1. Introduction

• Sparse coding.
• Structured sparsity (e.g., disjoint groups, trees): increased performance in several applications.
• Our goal: develop a dictionary learning method, which
  – enables general overlapping group structures,
  – is online, fast, memory efficient, adaptive,
  – applies non-convex sparsity inducing regularization:
    – fewer measurements,
    – weaker conditions on the dictionary,
    – robust (w.r.t. noise, compressibility).

2. Problem

Task:
• Group structure inducing on the hidden representation \( x \) through regularization:

\[
\begin{align*}
\ell(x) & = \frac{1}{2}\|Ax - b\|_2^2, \\
\ell(x) & = \frac{1}{2}\|Dx - x^0\|_2^2, \\
\ell(x) & = \frac{1}{2}\|Dx - x^0\|_2^2
\end{align*}
\]

• Loss for a fixed observation:

\[
\ell(x_i, D_{ij}) = \frac{1}{2}\|D_{ij}x - x_i\|_2^2, \quad i < j.
\]

• Goal: minimize the average loss of the dictionary:

\[
\sum_{i,j} \|D_{ij}x - x_i\|_2^2 / \sum_{i,j} |D_{ij}|
\]

• Possible dictionary representation constraints:

\( D = \Omega(x) \), where \( \Omega(x) \) is the set of all possible group structures on \( x \).

3. Special cases

1. \( x = [x_1, \ldots, x_N] \): fully observed OSDL task.
2. \( x = [x_1, \ldots, x_n] \): non-convex sparsity inducing regularization:

3. \( x = [x_1, \ldots, x_N] \): online, fast, memory efficient, adaptive.

4. \( x = [x_1, \ldots, x_N] \): general overlapping group structures.

5. \( x = [x_1, \ldots, x_N] \): non-negative D and \( \alpha \).

6. \( x = [x_1, \ldots, x_N] \): constrained D, non-negative \( \alpha \).

4. Optimization

Online optimization of dictionary \( D \) through alternations:

1. \( (x_i, D_{ij}, D_{ji}) \) \( \rightarrow \alpha \) \( x = \arg\min \ell(x_i, D_{ij}) \).

2. \( (x_i, D_{ij}, D_{ji}) \) \( \rightarrow \alpha \) \( D_{ij} = \arg\min \ell(x_i, D_{ij}) \).

Solution ideas:
• Iterated weighted least squares using the variational property of \( \ell \).
• Block-coordinate descent optimization: update column \( D_{ij} \) while keeping the others fixed.
• Utilization of the cost \( \ell \) can be efficiently updated online (matrix recursions).

5. Numerical experiments

5.1 Inpainting of natural images

We focused on the following questions:
• Structured (towards) vs. unstructured dictionary for inpainting.
• Efficiency in case of missing observations.
• Inpainting of full images using dictionaries learned on partially observed patches.

First experiment (complete observation):
• Increasing neighbor size: \( 1 \times 1 \) \( \rightarrow \) \( 3 \times 3 \).
• MSE grows slowly.
• \( D = \frac{1}{2} \) in Fig. 3.(d)-(f).

Second experiment (neighbor size: \( 3 \times 3 \), missing pixels: \( p_s \leq 0.9 \))
• Up to about \( p_s = 0.8 \), MSE grows slowly.

Third experiment (neighbor size: \( 3 \times 3 \), missing pixels: \( p_s = 0.9 \))
• Task: inpainting of an unseen image.

5.2 Online structured non-negative matrix factorization on faces

• Online, \( D \)-NMF: special case of OSDL.
• Illustration: color FERET large-scale (149 \times 129) facial dataset.
• \( D \) complete, 8-level binary tree (\( K = 2^8 \)).

Result: sliding average, Fig. 4(a), \( F_2 = 0.7 \), PSNR = 29 dB.

Figure 3: Group-structured D-\( A \) (a)-(d): complete, increasing neighbor size. (d)-(f): increasing incompleteness.

Figure 4: (a)-(d): full image inpainting. Illustration: top: observed, bottom: estimated. (b): structured NMF-dictionary, training samples at the upper left corner.

Online Group-Structured Dictionary Learning

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The project is supported by the Department for Energy grant number (KCK-2015-12-1388).

The financial support of the European Union is acknowledged through the European Social Fund. Project Brand: TÁMOP-4.2.2.C-11/1/KONV-2012-0022.