method for determination of these solutions. They allow to investigate in a direct way the physics of various constitutive models (for example, in suggesting specific experimental tests); to understand in depth the qualitative characteristics of the differential equations under investigation (for example, giving explicit appreciation on the well-posedness of these equations); and they provide benchmark solutions of complex problems. Compared with isotropic object, the application of the optical tensor tomography becomes much more difficult in the case of single crystal fibers. The first difficulty associated with natural birefringence. The next difficulty is connected with the solution of the inverse thermoelastic problem. In the case of plane elastic strain, residual stresses in a cubic single crystal can be reconstructed completely. Yttrium aluminum garnet (YAG) single crystal has isotropic mechanical properties. It is used not only as laser rod but also as fiber for higher – power lasers. Method of the reconstruction of
residual stresses in these articles almost the same as in glass fibers.

Assembly Hall, Laboratory Building, MIPT

Conference program

Monday, November 30th

Section «Inverse problems and integral geometry»

15:05: **V. A. Sharafutdinov (Sobolev Institute of Mathematics, Russia)**

«The linearized problem of magneto-photoelasticity»

The equations of magneto-photoelasticity are derived for a nonhomogeneous background isotropic medium and for a variable gyration vector. They coincide with Aben's equations in the case of a homogeneous background medium and of a constant gyration vector. We obtain an explicit linearized formula for the fundamental solution under the assumption that variable coe–cients of equations are su–ciently small. Then we consider the inverse problem of recovering the variable coe–cients from the results of polarization measurements known for several values of the gyration vector. We demonstrate that the data can be easily transformed to a family of Fourier coe–cients of the unknown function if the modulus of the gyration vector is agreed with the ray length.

15:40: **Coffee break**

16:00: **A. S. Demidov (Moscow State University, Russia)**

«Sur le problème inverse magnéto-encéphalographie»

(See page 22)

16:35: **A. Jollivet (Université de Lille 1, France), V. A. Sharafutdinov (Sobolev Institute of Mathematics, Russia)**

«An inequality for the zeta function of a planar domain»

We consider the zeta function ζ_Ω for the Dirichlet-to-Neumann operator of a simply connected planar domain Ω bounded by a smooth closed curve.
We prove non-negativeness and growth properties for
$\zeta_\Omega(s) - 2\left(\frac{L(\partial\Omega)}{2\pi}\right)^s \zeta_R(s) \quad (s \le -1)$, where $L(\partial\Omega)$ is the length of the boundary curve and ζ_R stands for the classical Riemann zeta function.
Two analogs of these results are also provided.

17:15: **N. I. Klemashev (Moscow State University, Russia)**

«The inverse problem in Pareto's theory of consumer demand»

The direct problem in Pareto's demand theory is to find the inverse demand functions of a rational consumer given their utility functions and expenditure levels. The inverse problem is to recover the utility function given the inverse demand functions. The inverse problem does not always has a solution which makes it ill-posed. I consider an approach to regularization of the inverse problem based on the model of temporal dictator with additional requirement of positive-homogeneity of utility function. I show that testing consistency of this model with economic data is an NP-complete problem.

Assembly Hall, Laboratory Building, MIPT

Conference program

Monday, November 30th

Section «Inverse problems and integral geometry» 17:35: **E. G. Molchanov (MIPT, Russia)** **«Finding elasticity of substitution in the recourse distribution problem»** The process of globalization has led to the fact that in Russian economics imported goods began to compete with domestic counterparts. On the other hand, the Leontief input-output model is based on empirical hypotheses that the structure of consumption of final goods and factors of production is permanent. It is one of the root causes of conventional models identification problem in recent years: Interbranch balance statistics has not been published since 2003. For the possible modification of economic models is necessary to determine substitutability of imported and domestic factors of production Determination of the elasticity of substitution in terms of imperfection statistics leads to the inverse problem of resource allocation. Solving the inverse problem of resource allocation results the study the moment problem. The solubility of this moment problem is equivalent to the question of belonging of a vector special type cone – the conical hulls of cutting region's spectral vertices. This report will discuss the Numerical results of substitution elasticity coefficient for some sectors of the Russian economy.	Assembly Hall, Laboratory Building, MIPT
Section «Nonlinear PDEs and integrability» **Chair: A. K. Pogrebkov** 13:20: **A. V. Faminskii (Peoples' Friendship University of Russia, Russia)** **«On global well-posedness results for Zakharov-Kuznetsov equatio»** Zakharov-Kuznetsov equation in two and three spatial dimensions is considered. This equation describes propagation of nonlinear waves in dispersive media moving in one direction with deformations in the transversal directions. The initial value and certain initial-boundary value problems are studied and results on global well-posedness are established. Large-time decay of solutions is also considered. 13:55: **E. Lakshtanov (University of Aveiro, Portugal)** **«On reconstruction of complex-valued once differentiable conductivities»** The classical $\bar{\partial}$-method has been generalized recently by Lakshtanov, Novikov, Vainberg to be used in the presence of exceptional points. We apply this generalization to solve Dirac inverse scattering problem without assumptions on smoothness of potentials. As a consequence, this provides an effective method of reconstruction of complex-valued one time differentiable conductivities in the inverse impedance tomography problem . 14:30: **O. S. Rozanova (Moscow State University, Russia)** **«On a stochastic representation of solutions to systems of quasilinear equations»** Basing on a method of stochastic perturbation of a Langevin system associated with the multidimensional Hopf equation we deduce an explicit asymptotic formula for smooth solution to the Cauchy problem for several important quasilinear system and equations. In particular, this asymptotical representation is possible for any genuinely nonlinear hyperbolic system of equations written in the Riemann invariants and nonlinear wave equation.	202, LaboratoryBuilding, MIPT

Conference program

Monday, November 30th

Section «Nonlinear PDEs and integrability»

15:05: **V. N. Razzhevaikin (Dorodnicyn Computing Centre of RAS, Russia)**
«Asymptotic behavior of solutions of reaction density dependent diffusion equations and its applications»
In the report several results of studying of travelling wave solutions of the reaction - diffusion equation with density dependent diffusion are presented. Among them are:
1)Theorems about phase plane structure of travelling wave solutions.
a)Trigger case. b)Kolmogorov's case. c)Chains of waves.
2)Theorems about phase plane convergence of monotonous solutions to chains of travelling waves.
3)Theorems about stabilization to the dominating homogeneous steady state.
4)Applications to a genetic model. 5)The leader theorem.
15:40: **Coffee break**
16:00: **M. V. Pavlov (Lebedev Physical Institute of RAS, Russia)**
«Integrability of the Benney System»
We consider a link between the Benney system describing propagation of long waves on finite depth fluid and the Vlasov kinetic equation. This link connecting the Benney system and the Vlasov kinetic equation is a singular integral transformation. Thus any given solution of the Benney system can be recomputed into corresponding solution of the Vlasov kinetic equation. An open problem is: if we know solutions of the Vlasov kinetic equation --> how to construct solutions of the Benney system?
16:30: **Shuhua Zhang (Tianjin University of Finance and Economics, China)**
«Modeling and computation of mean field equilibria in producers' game with emission permits trading»
In this talk, we present a mean field game to model the production behaviors of a very large number of producers, whose carbon emissions are regulated by government. Especially, an emission permits trading scheme is considered in our model, in which each enterprise can trade its own permits lexibly. By means of the mean field equilibrium, we obtain a Hamilton-Jacobi-Bellman (HJB) equation coupled with a Kolmogorov equation, which are satisfied by the adjoint state and the density of producers (agents), respectively. Then, we propose a so-called fitted finite volume method to solve the HJB equation and the Kolmogorov equation. The efficiency and the usefulness of this method are illustrated by the numerical experiments. Under different conditions, the equilibrium states as well as the effects of the emission permits price are examined, which demonstrates that the emission permits trading scheme influences the producers' behaviors, that is, more populations would like to choose a lower rather than a higher emission level when the emission permits are expensive.
17:15: **A. P. Antonova (Peoples' Friendship University of Russia, Russia)**
«On regularity properties of solutions to Zakharov-Kuznetsov equation»
The report addresses the question of interior regularity of weak solutions of the Cauchy problem and initial-boundary value problem for Zakharov-Kuznetsov equation. There are established results about the existence of generalized derivatives of these solutions and continuous in the norm of Hölder derivatives. The basis for the study is based on the properties of the fundamental solution of the corresponding linearized equation.
17:35: **M.A. Opritova (Peoples' Friendship University of Russia, Russia)**
«Regularity and large-time decay of solutions to generalized Kawahara equation»
In the present paper, we study Cauchy and Initial boundary value problem for generalized Kawahara equation. In the research we found the solution of the linear problem and the solution's estimate of the fundamental problem, research the question of existence, uniqueness, internal regularity and decrease with increasing time.

202, LaboratoryBuilding, MIPT

Conference program

Tuesday, December 1st

Plenar section

Chair: S. I. Kabanikhin

10:00: **I. A. Taimanov (Sobolev Institute of Mathematics, Russia)**

«On a numerical study of the discrete spectrum of two-dimensional Schroedinger operators with soliton potentials»

The discrete spectra of certain two-dimensional Schrodinger operators are numerically calculated. These operators are obtained via the Moutard transformation and have interesting spectral properties, i.e. their kernels are multi-dimensional and the deformations of potentials via the Novikov-Veselov equation (a two-dimensional generalization of the Korteweg-de Vries equation) lead to blowups. The calculations supply the numerical evidence for certain statements on integrable systems related to the 2D Schrodinger operator. The numerical scheme is applicable to a general 2D Schrodinger operator with fast decaying potential. The talk is based on a join article with A.N. Adilkhanov.

10:35: **M. I. Belishev (PDMI RAS, Russia)**

«Intrinsic geometric world of a symmetric semibounded operator»

We deal with a problem of reconstruction of a Riemannian manifold Ω from its boundary (dynamical and/or spectral) inverse data. It is shown that the problem can be solved by constructing a specific functional model of a symmetric semi-bounded operator L_0, the model being determined by the inverse data. A basic element of the construction is the so-called wave spectrum Ω_{L_0} of L_0, which is introduced via the trajectories of a dynamical system governed by the wave equation $u_{tt} + L_0^{*u} = 0$. The spectrum Ω_{L_0} is endowed with relevant topology and metric, which turn it to a Riemannian manifold $\widetilde{\Omega}$ such that $\widetilde{\Omega} \overset{isom}{=} \Omega$ holds. Thus $\widetilde{\Omega}$ provides the solution to the reconstruction problem.

11:10: **S. Kuksin (Universite Paris-Diderot, France)**

«Averaging for weakly-nonlinear PDE in finite volume»

Consider any linear Hamiltonian PDE in any space dimension, with pure imaginary discrete spectrum, and consider its epsilon-small nonlinear perturbation (Hamiltonian or not). I will present a theory which describes behaviour of solutions for the perturbed equation on time-intervals of order 1/ε. This theory is an analogy for PDE of the Bogolyubov averaging theory (applicable to ordinary differential equations).

11:45: **G. M. Henkin (Université Paris-VI, France), A. A. Shananin (MIPT, Russia)**

«On the Cauchy-Gelfand problem»

We consider question stated by Gelfand about the asymptotic behavior in time of solutions of the Cauchy problem for quasilinear first order equation with the initial conditions of the Riemann type. Based on the method of vanishing viscosity with uniform estimates without a priori assumptions about monotonicity of the initial data we obtain the exact asymptotics in the Cauchy-Gelfand problem and describe the parameters of the initial data, responsible for the localization of the shock waves.

12:20: **Coffee break**

Assembly Hall, Laboratory Building, MIPT

Conference program

Tuesday, December 1st

Section «Inverse problems of mathematical physics»

Chair: V. A. Sharafutdinov

13:20: **V. I. Agoshkov (Institute of Numerical Mathematics of RAS, Russia)**

«Study and numerical solution of inverse problems of the theory of tides»

The methods of description of tide-forming forces are discussed. Based on the variational data assimilation of satellite altimetry data, a solution algorithm for the inverse problem of reconstruction of ‘self-attraction’ forces and a method for approximate solution of this problem are proposed. A mathematical model of the dynamics of oceans and seas is considered subject to tide-forming forces. The numerical study of the effect of tide-forming forces on the Indian Ocean dynamics is performed.

13:55: **A. Jollivet (Université de Lille 1, France)**

«Inverse scattering in classical mechanics»

We consider in R^n, $n\geq 2$, a classical particle moving in a smooth and long range static lectromagnetic field F decomposed as a sum of a known long range tail F^l and an unknown short range part F^s .We define a scattering map and we address the question of reconstructing the force field F from the scattering map. We will review two types of results: First at high energies meaning the reconstruction of the force field from the high energies asymptotics of the scattering map, and then at fixed energy. In this latter case interesting questions remain opens to our knowledge.

14:30: **Coffee break**

14:50: **A. S. Shurup, O. D. Rumyantseva (Moscow State University, Russia)**

«Numerical simulation of the functional approach for recovering vector fields in acoustic tomography»

We consider the numerical implementation and discuss the possibilities of the two-dimensional functional-analytic algorithm for the reconstruction of vector inhomogeneities in the problems of acoustic tomography. As examples of vector inhomogeneities, one can consider the blood flow velocity in medical applications or the speed of currents in oceanographic problems. It is shown that reconstruction of a full vector field consisting of solenoidal and irrotational components is possible by using several (at least two) frequencies. In this case, together with the reconstruction of the solenoidal component, it becomes possible to reconstruct the function of divergence of the full vector field of the current speed and then to reconstruct the irrotational component based on the estimated divergence. As a result, the full vector field is obtained by summation of the solenoidal and irrotational components. Moreover, the discussed algorithm allows the joint reconstruction of different components of the scatterer, i.e. the vector and scalar inhomogeneities; the scalar components are presented by inhomogeneities of the sound speed and absorption coefficient. Meanwhile, the considered scatterer can significantly distort the incident field, being beyond the Born approximation.

BioPharm Building, MIPT

Conference program
Tuesday, December 1st

Section «Inverse problems of mathematical physics»

15:25: **A. V. Gasnikov (MIPT, Russia)**

«Inverse problems in mathematical modeling of traffic flows»

We propose universal primal-dual approach of decription equillibriums in large class of hierarchical congestion population games. In the very core of our approach lies hierarchy of enclosed to each other transport networks. We consider in different time-scales corresponding logit dynamics on this networks. This dynamics reflect restricted rationality of the agents. We reduce searching of equillibrium configuration to the multi-level convex optimization problem (in other words we formulate variational principle for these class of the games/models). Then we formulate dual problem. This problem has natural interpretation in turn. This problem is more computationaly attractive then the primal one. So we propose an effective primal-dual method for this problem. This method is based on the composite fast gradient method. Due to primal-duality we can recover the solution of the primal problem from the dual iterative process. To fullfil one iteration of this method we have to solve rather complex problem of calculation characteristic function on graph. We propose (based on some ideas of Yu. Nesterov) special technique (smooth version of Bellman--Ford approach to the shortest path problem) that allows to do it in a cheap maner. It also should be mentioned that we actively use different tricks of small parametrs methods.

Results will be published in Journal Trudy MIPT. 2015. V. 7. no. 4. (in Russian)

16:00: **A. D. Agaltsov (Ecole Polytechnique, France)**

«A global uniqueness result for acoustic tomography of moving fluid»

We consider a model time-harmonic wave equation of acoustic tomography of moving fluid in an open bounded domain with variable sound speed, fluid velocity, density and absorption coefficient. We give global uniqueness results for the related inverse boundary value problem for the cases of boundary measurements given for one, two and three frequencies. We also give an example of non-uniqueness for the case of boundary measurements given for all frequencies.

16:20: **A. V. Podoroga, I. V. Tikhonov (Moscow State University, Russia)**

«Computer simulation for quasi-linear equations of traffic flow»

A mathematical description of traffic flows is discussed. We present computer program «Cars», which simulates the behavior of big clusters of cars on a single lane road. A connection between microscopic settings of the program and macroscopic parameters of the traffic flow is considered. We compare the calculation results with predictions of the theory of quasi-linear differential equations for some typical cases (Greenberg, Nagel-Schreckenberg). Some special effects connected with kinematic traffic waves are provided.

BioPharm Building, MIPT

Conference program

Tuesday, December 1st

Section «Nonlinear PDEs and dynamical systems»

Chair: J.-C. Saut

13:20: **A. N. Sobolevski (IITP RAS, Russia)**

«Evolution equations in cosmology: hydrodynamic and kinetic formulations»

The present stage of large-scale cosmological evolution is dominated by gravitation and can be described with mathematical models based on general relativity (globally) or even on Newtonian gravity (our local cosmic neighbourhood). Their relative simplicity allows to pose and to approximately resolve the problem of cosmological reconstruction, i.e., of recovering the past dynamical history of some neighbourhood of the Milky Way from information on the present distribution of matter, without having to know the velocities of matter elements.

In this talk we will show how the cosmological reconstruction turns out to be closely connected to optimal transportation theory, review the approach to cosmological reconstruction based on the Zeldovich approximation (the Monge-Ampere-Kantorovich approach) and its possible development based on a full hydrodynamic or kinetic description of the large-scale cosmological evolution.

13:55: **D. V. Tunitsky (Institute of Control Sciences of RAS, Russia)**

«On global solvability of one-dimensional nonlinear wave equations»

The talk is devoted to global solvability of initial value problem for one-dimensional nonlinear wave equation. It is proved that in the class of multi-valued solutions the problem under consideration has the unique maximal solution.

14:30: **Coffee break**

14:50: **A. A. Belolipetskiy (Dorodnicyn Computing Centre of RAS, Russia), A. M. Ter-Krikorov (MIPT, Russia)**

«Modified Kantorovich theorem and asymptotic solution of the singularly perturbed systems of ordinary differential equations»

In Banach space it is considered functional equation $Ax\text{-}f(x,\varepsilon)=0$ with small parameter ε. It is supposed that $f(x,\varepsilon)$ is analytical function in neighborhood of x_0, linear operator $(A\text{-}f_x(x_0,0))^{-1}$ is bounded and exists function $x_n^0(\varepsilon)$ satisfying equation with an accuracy of $O(\varepsilon^{n+1})$. The modified Newtonian sequence with initial element $x_n^0(\varepsilon)$ is constructed. For small values of parameter ε the existence of the limit of thisNewtonian sequence is based on NK- theorem which is proofed in this work. NK-theorem is a new version of L.V. Kantorovich's theorem , supporting the convergence of iterative sequence of Newton. The limit of this sequence is the solution of the equation $Ax\text{-}f(x,\varepsilon)=0$. The deviation of limit of the Newtonian sequence from the initial state $x_n^0(\varepsilon)$ is $O(\varepsilon^{n+1})$ that proves the asymptotical behavior of initial approximation $x_n^0(\varepsilon)$. The proposed technique is applied tosingular perturbed systems of ordinary differential equations (ODE) for construction asymptotic approximations for a solution on the target or infinite time interval. But that technique may be applied to wider classes of functional equations with small parameter.

BioPharm Building, MIPT

Conference program

Tuesday, December 1st

Section «Nonlinear PDEs and dynamical systems» 15:25: **L. A. Beklaryan, A. L. Beklaryan (Central Economics and Mathematics Institute of RAS, Russia)** **«On traveling wave type solutions in infinite dimensional dynamical systems»** (See page 23) 16:00: **A. S. Bratus (Moscow State University of Railway Engineering, Russia)** **«Distributed replicator system and its application to biology»** We consider the system of nonlinear partial equation that described the behavior of interaction and growth of macromolecules. Formally the system has form of reaction-diffusion system nevertheless from the mathematical point of view the obtained system is not the classical system of partial differential equation since the mean fitness is the functional on the set of solution of the system. From the point of view of biology introducing the space should provide new possibilities for successful coexistence and surviving of the macromolecules with compere to the case of ordinary differential equation. In the case of small diffusion coefficients spatially nonhomogeneous local solutions appear. The examples of different replicator system from biology are considered.	BioPharm Building, MIPT
Section «Numerical methods» (school for young scientists) **Chair: A. A. Shananin** 13:20: **A. Lagunovskaya (Keldysh Institute of Applied Mathematics of RAS, Russia)** **«On the relationship between simulation logit dynamics in the population game theory and mirror descent method in the online optimization using the example of the shortest path problem»** This presentation describes a mirror descent method for the stochastic online optimization problems on the simplex. Based on this method the optimal strategies of the users of the transport network choosing a path are constructed. The behavior of all users following such strategies generates simulation logit dynamics in the population game corresponding to the Beckman model of the the equilibrium flow distribution over paths. Thus, a specific example (The Shortest Path Problem) is used to demonstrate a link between online optimization and population game theory. 13:40: **D. Kamzolov, A. Suvorikova (MIPT, Russia)** **«Fast computation Wasserstein Barycenters»** With the development of technologies in the field of computer vision and image processing, Wasserstein barycenter has become widely used. This work proposes a new method of searching Wasserstein barycenter, based on an universal gradien method with inexacte oracle. 14:00: **D. Kamzolov (MIPT, Russia), A. Anikin (Institute for System Dynamics and Control Theory SB RAS, Irkutsk, Russia)** **«Effective numerical methods for huge-scale linear systems with double-sparsity and applications to pagerank»** In this work we propose three methods for solving the PageRank problem for the matrices with both row and column sparsity. All the methods leads to the convex optimization problem over the simplex. The first is based on the gradient descent in L1 norm instead of Euclidean one. The idea behind the second method is Frank–Wolfe conditional gradient algorithm. The last one is randomized version of mirror descent algorithm. We proof converges rates for these methods for sparse problems as well as numerical experiments support their effectiveness.	202, LaboratoryBuilding, MIPT

Conference program

Tuesday, December 1st

Section «Numerical methods» (school for young scientists)

14:20: **D. Merkulov (MIPT, Russia)**

«Convex online optimization with noisy first order oracle»

We consider the stochastic online optimization problem. We assume that only noise stochastic gradients can be obtained from the oracle. We use two assumptions about noise, which goes back to the Juditski-Nemirovski'11 and to the Devolder-Gellineur-Nesterov'11. We demonstrate how to generalize this concept in online context. Additionally, we investigate corresponding theorem about mirror descent convergence in stochastic online context with inexact oracle. All estimations are unimprovable up to a logarithmic factor. We also consider applications to the contextual bandits and SLT.

14:40: **V. Dryuma (Institute of Mathematics and Informatics AS, Moldova)**

«New solutions of the Euler system of equations for incompressible fluid»

(See page 24)

15:00: **A. Chernov (MIPT, Russia)**

«Primal-dual method for entropy-linear programming»

Proposed an efficient dual algorithm for ELP based on Fast Gradient Method (FGM). The basic idea - to solve properly regularized dual problem. The other approach based on the primal-dual properties of FGM.

15:20: **S. Omelchenko (MIPT, Russia)**

«The evolutionary inference of the simplest model of splitting traffic flow on public and private ones»

In the talk we desribe the problem of splitting the traffic flow to public vs private. Based on the assumptions of the price-evaluation of personal time of each person and of the functions of the cost of private and public transport, it was proposed an iterative pocedure that defines the percentage of people who choose private trasport daily. Besides it was formulated and proved the theorem, which says that this iterative process converges to a unique fixed point. The results were tested with the numerical experiment.

15:40: **M. Mendel (MIPT, Russia)**

«Inverse problem of demand IP-matrix calculation from link loads»

The problem of traffic matrix estimation from link loads for IP networks will be stated. Mathematical models used for solving this problem will be formulated. The results of numerical experiments obtained by fast gradient method for dual problem will be presented.

16:00: **D. Grigorievykh (MIPT, Russia)**

«Mathematical modeling of cracks in the solid deformable bodies using hexahedral mesh»

The article discusses the possibility of modeling cracks and irregularities in solid deformable bodies using grid-characteristic method on one Lagrangian hexahedral grid. The problems of elastic waves pass through the fluid- or gas-saturated cracks and irregularities were discovered as well as the processes of formation of new cracks. The basis of the model of cracking taken from a well-known model Mainchen-Sak.

16:20: **A. Aleksandrova (Peoples' Friendship University of Russia, Russia)**

«Running wave speed analysis in one Polterovich-Henkin model modification»

Modification of the Polterovich-Henkin Model of Expanding New Technologies is considered tacking into account elimination of productive capacity. The equation is simplified to quasi-linear equation with the help of automodel reduction. In this paper convergence of the solution to the traveling wave will be proved and the wave propagation speed will be found.

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Conference program

Wednesday, December 2nd

Plenar section

Chair: R.Novikov

10:00: **K. Khanin (University of Toronto, Canada)**

«On global solutions to the random Hamilton-Jacobi equation»

We shall discuss a problem of existence and uniqueness of global solutions to the random Hamilton-Jacobi equation. While situation in the spatially periodic case is well understood by now the most interesting non-compact translation invariant case is largely open. In this talk we shall present partial results and conjectures in the non-compact setting. A connection with the problem of KPZ universality will also be discussed.

10:35: **I. B. Petrov (MIPT, Russia)**

«Numerical investigation in solid mechanics of direct and inverse problems»

A lot of different physical phenomena can be described in the frame of solid mechanics with igh precision. Unfortunately, in most practical cases solution can't be obtained analytically. Due to the rapid growth of computer systems performance last years the problem of development of new precise numerical methods is very important. In this report the latest results of numerical simulation of geological, geophysical and some others problems with grid-characteristic and Galerkin's numerical methods are presented.

11:10: **A. K. Pogrebkov (Steklov Mathematical Institute of RAS, Russia)**

«Integrable discretizations of integrable PDE's»

We present a method of derivation of the linear differentiable equations that admit "nonlianerization" to integrable (in fact, quasilinear) differential equations. Developing approach based on the commutator identities on associative algebras, we suggest procedure of discretization of such integrable equations that preserves property of integrability.

11:45: **Coffee break**

12:00: **R. G. Novikov (Ecole Polytechnique, France)**

«Introductory lecture for students «Inverse scattering and applications»

We give an introduction into the domain of inverse scattering problems by considering such problems for the three-dimensional Schrodinger equation. This lecture is based on classical results going back to M. Born (1926), B. Lippmann, J. Schwinger (1950), L. Faddeev (1956), S. Agmon (1975) and on very recent results of R. Novikov (2015).

14:00: **Coffee break**

BioPharm Building, MIPT

Section «Modeling in Continuum Mechanics and Inverse Problems using HPC»

Chair: I. B. Petrov

15:00: **O. Ya. Voinov (MIPT), V. I. Golubev (MIPT), M. S. Zhdanov (Univ. of Utah)**

«Migration imaging in acoustic media using Kirchhoff and Born methods»

In this research work Kirchhoff and Born methods for migration of acoustic data were applied to layered media with curvilinear boundaries. Effective algorithms for creation of seismic images were developed and compared on synthetic data. Main drawbacks of algorithms were analyzed. The preliminary results of generalization on 3D elastic case were discussed.

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Section «Modeling in Continuum Mechanics and Inverse Problems using HPC»

15:20: **A.V. Favorskaya, D. I Petrov, N. I. Khokhlov, I. B. Petrov (MIPT)**
«Numerical modeling of seismic prospecting in the Arctic by grid-characteristic method»
Oil exploration in the Arctic has its own features. For example, the signals from the source propagate through the sea and the ice and they influence to the measured or calculated seismic response. We simulate wave propagation in media with linear-elastic and acoustic layers. The complete system of equations describing the state of a linearly elastic body and a system of equations describing the acoustic field are solving. The use of the grid-characteristic method provides correctly describing of wave processes and to obtain all types of seismic waves. Correct the contact and boundary conditions, including the contact condition of between acoustic and linear-elastic layers are used.

15:40: **A. V. Favorskaya (MIPT), M. S. Zhdanov (University of Utah)**
«Elastic Kirchhoff Integrals and Migration»
The classical Kirchhoff integrals are widely used for solving boundary value problems for the wave equation. There are also generalized Kirchhoff integrals, describing the solution of the boundary problem of the theory of elasticity in a closed form. In this work we obtain the expressions for the integrals for solving the boundary value problem of elasticity theory on the basis of the relevant Kirchhoff integrals. These Rayleigh integrals allow to generalize the classical methods developed for the solution of the scalar wave equation for the case of waves in an elastic medium. We made a comparison of solutions obtained with the help of Rayleigh integrals and using grid-characteristic method.

16:00: **K. Beklemysheva, A. Ermakov, A. Vasyukov (MIPT)**
«Modeling non-destructive testing of composite materials for delamination identification»
Non-destructive testing (NDT) is an important method of ensuring integrity of engineering structures. NDT techniques based on ultrasonic measurements are among the most commonly used ones. The quality of NDT ultrasonic analysis depends on correct interpretation of elastic waves in the specimen in question. Composite materials have complex internal structure and rheological properties, so understanding the nature of reflected waves on receiver requires supporting numerical modeling tools. This work describes a numerical method of modeling dynamic wave processes in heterogeneous anisotropic media using explicit boundary conditions between different structural parts of the specimen. The method is applied to the problem of modeling ultrasonic NDT procedure for composite material with delamination.

16:20: **Biryukov V. (MIPT), Miryaha V. (MIPT), Sannikov A. (IHEP)**
«Numerical simulation of impact of ice formation on Arctic offshore objects.»
We use elastoplastic rheology and different models of destruction to describe behavior of an ice formation under collision with offshore object.
To simulate deformation processes we apply discontinuous Galerkin method on moving Lagrangian meshes.
Using numerical simulation global load from ice to column of offshore object was obtained.

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Conference program

Wednesday, December 2nd

Section «Modeling in Continuum Mechanics and Inverse Problems using HPC»

910, Applied Mathematics Building, MIPT

16:40: **N. B. Yavich (MIPT), M. S. Malovichko (SPbSU), M. S. Zhdanov (Utah)**
«Efficient algorithm for modeling low-frequency electromagnetic field and its applicability to inversion of geophysical data»
One of the most challenging steps in low-frequency electromagnetic (EM) modeling is design of an efficient solver for the resulting discrete system of algebraic equations. In this presentation, we consider the finite-difference discretization and discuss a novel solver based on a spectrally-equivalent preconditioner. We present practical examples of 3D magnetotelluric modeling in geothermal exploration and illustrate performance of the designed solver.

17:00: **M. S. Malovichko (MIPT), N. I. Khokhlov (MIPT), M. S. Zhdanov (Univ. of Utah), I. B. Petrov (MIPT)**
«Integral approximations in seismic modeling»
The Born approximation is known to be inaccurate even for moderate frequencies if the seismic model contrast is high. However, it is still widely uved in the framework of full-waveform seismic inversion. A number of fast and far more accurate approximations were proposed in the field of electromagnetic scattering over the last decades. We apply several solutions of this kind to acoustic modeling for both lossy and lossless media and evaluate their numerical merits. We estimate applicability of these approximate solutions in the context of full-waveform inversion.

17:40: **Coffee break**

Section «Nonlinear PDEs and dynamical systems»

Chair: S. Kuksin

BioPharm Building, MIPT

15:00: **N. S. Petrosyan (MSTU "STANKIN")**
«Asymptotic time-behavior of the solution of the mixed problem for quasilinear scalar conservation law»
We study the behavior as time increases indefinitely of generalized entropy solution $u(t,x)$ of the mixed problem in the domain $t>0, x>0$ for the equation $u_t+(f(u))_x=0$. We suppose that the initial function $u_0(x)$, $x>0$, is measurable, bounded and has limiting mean value u^+ that is uniform with respect to translations. We impose a zero boundary condition $u(t,0)\equiv 0$, $t>0$. The flux function $f(u)$ is assumed to be strictly convex $f''(u)>0$ and $f(0)=f'(0)=0$. Using the explicit formula of Ph.LeFloch, which generalizes a representation of Lax and Oleinik, we prove that either the solution $u(t,x)$ converge to u^+, if $u^+\le 0$, or the solution $u(t,x)$ exits to the rarefaction wave, if $u^+>0$, as time t tends to infinity.

Section «Nonlinear PDEs and dynamical systems»

15:35: **A. Kazeykina (Université Paris-Sud, France)**

«On the well-posedness of the Cauchy problem for the Novikov-Veselov equation»

Novikov-Veselov equation (NV) is a two-dimensional generalisation of the Korteweg de Vries equation integrable via the inverse scattering transform for the two-dimensional stationary Schrodinger equation at fixed energy E. As energy E tends to infinity, the NV equation, after an appropriate rescaling, reduces to the Kadomtsev-Petviashvili equation (KPI and KPII respectively). In this talk we review recent advances on the well-posedness of the Cauchy problem for the NV equation. We show, in particular, that unlike the closely related KP equation, the NV equation does not change its character from semilinear to quasilinear as the energy parameter E changes sign.

16:10: **V. E. Karpov, A. I. Lobanov (MIPT, Russia)**

«Mathematical model of the fibrin polymerization and phase change in blood»

Blood coagulation system (BCS) modeling is an important issue with a plenty of applications in medicine and biophysics. The BCS main function is to form a localized clot at the site of injury preventing blood loss. Mutual influence of fibrin clot consisting mainly of fibrin polymer gel and blood flow is an important factor for BCS to function properly. The process of fibrin polymer mesh formation has not adequately been described by current mathematical models. That is why it is not possible to define the borders of growing clot and model its interaction with a blood flow. The main goal of this work is to propose physically well-founded mathematical model of fibrin polymerization and gelation. The proposed model defines the total length of fibrin polymer fibers in the unit volume, determines a position of the border between gel and liquid and allows evaluating the permeability of growing gel. Without significant structural changes the proposed model could be modified to include the blood shear rate influence on the fibrin polymerization and gelation. The current work also presents an approach to the sol-gel transition description while fibrin polymerization is similar to the generalized Stefan problem on the phase transition border motion. The main scheme of chemical reactions of polymerization processes was described in [1]. We took into account phase change in moving ideal incompressible fluid. The motion of the fluid is describing by Navier–Stokes equations. We used «vorticity» – «stream function» variables to simplify the equations. The main difficulties in the above approach are the correct conditions on sol-gel phase bound. In present work the boundary conditions for 2D problem are describing. The work was supported by RFBR, grant No 15-51-45109.

16:45: **N. K. Volosova, S. P. Vakulenko, K. A. Volosov (Moscow State University of Railway Engineering, Russia)**

«On the theory of method of quality control of the railroad track from a moving train»

In this article a wide class of exact solutions is built for system of PDE model Timoshenko's beam theory with boundary and initial conditions. The right part of PDE includes Dirac delta function. The method of the factorization was used. We first establish that the problem can be reduced to solution sequence of two mixed problems for inhomogeneous: mixed problem for Klein- Gordon - Fock PDE. It was demonstrated that the problem has two gauges (two basic speed, two basic frequencies). Semianalytic numerical method was proposed for solving of these problems. In the case of the model of rolling of the wheelset over the rails with - like wearing - down (influence of the wheel radius) we receive the standard mixed problem for unsteady - state equation of fourth order. The construction of apparatus for condition monitoring and diagnostics of 120 thousands of kilometers of railway based on the analysis of vibration spectrum of dynamic processes during rolling stock movement doesn't have any alternative methods in Russian Federation. The similar methods are used for a long time in aircrafts operation, other technical fields and biomedical science. This method can compliment the known local methods used for Russian Railways.

17:40: **Coffee break**

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Conference program
Wednesday, December 2nd

Section «Inverse problems of mathematical physics»
Chair: L. N. Pestov

15:00: **A. A. Gavrikov, D. U. Knyazkov, A. V. Romanova, V. V. Chernik, A. S. Shamaev**
«Direct and inverse problems of sea surface electromagnetic tomography»
It is considered a problem of reconstructing a sea surface roughness spectrum from an electromagnetic inspection data. An identification of sea surface form is of great importance for determination of near-surface wind dynamic characteristics, weather prediction, identification of artificial and natural deep-water processes. The inspection is most often performed using an airplane or an artificial Earth satellite [1]. What is significant here is the fact that inspection wave wave-length and surface roughness scale are of the same order.
A method of parallel sections [2] was used for sea surface radiation simulation. This method was implemented as a computer program. A sensitivity of self-radiation parameters to surface harmonics amplitude variations was examined with help of this program. These harmonics was resonant regarding radiation wave-length as well as non-resonant. Wave-lengths of the latter could be substantially bigger or substantially smaller then the radiation wavelength. A computing cluster HybriLIT located in Laboratory of information technologies JINR, Dubna, Russia was used for the calculations.
It was considered an inverse problem of reconstruction of radiating surface spectral properties from a set of energy defect values acquired at different angles and measurement frequencies. A formula of dependence of an energy defect values from a surface form was written with help of local perturbations method. Then a gradient from the energy defect by radiating surface Fourier decomposition harmonic amplitudes was constructed in an explicit analytical form. These expressions were used for solving the inverse problem with the use of gradient optimization of RMS residual between experimental and calculated data. This approach was implemented in a computer program. A number of calculations was performed using this program, that demonstrated a convergence of the surface spectrum search algorithm to a set of coefficients specified as an experimental one.
1. Nesterov S.V., Shamaev A.S., Shamaev S.I. Methods, algorithms and tools of aerospace computer tomography of the near-surface Earth layer. Moscow: Naychnij mir. 1996. 272 p. (in Russian)
2. Il'inski A.S. A method of investigating wave diffraction problems on a periodic structure // USSR

15:35: **A. G. Yagola (Moscow State University, Russia)**
«Error estimation for ill-posed problems»
It is very well known that in a general case it is impossible to provide error estimation for ill-posed problems without an additional a priori information about the unknown solution.
In this presentation two types of a priori information are considered:
1) the solution is sourcewise represented;
2) the solution is an element of a given compact set.
I thank the RFBR for a financial support (grants 14-01-00182, 14-01-91151NSFC).

16:10: **V. A. Sedaykina (Immanuel Kant Baltic Federal University, Russia)**
«Simulation of acoustical imaging in semigeodesic coordinates by BC-method»
We consider a wave equation in two-dimensional Euclidian space with unknown velocity and a set of point sources located at a part of the line x_2=0. The numerical problem of migration imaging is considered. The migration images are calculated and presented in semi-geodesic coordinates for some velocity models. The imaging is based on the Boundary Control method.

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Conference program

Wednesday, December 2nd

Section «Inverse problems of mathematical physics»	202, LaboratoryBuilding, MIPT
16:30: **P. A. Mikheev (SRISA/NIISI RAS, Russia)** **«The application of gradient method to electromagnetic wavefront optimization»** An ill-posed inverse problem was considered - determination of a function, that would possess the required characteristics after convolution with a given kernel. The application of the gradient method to this problem was described, and an effective algorithm of gradient calculation was developed. The results obtained can be applied in synthesis of electromagnetic waves with desired wavefronts. 17:40: **Coffee break**	

18:00: Closing, BioPharm Building, MIPT

On inverse problem magnetoencephalography

M.A. Galchenkova [1], A.S. Demidov [1,2], A.S. Kochurov [2]

[1] MIPT FNBIK, [2] MSU Faculty of Mechanics and Mathematics

An inverse MEG-problem [1] it is the problem of finding the distribution of pulses $Q = (Q_1, Q_2, Q_3)$ ele field in the region Y, corresponding to the cerebral cortex according to the magnetic field induced by t $B = (B_1, B_2, B_3)$, measured in the field $X \neq Y$, corresponding dressed helmet on his head with sensors.

According to the Biot-Savart law, $B(x) = \int_Y K(x,y)\,Q(y)\,dy$ for $x = (x_1, x_2, x_3) \in X$, where (in SI unit

$$K(x,y) = \frac{\mu}{4\pi}\begin{pmatrix} 0 & K_{12}(x,y) & -K_{31}(x,y) \\ -K_{12}(x,y) & 0 & K_{23}(x,y) \\ K_{31}(x,y) & -K_{23}(x,y) & 0 \end{pmatrix},$$

and

$$K_{12}(x,y) = \frac{x_3 - y_3}{|x-y|^3}, \qquad K_{31}(x,y) = \frac{x_2 - y_2}{|x-y|^3}, \qquad K_{23}(x,y) = \frac{x_1 - y_1}{|x-y|^3}.$$

Of interest are those vector functions $Q : Y \ni y \mapsto Q(y)$ that delivers a local minimum of the functional

$$F(Q) = \int_X \left\| B(x) - \int_Y K(x,y)\,Q(y)\,dy \right\|^2 dx.$$

This demand leads to the following system of integral equations of the 1st kind

$$\int\limits_{z \in Z = Y} \sum_{m=1}^{3} A_{jm}(y,z) Q_m(z)\,dz = f_j(y), \qquad j = 1, 2, 3,$$

where

$$A_{jm}(y,z) = \int\limits_X \sum_{l=1}^{3} K_{jl}(x,y)\,K_{lm}(x,z)\,dx, \qquad \text{and} \qquad f_j(y) = \int\limits_X B_j(x) \sum_{l=1}^{3} K_{jl}(x,y)\,dx.$$

In the simplest one-dimensional case the system (1) takes the following form

$$\int_{-1}^{1} A_j(y,z) Q_j(z)\,dz = f_j(y), \qquad y \in [-1,1], \qquad j = 1,2,3,$$

where

$$A_1(y,z) = -\int_{-N}^{N} \frac{\varepsilon^2}{r(x,y,z)} dx, \quad A_2(y,z) = -\int_{-N}^{N} \frac{\varepsilon^2 + (x-y)(x-z)}{r(x,y,z)} dx, \quad A_3(y,z) = -\int_{-N}^{N} \frac{(x-y)(x-z)}{r(x,y,z)}$$

$x \in [-N, N]$, $r(x,y,z) = \Big((x-y)^2 + \varepsilon^2\Big)^{3/2} \Big((x-z)^2 + \varepsilon^2\Big)^{3/2}$, and ε there is a small positive number.

The problem is reduced to a system of pseudo-differential equations [2].

This work was partially supported by RFBR (grant 13-01-00642).

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[1] M. Hamalainen et al (1993) Magnetoencephalography — theory, instrumentation, and applicatior noninvasive studies of the working human brain, Reviews of Modern Physics, Vol. 65, No 2, 413-497.

[2] M.I. Vishik and G.I. Eskin (1967) Elliptic equations in convolution in a bounded domain and their applica Russian Mathematical Surveys, 1967, 22:3, 13–75.

ABOUT SOLUTIONS OF TRAVELLING WAVE TYPE FOR INFINITE-DIMENSIONAL DYNAMICS SYSTEMS

L.A.Beklaryan, A.L.Beklaryan
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Many applied problems lead to studing solutions of travelling waves type for infinite-dimensional dynamic systems. In the report the approach is presented by which the solutions of travelling waves type are studied for a wide class of such systems. The approach will be shown on an example of finite difference analogue of the wave equation with nonlinear potential

$$m_i \ddot{y}_i = y_{i+1} - 2y_i + y_{i-1} + \phi(y_i), \qquad i \in \mathbb{Z}, \quad t \in \mathbb{R}. \tag{1}$$

In case of a homogeneous environment (i.e. $m_i = m, \quad i \in \mathbb{Z}$) it is possible to describe all space of the solutions of travelling waves type at the general assumptions on potential in the form of Lipshchitz's condition.

The solutions of travelling waves type for system (1) are realised as the solutions of induced one-parametrical family functional-differential equations of pointwise type.

In case of the nonhomogeneous environment for system (1) there are not the solutions of travelling waves type, witch is different from stationary, or rectilinear uniform movements. In this connection, the new class of solutions in the form of quasisolutions of travelling waves type is defined.

Such quasisolution of travelling waves type for system (1) is "correct" expansion of concept of travelling waves type solution and that coincides with it in case of a homogeneous environment. The quasisolutions of travelling waves type for system (1) are realised as the impulse solutions of induced one-parametrical family functional-differential equations of pointwise type.

On solving of $3D$ -Euler system of equations

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Abstract

$3D$- Euler system of equations of incompressible liquid written in the vortex form

$$rot[\vec{H} \times rot\ \vec{H}] = 0,\ div\vec{H} = 0, \tag{1}$$

where $\vec{H}$ and $rot\vec{H}$ are the vectors with the components $\vec{H} = (U, V, W)$ and $rot\vec{H} = (W_y - V_z, U_z - W_x, V_x - U_y)$, $div\vec{H} = U_x + V_y + W_z$, admits the solutions when the vector of velocity $\vec{H}$ satisfies to the conditions [1]

$$rot\vec{H} = \nu\ \vec{H},\ div\vec{H} = 0, \tag{2}$$

where ν is in general case an arbitrary function $\nu = \nu(x, y, z)$. To integrate the system (2) we use the components of velocity [4]

$$U = M_{yz} - N_y - K_z,\ V = -1/2\, M_{xz} - 1/2\, L_z + N_x,\ W = K_x - 1/2\, M_{xy} + 1/2\, L_y \tag{3}$$

which are expressed through the four arbitrary functions K, L, M, N.

By such a way the following results were obtained

1. Generalization of classical equations of the (A, B, C) -flow.

The components of velocity and the pressure are of the form

$$U = 1/2\, C_1\ \sin(y) + 1/2\, E_1\ \cos(y) + 1/2\, F_1\ \cos(z) + 1/2\, H_1\ \sin(z),$$

$$V = 1/2\, F_1\ \sin(z) - 1/2\, H_1\ \cos(z) - A_3\ \sin(x) + B_3\ \cos(x),$$

$$W = A_3\ \cos(x) + B_3\ \sin(x) + 1/2\, C_1\ \cos(y) - 1/2\, E_1\ \sin(y),$$

$$\begin{gathered}
P = (-1/4\, C_1\ A_3 - 1/4\, E_1\ B_3) \cos(x + y) + (1/4\, E_1\ B_3 - 1/4\, C_1\ A_3) \cos(x - y) + \\
+ (-1/8\, E_1\ F_1 + 1/8\, C_1\ H_1) \cos(z + y) + (-1/8\, C_1\ H_1 - 1/8\, E_1\ F_1) \cos(-z + y) + \\
+ (-1/8\, C_1\ F_1 - 1/8\, E_1\ H_1) \sin(z + y) + (-1/4\, A_3\ H_1 + 1/4\, B_3\ F_1) \sin(x - z) + \\
+1/4\, {C_3}^2 + 1/4\, {E_3}^2 - 1/16\, {C_1}^2 - 1/16\, {E_1}^2 + C_1 - 1/16\, {F_1}^2 - 1/16\, {H_1}^2 + \\
+ (-1/4\, A_3\ H_1 - 1/4\, B_3\ F_1) \sin(x + z) - 1/2\, {B_3}^2 + (1/4\, A_3\ F_1 + 1/4\, B_3\ H_1) \cos(x - z) + \\
+ (-1/4\, E_1\ A_3 - 1/4\, C_1\ B_3) \sin(x - y) + (-1/4\, C_1\ B_3 + 1/4\, E_1\ A_3) \sin(x + y) + \\
+ (-1/4\, A_3\ F_1 + 1/4\, B_3\ H_1) \cos(x + z) + (1/8\, E_1\ H_1 - 1/8\, C_1\ F_1) \sin(-z + y).
\end{gathered}$$

*Work supported in part by Grant RFFI, Russia-Moldova

2. Periodic solutions.

The components of velocity are of the form

$$W = K_x - 1/2\sin(m_1 x)m_1 \sin(m_2 y)m_2 \cos(\sqrt{1-{m_1}^2-{m_2}^2}z)-$$
$$-1/2\cos(l_1 x)\sin(l_2 y)l_2 \cos(\sqrt{1-{l_1}^2-{l_2}^2}z),$$
$$V = -1/2\sin(m_1 x)m_1 \cos(m_2 y)\sin(\sqrt{1-{m_1}^2-{m_2}^2}z)\sqrt{1-{m_1}^2-{m_2}^2}+$$
$$+1/2\cos(l_1 x)\cos(l_2 y)\sin(\sqrt{1-{l_1}^2-{l_2}^2}z)\sqrt{1-{l_1}^2-{l_2}^2}-$$
$$-\sin(n_1 x)n_1 \cos(n_2 y)\cos(\sqrt{1-{n_1}^2-{n_2}^2}z),$$
$$U = \cos(m_1 x)\sin(m_2 y)m_2 \sin(\sqrt{1-{m_1}^2-{m_2}^2}z)\sqrt{1-{m_1}^2-{m_2}^2}+$$
$$+\cos(n_1 x)\sin(n_2 y)n_2 \cos(\sqrt{1-{n_1}^2-{n_2}^2}z) - K_z, \tag{4}$$

where (l_i, n_i, m_i)- are the parameters $(l_1^2+l_2^2+l_3^2=1,...)$ and the function $K = K(x,y,z)$ is determined from the linear system of p.d.e..

3. Localized nonsingular solutions.

The components of velocity and the pressure have the form

$$U(x,y,z) = \frac{A\,(xz-ay)}{(a^2+x^2+y^2+z^2)^2},\quad V(x,y,z) = \frac{A\,(yz+ax)}{(a^2+x^2+y^2+z^2)^2},$$
$$W(x,y,z) = \frac{1}{2}\,\frac{A\,(a^2-x^2-y^2+z^2)}{(a^2+x^2+y^2+z^2)^2},\quad \nu(x,y,z) = -4\,\frac{a}{a^2+x^2+y^2+z^2}. \tag{5}$$
$$P(x,y,z) = -\frac{1}{8}\,\frac{A^2}{(a^2+x^2+y^2+z^2)^2} + F_1. \tag{6}$$

Remark 1 *To obtain solutions of the Euler system of equations in the form (5-6) were used the results of the article [3] where mapping of the Hopf $S^3 \to S^2$ is applied for construct of localized topological solitons of the equations of magnetohydrodynamics.*

Remark 2 *The unsteady Euler equations*

$$rot(\vec{H})_t - rot[\vec{H}\times rot\ \vec{H}] = 0$$

in that case if the functions of velocity $\vec{H}(x,y,z,t) = [U,V,W]$ and the pressure P are of the form $U(x,y,z,t) = v + U(x-vt, y-vt, z-vt), V(x,y,z,t) = v + V(x-vt, y-vt, z-vt), W(x,y,z,t) = v + W(x-vt, y-vt, z-vt), ..P(x,y,z,t) = P(x-vt, y-vt, z-vt)$ with the parameter v has form of the system of equations (1). This means that the functions (5-6) after change of the variables can use for construction of localized solutions of the 3D Euler equations of the traveling wave type.

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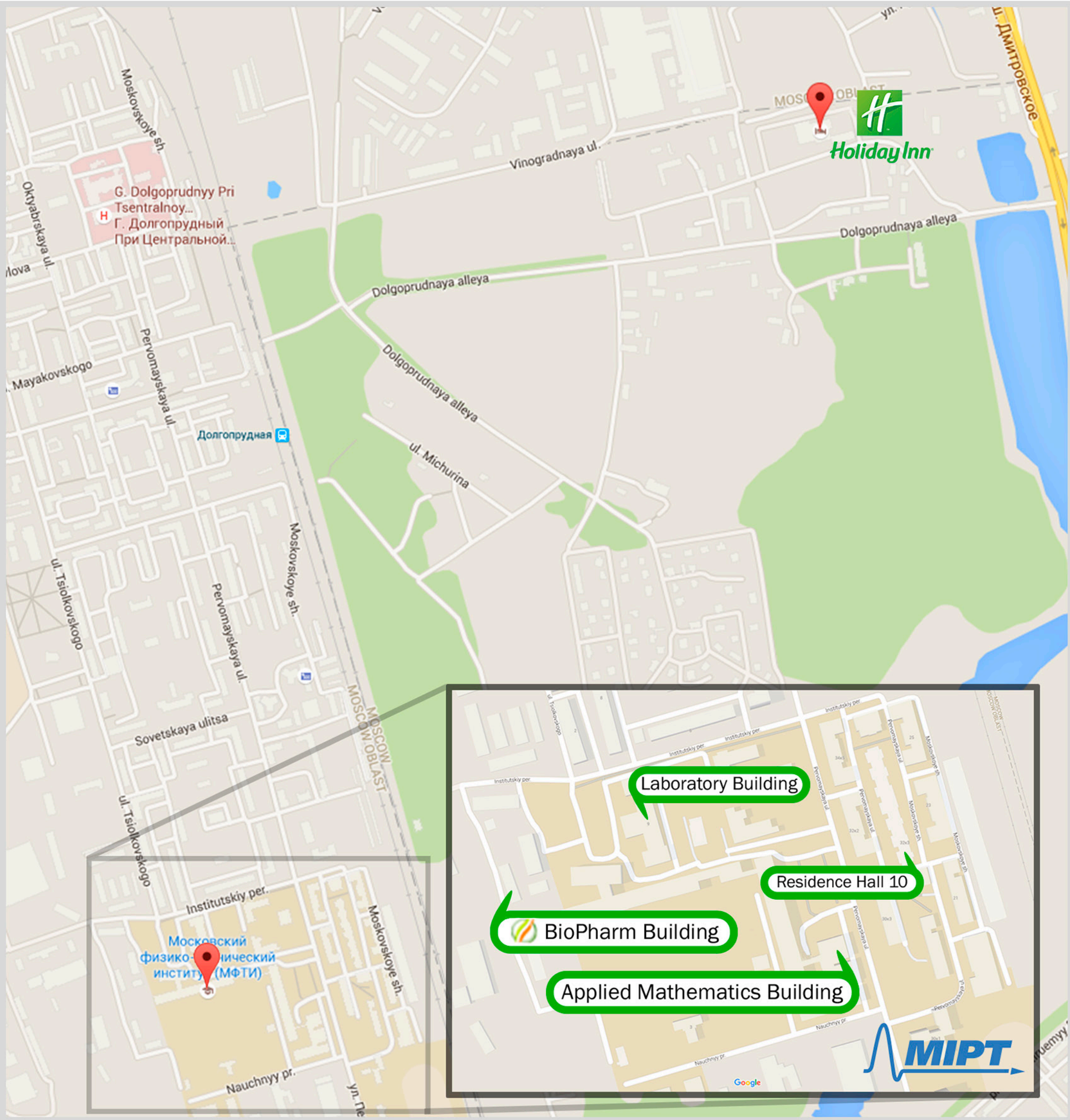

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